Against the gods – the remarkable story of risk
Peter Bernstein

A survey of mathematical genius through the ages, and how they dealt with risk

This book is itself a work of genius. Only ten pages in, I felt compelled to go to Wikipedia to learn who Peter L Bernstein is. He was born rich, attended the best schools, knew the leading mathematicians, statisticians and risk experts of his age, and contributed significantly to the literature himself. This, his most famous book, appeared in 1996 when he was already 77 years old and had completed what for most men would have accounted for three or four careers. It is his gift to posterity, and what a remarkable gift!

He writes short but amazingly poignant biographies of the men who developed mathematics from its very inception. The Greeks and Romans were no slouches – they were able to determine that the world was round and pull off engineering feats that required a pretty good knowledge of arithmetic. They were handicapped by their numbering systems, all of which resembled Roman numerals. You simply cannot apply an algorithm to find the product of LXXXII x XCVI  the way that you can 82 x 96.

The study of probability and risk has always been a mathematical exercise. The deepest early studies concerned games of chance. What are the chances that a pair of fair dice will come up totaling three? What are the odds that that will happen in two rolls? Working this out takes a fair amount of arithmetic. The Romans had the abacus, and it largely defined the limits of what they could do with numbers. It turns out that in the West, only in the year 1202 did a fellow named Fibonacci write a book on how to do arithmetic using Arabic numerals. It caught on slowly – Bernstein writes that even in 1500 very few people could use them.

Actually, Fibonacci had been preceded by almost 1000 years by a Greek named Diophantus, who not only extolled the virtue of using a real numbering system in place of letters, but had developed the rules of algebra including the use of letters for variables. The work survived – but nobody read it until the 1700s, when they were astounded to find that it embraced concepts such as negative numbers.

The Arab world was at the forefront of civilization at the end of the first millennium. Bernstein writes about one Arab, whose name has been given to "algorithm," and Omar Kayyam as the great mathematicians of that age.

Bernstein's history jumps to the Renaissance, at which time the world took a new view of the kinds of problems that ought even to be addressed, the appropriate objects of intellectual curiosity. A lot of them were mathematical. Bernstein introduces an example that he will follow for several centuries. It is easy to state: A and B are playing a game called balla. They agree to continue until one has one six rounds. The game actually stops when A has won five and B three. How should the stakes be divided? In other words, is the probability that A, who is leading, would actually be the first to win six games? He keeps us in suspense for half of the book with this question. The answer is not difficult from a mathematical respective, but looking for the answer requires a modern frame of mind that did not exist in the early Renaissance.

He follows with short biographies of many colorful characters who happened to be mathematical geniuses.  Cardano was an obsessive gambler who used algebra to figure his odds. He liked to make arcane bets, such as the chances of failing to throw a six in five roles of a fair die, on which he could calculate the chances of winning and his opposite number could not.

Galileo was also a mathematician. He wasn't a gambler, but his patron was, so Galileo did what it took to give the Duke and edge at the gaming tables.

Blaise Pascal was a mathematician, philosopher, and religious nut. Bernstein describes Pascal's development of what has become known as Pascal's triangle, the binomial expansion, and its many applications to probability and risk.  Pascal was in correspondence with Fermat, famous for his last theorem, and the mathematician/gambler Chevalier de Méré.  The three advanced the science of probabilities, defined as the likelihood of an event occuring within a given set of constraints.  They started to recognize the distinct notion of risk, where the constraints are not known.

I used Pascal's triangle last year. The former president of our Rotary club claimed he had paid his dues all along. The books showed that over the past 2 1/2 years there had been 67 dues payments by other club members recorded without any problems, but none by him. Pascal's triangle provided the odds that there had been five errors in 72 payments, all attributable to the same individual: one in 76 million. I didn't believe him. Unfortunately, as is often the case in human affairs and as Bernstein gets into in later chapters, irrationality won the day.  He was readmitted and I quit the club.

At this point Bernstein's history makes a major jump from the theoretical to the actual. An Englishman named John Graunt compiled actuarial tables for the City of London from 1604 to 1664. It was a massive manual undertaking, looking at the church records from every parish in the city. It provided a great many unintuitive observations, among them the fact that deaths by syphilis were probably attributed to something else. There were systematic problems – not everybody belonged to an Anglican parish. Nevertheless, he was able to establish actuarial tables presenting life expectancy at age is from birth to 76. This was of a great deal of interest as the English crown was raising money by selling annuities.

Edmund Halley, the fellow whose name is associated with a comet, saw the problems inherent in doing such a study in London. It was a big city where people came and went. Inspired by Graunt, he made the same kind of study for the smaller, stable, inland city of Breslau (now Wroclaw) that served as Europe's best source of actuarial estimates for many decades to come.

The story then moves to insurance. This was the age of exploration and Dutch and English were sending ships all over the world. It was a highly profitable but risky business. Out of it grew Lloyd's of London, from a group of underwriters gathering at Lloyd's coffee shop by the wharves. Insurance required forecasting not only the risk of loss, but the probable gain from a voyage. Merchants needed to decide whether the cost was worth the risk. At the same time in Italy, the Monte de Paschi bank was ensuring crops. It had the advantage of serving many geographical regions, and was thus able to balance risk as the losses in one would almost invariably be offset by profits in another. Note that this is the same bank, the world's oldest, which is failing as I write this in 2016. The nature of risk is changed.

His focus shifts to the family of Swiss prodigies, the Bernoullis, who introduced the human factor. Daniel Bernoulli observed that a gain of $100 was worth more to a man who had only $100 to start with than to a man who started with a $1,000. Conversely, the loss of $100 would be more painful to the first. He called this concept "utility."

He exemplifies utility in his "Petersburg paradox." Peter tosses a coin and he will continue to toss it until it comes up heads. He will pay Paul one ducat if heads comes up on the first toss, two if it comes up on the second, four on the third, and so on. With each additional throw on which a head does not show, the money he owes Paul doubles. The question: how much would somebody pay Paul for the privilege of taking his place in this game?

This is the paradox. The expected value of the proposition is infinity, but Bernoulli noted that any reasonable man would settle for 20 ducats. It is an issue of utility. Past a certain point, additional money has less and less utility. Bernstein quickly jumps to the stock market. Thirty years ago Microsoft and Oracle were in the position to make seemingly endless amounts of money. Today it is Google, Facebook and Amazon. How much are there stocks worth? It is hard to come up with an objective answer.

Bernoulli added another concept, that of human capital. Even a beggar has a certain capital – the amount that he can bring in out on the street. If he brings in $2,000 a year begging, would it be worthwhile to give up begging for $10,000? This is another hard question to answer. Daniel Bernoulli's addition of the concept of utility has changed our perception of risk and probability, and figures in in some way to all subsequent work.

The perpetual question in statistics is "How sure are you?" In the social sciences statisticians are usually content to consider a statistical observation to be valid if the chances are less than 5% that the observation would occur by random chance. When they get down to 1% they are very happy. A practical question would be, how many tosses of a coin do you have to observe before you decide that it is loaded? If you saw a coin come up heads eight times out of 10, would you want to gamble against the person that owned that coin?

Bernstein's anecdote is that in one of the German air raids on Moscow World War II, a professor statistics surprised his friends when he showed up at the air raid shelter. He had never been there before. He had contended "There are 7 million people in Moscow, why should I expect them to hit me?" His friends asked him why he changed his mind. He explained "There are 7 million people in Moscow and one elephant. Last night they got the elephant."

Here Bernstein introduces Jacob Bernoulli, uncle of Daniel. How do you know? What is moral certainty? Is 5% enough?  When is it not enough? Jacob asked the practical question of how much longer a person of any given age was likely to live. He is credited with developing the law of large numbers. The more observations you have, the more likely you are that the average is closed the true average, and the more representative the distribution.

We returned to the Frenchman de Moivre, who developed the concept of the mean and the standard deviation, though they were not called that at the time. He addressed real-life problems. Suppose a pin factory wants to allow no more than 1/10 of 1% to be defective. How many pins should they sample?

Next in line is Thomas Bayes, the author of Bayes' theorem. The idea is that prior knowledge must be taken into account in a probability. The example I like best does not come from Bernstein. Suppose that 80% of the taxis in the city are blue and 20% are green. There has been an accident. A witness says it was a green taxi. Examining the witness reveals that he is accurate 70% of the time. What are the chances it was a green taxi involved in the accident? Bayes says you have to combine the probabilities. 30% of 80% is greater than 70% of 20%. Chances are it was a blue taxi.

Bernstein next takes a bit of an aside to discuss Karl Gauss. Gauss was such a remarkable mathematician and made such tremendous contributions to calculus and numerous other areas that he cannot be overlooked. His contribution to the study of probability was not that great, even though the bell curve is named for him: the Gaussian distribution.

Bernstein makes a break as he begins chapter 9, stating "up to this point, our story has been pretty much about numbers." This is true – mathematics, and the interesting mathematicians that develop them. The second half of the book concerns the more modern authors, and it goes into statistics and psychology rather than mathematics.

Francis Galton, Darwin's cousin, laid the foundation for the field of statistics. You gather a sample, you measure it, and you draw statistical inferences from your measurements. He measured everything. Interestingly, although his most famous book was entitled "Hereditary Genius," it was some of his colleagues and successors that developed intelligence testing. Galton contributed to a great many areas of science.  He is best remembered as the father of the eugenics movement, which advocated that smart people breed copiously and the not so smart be encouraged not to. In the course of his studies he observed that the sons of geniuses were only rarely themselves geniuses. There were families, such as his own, the Bernoullis and the Rothschilds, in which genius seemed to run. But all of them experienced a "regression to the mean," as parents who were extreme by any measure tended to have children who were less extreme. Galton demonstrated why this must be so. If it were not so, each succeeding generation would have greater extremes than its predecessor.

The Frenchman Quetelet was so enamored of the normal distribution that he went through life looking for applications. The bell curve seem to fit so many things that one observed in life. Galton borrowed extensively from his work in developing his own theories.

Karl Pearson, another imminent mathematician and Galton's biographer, was the first name mentioned when I studied statistics. He developed the idea of statistical correlation and the idea of measuring intelligence. He observed as a teacher in English public (that is, private) schools that talent in English, French, music, mathematics and history all correlated. If a student was good in one, the chances were that he would be good in others as well. The correlations between certain subjects, mathematics and physics for instance, were higher than those between perhaps French and music.

Chapter 10 begins the discussion of financial markets. Obviously, it is of intense interest to determine which way any particular stock or market is headed. A number of observers noted that stock prices take a random walk – there is no predictable pattern to trading activity over the course of a short period of time. They also observed that regression to the mean is a rule. If the stock market goes up, it will go back down. However, they note also that the timing is unpredictable. The stock market went down in 1929, not to emerge for another decade or more. Interest rates were stable for decades, and then soared in this 1970s and 80s. Then they started to decline and have remained at historic lows. Anybody betting on a regression to the mean would have gone broke before making money.

Bernstein returns to the distinction between probability and risk. If you are playing roulette, poker or craps, you can compute the odds before you make your bet. Playing the stock market, buying insurance and similar transactions there are unknown risk factors. Past experience is a guide to future expectations, but many events can disrupt the continuity. Fortunes have been lost on Wall Street by assuming regularities that do not exist. Stocks and bonds go in opposite directions – except when they don't! The risks in foreign markets are not correlated with one another – except when they are! Fortunes are lost, as in Long-Term Capital Management, and gained, as in The Big Short.

There is a section on chaos theory, the randomness of it all. There is a lot written about John Maynard Keynes, a household name, and his contemporary Frank Knight. These men's insight was that markets are not impersonal. One player's actions affect the other players. There are sections on game theory. There is a particularly enlightening piece about Federal Reserve board member Alan Blinder, analyzing the different motivations of the Federal Reserve and the politicians – the Treasury Department and the Congress. The Federal Reserve wants to take credit for stability and low inflation. The Congress wants to take credit for something-for-nothing programs - spending money.

Blinder's analysis demonstrates why these two have been unable to come up with an optimal solution – far from it. As is painfully obvious in 2016, the politicians are unwilling to even come close to balancing the budget, and the Federal Reserve accommodates them by continuing to expand the money supply.

There are chapters on what one would call the problems of agency. People who manage money are judged in the short term, not the long-term. They are judged by the "prudent man rule". A person takes a bigger risk going against the herd than making the same mistake as everybody else. Bernstein explains why this leads to herd behavior on Wall Street, the markets invariably overshooting on both the upside and the downside.

The last great body of work discussed in the book concerns Daniel Kahneman and Amos Tversky, who was still alive at the writing. This was before Kahneman's magnum opus, Thinking Fast and Slow, although Bernstein does a good job of explaining the irrationality of human behavior, even investor behavior. He has a number of examples to demonstrate that the way a question is phrased greatly affects how an investor will react to an identical set of circumstances. It talks about the "endowment effect," the greater reluctance to part with something if you think you own it than if you simply have a chance to acquire it.

That is pretty much where Bernstein leaves the study of irrationality. The single thinker whose name include a successor to these would be Nassim Nicholas Talib, whose best known book is The Black Swan. Taleb goes on at length about how inadequate statistics are as a tool for predicting the markets. New situations come into play that are simply not represented by the historical data being analyzed. I will close this review and saying that as I write this, most markets in the world, and these include the major stock markets, bond markets, precious metals markets, and even oil and ordinary commodities markets have the appearance of being manipulated by government, major banks and central banks. The way this is phrased is that "there is no price discovery anymore." Prices are all being manipulated to serve the political ends of the powerful: the political establishment, the banks, large companies, and even the academic community.

As I write this in December 2016 Britain has just voted to exit the European Union and the United States is just elected Donald Trump, both events absolutely in spite of the above-named powers that be. It appears that the manipulation is coming to an end. Although one cannot tell what comes next, it seems quite obvious that statistics will not be an adequate tool to predict it. Statistical observations from the past few decades are simply useless for predicting the next decade. As Taleb puts it, we are in line for a number of "black swan" events which could not be foreseen by the methodologies that Bernstein covers in this book.