Mathematically Correct

2+2=4

"There is a mathematically correct solution"

This web site is devoted to the concerns raised by parents and scientists about the invasion of our schools by the *New-New Math* and the need to restore basic skills to math education.

Mathematically Correct is the informal, nationwide organization that fights the Establishment on behalf of sanity and quality in math education. -- David Gelernter, NY Post

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Mathematics achievement in America is far below what we would like it to be. Recent "*reform*" efforts only aggravate the problem. As a result, our children have less and less exposure to rigorous, content-rich mathematics .

The advocates of the new, fuzzy math have practiced their rhetoric well. They speak of higher-order thinking, conceptual understanding and solving problems, but they neglect the systematic mastery of the fundamental building blocks necessary for success in any of these areas. Their focus is on things like calculators, blocks, guesswork, and group activities and they shun things like algorithms and repeated practice. The new programs are shy on fundamentals and they also lack the mathematical depth and rigor that promotes greater achievement.

Concerned parents are in a state of dismay and have begun efforts to restore content, rigor, and genuinely high expectations to mathematics education. This site provides relevant background and information for parents, teachers, board members and the public from around the country.

Site Index

Hot Topics

An Open Letter in Support of California's Standards System for K-12 Education

In an open letterdated July 7, 2006, former governors Gray Davis and Pete Wilson expressed unambiguous support for California's K-12 academic content standards, curriculum frameworks, instructional materials, and tests aligned to the standards. They warned that dismantling California's system for K-12 education would be *a disastrous step backward.*

Save Our Children from Mediocre Math

Save Our Children from Mediocre Math (SOCMM) is a non partisan organization that advocates ensuring Conejo Valley Unified School District parents have a *choice*to have students instructed in mathematics using a State Board of Education approved curriculum.

Kids Do Count!

Great Connected Math News! We know of two Alpine Jr. High Schools that have completely thrown out Connected Mathematics(excepting one teacher) because it failed so badly!

Rebuttal to Johnny Lott's "Stalkers"

Johnny Lott's condemnation of the open letter as a form of "stalking" appears to be an attempt to side step legitimate criticisms of the current direction of mathematics education in the United States.

California's Algebra Crisis

California has had its share of educational crises—such as whole language and fuzzy math. Despite recent improvements, the state is still in the grips of an algebra crisis.

Review of the Interactive Mathematics Program (requires Adobe Reader)

IMP is a program designed to retain the attention of students who will either not attend college or will major in non math fields. It lacks the depth of study for students who will study math in college. It is not a college prep math curriculum.

NYC HOLD (Honest Open Logical Debate on Mathematics Education Reform) Web Site

NYC HOLD is a nonpartisan advocacy organization that provides parents, educators, mathematicians and other concerned citizens opportunities to work together to improve the quality of mathematics education in the New York City schools.

> *We have followed our children's experience and progress in NCTM Standards-based mathematics programs and have grown increasingly concerned. We have studied the materials and teaching approaches in our children's schools. Some of us have researched the programs and their use in other regions and found that we are not alone in our concerns, rather, our experiences and worries are shared by parents across the country.*
>
> *We have been dismayed and frustrated by teachers' reports that their hands are tied, that they're not free to teach with the materials and methods they believe best suited for our children. We have learned that mathematicians and scientists have confirmed our suspicions that the programs lack adequate skills development, important topics, and the rigor necessary to prepare our children for advanced high school math and science courses and pursuit of college math-based courses and majors.*

Is Los Angeles dictating bad use of a good math book?

Is LA turning a silk purse into a sow's ear? An Open Letter to the superintendent and school Board suggests LA's Algebra I pacing plan is disastrous and undermines both the California math standards and state approved textbooks.

A Plan for Improving the Quality of Exposition in High School Mathematics by Frank B. Allen

In order to raise the level of student achievement in secondary school mathematics, which everyone agrees is urgently necessary, there must be major improvements in the expository procedures employed by teachers.

How the NCEE Redefines K-12 Math: An Analysis of the NCEE Math Performance Standards by Bill Quirk

The NCEE is satisfied with entry level math. Their "vision" is limited to the needs of everyday life. The concept of prerequisite knowledge is never mentioned. They're not concerned with setting the stage for learning more advanced math. Many of their high school math examples belong at the elementary school level. They claim to emphasize conceptual understanding, but give no evidence that they understand how math ideas are connected. They appear blind to the vertically-structured nature of the math knowledge domain.

Mathematics Education in California

- The California Mathematics Standards
- Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve [requires Adobe Acrobat reader]
- Practice Problems for the California Mathematics Standards Grades 1-8 [requires Adobe Acrobat reader]
- More about California

A review of *Geometry: tools for a changing world*

David E. Joyce provides this review of the Prentice-Hall Geometry text, noting ... *It's the content that bothers me, in particular, the lack of logical content.*The review covers each chapter in a way that is especially informative for those required to use this text.

Does Two Plus Two Still Equal Four? What Should Our Children Know about Math?

Despite efforts to improve mathematics education in the United States, the August 2001 National Assessment of Educational Progress report found that a majority of children are still unable to perform at a basic level in mathematics and that an achievement gap between white and minority students continues to persist in that subject. The link provides information and transcript of the seminar at AEI.

Stand and Deliver Revisited by Jerry Jesness

The untold story behind the famous rise -- and shameful fall -- of Jaime Escalante, America’s master math teacher.

TERC Hands-On Math: A Snapshot View by Bill Quirk

A review of the TERC (Investigations in Number, Data, and Space) mathematics program shows:

- TERC Omits All Standard Computational Methods
- TERC Omits Standard Formulas
- TERC Omits Standard Terminology

Also read the full report, TERC Hands-On Math: The Truth is in the Details.

A Brief History of American K-12 Mathematics Education in the 20th Century by David Klein

Mathematics education policies and programs for U.S. public schools have never been more contentious than they were during the decade of the 1990s. The immediate cause of the math wars of the 90s was the introduction and widespread distribution of new math textbooks with radically diminished content, and a dearth of basic skills. This led to organized parental rebellions and criticisms of the new math curricula by mathematicians and other professionals.

New Front in the New York City Math Wars

In NYC, organized parent and teacher opposition to the new math programs began in District 2, one of the city's best; and now extends to District 3, District 10 and District 15. Bronx high school teachers have organized to express opposition to next years' requirement they

use only the Interactive Mathematics Project (IMP), an experimental high school math program. The teachers worry the program lacks important mathematical content necessary to prepare large numbers of their students for the Regents A exam and college level coursework.
The new programs, many without student texts, are based on a "constructivist" teaching philosophy, which discourages teachers from teaching mathematical rules and procedures. Instead, teachers guide students, through group activities, to their own "discovery" of personal solutions. Students are encouraged to seek help from each other, rather than from the teacher.

Content Review of CPM Mathematics

A point by point review of CPM Mathematics vis-a-vis the California Standards. *Much of Volume 1 actually detracts from developing algebraic competence. Almost all of the mathematical content is at the level of the Grade 7 standards or below ... This is not algebra and it is not college preparatory math, no matter what it calls itself. Eventually, Volume 2 starts teaching some algebra but it is too little and too late.*

California Mathematics Program Adoptions for 2001

The list of K-8 programs approved by the State Board of Education on January 10, 2001. These programs were seen as the best fit to the state Mathematics Standards[.pdf file 504k] and the guidelines in the Mathematics Framework[.pdf file 1748k].

California State Adopted Middle School Math Programs

This site provides reviews of textbooks adoped in California for middle school, including texts in algebra 1. The report helps to identify the strengths and weaknesses of these adopted programs.

California Teacher Training Bias Puts Politically Correct Methods Over Proven Methodology, Study Finds

This is the press release for the report from Pacific Research Institute for Public Policy. The full text[.pdf file 741k] of the report, *Facing the Classroom Challenge Teacher Quality and Teacher Training in California's Schools of Education*is also available.

How To Respond When Your School Announces a New-New Math Program, by Kevin Killion

What do you say? How do you respond when your school tells you that your child's math program is going to be replaced? What is your reaction when the replacements main advantages are a "Tokyo by Night" layout, fuzzy-headed but politically correct examples, oddball algorithms and methods (or no methods at all), and a big emphasis on writing essays and playing games?

The Math Wars, by David Ross

My disagreement with my father contained the essential elements of the current Math Wars, the debate that is going on today over the way that mathematics should be taught.

Excerpts from *Poor Performance Review*

Ralph A. Raimi, professor emeritus of mathematics at the University of Rochester, talks about MSPAP, the examination system Maryland uses to assess student performance. He notes that, *The system was of no value for its announced purpose*. For the full story see the Washington Times of Sunday, April 1, 2001.

Big Business, Race, and Gender in Mathematics Reform, by David Klein

Opposition to California's mathematics standards from reform leaders continues as of this writing. Former NCTM president Jack Price wrote in a letter published by the Los Angeles Times on May 10, 1998:

...if the state board had adopted world-class mathematics standards for the 21st century instead of the 19th century, there would have been a great deal of support from the 'education' community.

This sententious observation encapsulates the topics discussed in this essay. For the reformers, "world-class mathematics standards for the 21st century" eluded the Stanford mathematicians who wrote California's 1998 math standards. Missing are the greater emphasis on technology--an end in itself--and pedagogical directives harmonious with the reified "cognitive styles" of the racially diverse populations of the 21st century. The "19th century" arithmetic, algebra, geometry, and trigonometry highlighted in California's 1998 standards will have diminished value in the postmodern epoch of technological wonderments envisioned by math reformers. Perhaps the academic community should consider whether the discipline of mathematics education--much more so than mathematics--needs fundamental alterations for the 21st century.

NCTM Math in the NCEE America's Choice Performance Standards

- The NCEE Wants to Demonstrate "How Good is Good Enough"
- What's Emphasized in the ACPS? What's Missing?
- A Compact Version of the 38 ACPS Math Examples

2+2=5: Fuzzy Math Invades Wisconsin Schools, [.pdf file 80k] by Leah Vukmir

This level of parental outrage and concern is certainly not confined to McFarland. According to Parents Raising Educational Standards in Schools, a Wisconsin-based parent organization, math education has become the number one concern of parents calling for information and assistance. In the last two years it has supplanted the "Reading Wars" and is causing parents across Wisconsin and the nation to organize and rebel.

Standards in School Mathematics [.pdf file, 52k, see second page]

Ralph Raimi discusses the new *Principles and Standards for School Mathematics (PSSMP)*from the NCTM.

I warn you that these "principles and standards" cannot be appreciated by reading only a few pages. In the small the document sometimes sounds good. But if PSSM in the large informs our vision, then self-esteem is better than knowledge, dictionaries can replace a ready (memorized?) vocabular, and higher-order thinking skills will boil stones into soup.

Cognitive Child Abuse in Our Math Classrooms, By C. Bradley Thompson

Whole math must lead to a miasma of confusion, boredom, and despair. Rather than encouraging independent, conceptual-level thinking, it is thoroughly anti-conceptual. It dooms children to function on a primitive, perceptual level—i.e., to flounder in a chaotic sea of concretes with no objective principles to guide them. This is cognitive child abuse. Whole-math defenders are shrinking the cognitive capacities of their students to those of infants or even animals.

Romancing the Child by E. D. HIRSCH JR.

"The progressive way of running a school is essentially the opposite of what the 'effective schools' research has taught us." So says Hirsch in the new journal, Education Next

Open Letter on the Department of Education's List of Programs

The U.S. Department of Education issued a statement endorsing some of the worst mathematics programs available. This prompted a rebuttal endorsed by over 200 of the nation's leading mathematicians and has resulted in a congressional hearing.

Testimony to the House of Representatives regarding the Department of Education

Testimony from the appropriations subcommittee that deals with the Department of Education.

Reality Check 2001

Although high-stakes standardized tests are often controversial, Reality Check picks up few signs of public backlash. Neither parents, teachers, nor students themselves voice significant dissatisfaction with testing in their own schools. Large majorities of all groups express strong support for their own district's efforts to raise standards and for using standardized tests to enforce standards ...

Clinton's Proposed National Voluntary Mathematics Test

Noting the inadequate achievement in school mathematics in the United States, President Clinton proposed a voluntary national test as a way to combat the problem. Examinations can be an effective way to stimulate achievement gains, but the devil is in the details. Clinton's plan got off to a bad start. The first committee working on it was full of fuzzy math supporters, and the initial plans were dismal. The test design was shifted from the initial committee to the National Assessment Governing Board. The progress was often delayed and funding was restricted by congress.

The Truth About The REVISED NCTM Standards: Arithmetic is Still Missing! by William G. Quirk, Ph.D.

Similar to the original NCTM Standards, PSSM is vague about the major components of arithmetic mastery:

1. Memorization of of basic number facts
2. Mastery of the standard algorithms of arithmetic
3. Mastery of fractions

The NCTM has toned down the constructivist language, but they still stress content-independent "process skills" and student-centered "discovery learning". Similar to the NCTM Standards, PSSM emphasizes manipulatives, calculator skills, student-invented methods, and simple-case methods.

Mathematics "Council" Loses Hard-Earned Credibiility

By Frank B. Allen

The National Council of Teachers of Mathematics, now led by theoreticians from our Schools of Education, imposes policies that distort the teaching process and heavily impair the learning of school mathematics. Can the NCTM accept the challenge to Save Our Schools?.

Brown Center Report on American Education [.pdf file 43k]

The academic achievement of American students has risen since the 1970s but only at a snail’s pace. Performance in arithmetic remained static or declined slightly. Results for thirteen year olds suggest large numbers of students have not mastered the basic arithmetic skills that are necessary before moving on to algebra.

High Achievement in Mathematics: Lessons from Three Los Angeles Elementary Schools

In a paper commissioned by the Brookings Institution, David Klein describes characteristics and academic policies of three low income elementary schools in the Los Angeles area whose students are unusually successful in mathematics.

Excerpts from *Math Wars*

The Wall Street Journal Editiorial of January 4, 2000 addressed the so-called reform in mathematics education. The editorial concludes that *New Math will take its casualties, especially among the poor, adding to the already mounting costs of the decline in national educational standards.*

Why Education Experts Resist Effective Practices, by Douglas Carnine

American education is under intense pressure to produce better results. The increasing importance of education to the economic well-being of individuals and nations will continue feeding this pressure. In the past—and still today—the profession has tended to respond to such pressures by offering untested but appealing nostrums and innovations that do not improve academic achievement.

Basic Skills Versus Conceptual Understanding: A Bogus Dichotomy in Mathematics Education [.pdf file 54k], by H. Wu

The truth is that in mathematics, skills and understanding are completely intertwined. In most cases, the precision and fluency in the execution of the skills are the requisite vehicles to convey the conceptual understanding.

Knowing and Teaching Elementary Mathematics [.pdf file 73k], by Richard Askey

The U.S. Department of Education has announced the results of an exercise to identify "expemplary" and "promising" texts. *Connected Mathemaitcs*, a series for grades 6-8, is one the department has deemed exemplary. I do not understand why it deserves that rating. I am quite familiar with this series, as I reviewed it as part of a textbook adoption process. Regarding fractions, for example, *Connected Math*has some material on the addition and subtraction of fractions, but nothing as systematic as described by the Chinese teachers interviewed by Ma. There is less on multiplication of fractions, and *nothing*on the division of fractions. If our students go through grade 8 without having studied the division of fractions, where are our future primary teachers going to learn this? The criteria used by the Department of Education review should be rewritten now that Liping Ma's book has provided us with a model of what school mathematics should look like.

Reform Mathematics Education: *How to "Succeed" Without Really Trying*

The reform designs open the door to claims of successfully teaching mathematics without really doing so. The reform writings and methods are many and varied, but a common feature is that they end up obscuring the failure to teach mathematics. In reform mathematics education, the goal of success for all is not supported by achievement but rather by redefining success and, mostly, by obscuring failure.

Recent Directions in San Diego Mathematics Education

It is obvious that the district is planning to used dumbed-down mathematics in the focus schools. They are taking the approach we have fought so hard to avoid - lowing expectations while claiming otherwise.

San Diego Draft Framework Critique

Is San Diego trying to snatch defeat from the jaws of victory? After the release of state and district standards and a state framework that all align to set high expectations for student learning and clearly detail the course of mathematics instruction, San Diego has released a draft of a local framework that seems devoid of substance. See this review for more details.

New-New Math in Santa Monica

The curriculum problems in Santa Monica are complicated by an insidious form of discrimination where students in the more affluent neighborhood schools have the benefit of a State approved, State content aligned curriculum and the schools in less affluent southern part of the District have MathLand and CPM, neither of which is State approved for educational content ... With nearly one-sixth of the students in our elementary and middle schools in danger of retention at the end of June, 2000, giving parents complete and truthful information was an important step in helping children succeed. We formed a coalition of parents and other interested parties to address the issues at the District level, Santa Monicans Working for Equity and Excellence in Public Schools (SMWEEPS).

Mountain View Achievement

Mountain View Achievement is a group of concerned parents, teachers and community volunteers who believe that all children can achieve academic success when given appropriate opportunities and tools. We are dedicated to improving educational opportunities, and we invite every member of the community to join us in this endeavor.

Russian Mathematics

From Mathematics: A Text for 6th Graders
by Enn R. Nurk and Aksel E. Telgmaa
© 1995 Drofa, Moscow
Translated from the Russian by Willis Harte
University of Iowa College of Education

The Role of Long Division in the K-12 Curriculum

Reviews the reasons that most math educators today depreciate the topic and other topics in the curriculum that derive from it, methods for teaching long division in such a way that the underlying concepts can be understood by students, the ways in which these concepts develop in later mathematics course, and why they are so important.

Number Sense in California

By showing topic development across grade levels, it becomes easier to evaluate any particular piece of mathematics content relative to the California standards. The California grade level of content from textbooks or tests or other sets of mathematics standards can thus be more easily identified.

NYC MATH WARS

Community response in NYC and across the country has erupted in what have become known as the "math wars." Critical parents, joined by mathematicians and scientists advocate clarity and balance in math reform: urging the inclusion of grade by grade goals, explicit teaching of standard procedures, basic skill building and rigor along with the inclusion of some of the creative exercises in the new programs. The pendulum has swung too far and must be corrected.

The Mathematics Framework in Massachusetts

Massachusetts has been going through quite a struggle over arithmetic in their state mathematics framework. Links include background and the details of the story of the attack by Hyman Bass on the Massachusetts Deputy Commissioner of Education.

Site Index

- Where Did Fuzzy Math Come From?
- Fuzzy Math in California
- What is Changing in Math Ed?
- The writings of Frank B Allen

- The Nation

- The Department of Education's List of Programs
- Statements from Mathematically Correct
- On the NCTM Standards
- International Comparisons
- State Assessments
- The Mathematics Framework in Massachusetts
- The proposed National Voluntary Mathematics Test
- Other reports on math education

- California

- California Background
- Mathematics Standards
- The Mathematics Framework
- Textbooks
- Testing
- Districts Around the State
- Other California Goodies
- A Chronicle from the San Francisco Chronicle

- Mathematics Teachers and Professional Development

- **Mathematics Programs and Textbooks -- Reviews and Information**

- Science Corner

- Just for Parents

- What's a Parent to Do?
- True Standards that Parents Can Use
- Mathematically Correct Kindergarten to Geometry Standards
- Coping with Math Reform
- Terminology Every Parent Must Understand

- Glossary of Terms

- Web links of interest

- Math woes from around the country

- What Can You Do?

What Can You Do?

If you are concerned about the changes to the mathematics curriculum, you should:

- Make your opinions known
  - Use any method - letter, fax, telephone and email
  - Contact your teachers, principals, local board members, state board and state government
- Read
  - California Mathematics Academic Content Standards
  - Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve [requires Adobe Acrobat reader]
  - San Diego Mathematics Standards
  - What's a parent to do?
  - A Program for Raising the Level of Student Achievement in Secondary School Mathematics
  - The Schools We Need & Why We Don't Have Them
  - Angry Parents, Failing Schools

Governor Pete Wilson *Governor Gray Davis*

July 7, 2006

Honorable Arnold Schwarzenegger
State Capitol Building
Sacramento, CA 95814

Dear Governor Schwarzenegger:

It has come to our attention that Sacramento is awash in misinformation about the recent history and intentions of California's school reform strategy. While across the country many look to California's standards as the best, confusion in Sacramento threatens the core of California's program of school reform. Term limits have left Sacramento with little institutional memory of the goals and rationale of California's reforms. As the Governors who led this work, we believe it is time to remind the public of the facts.

In the attachment we have listed some of the incorrect claims being advanced about California's recent experience with standards and accountability and a case-by-case rejoinder that sets the record straight.

To consider the purpose of many of the programs now in place in California, it might be helpful to consider California's history *before* the introduction of standards. When California started down the road to building standards and assessments, there was no standard of excellence throughout California's public school system.

Indeed, national and international test scores released in the early 1990s provided a sobering message about the low overall achievement of California students. The state that once led the nation in education was in the 1990's struggling along at very low levels. State leaders realized that California had much work to do to correct this long slide. Achievement gaps almost certainly existed, but there was no way to measure them and without that, no way to hold schools accountable for changing them. And conversations about appropriate policy quickly turned into ideological warfare because no one had evidence about results. Even if one wanted to look for results, no results were available. While most people knew which schools were problems in each community, there was no mechanism to measure progress or failure.

The need for standards in that environment was clear and compelling: Standards provide a measure of excellence regardless of one's skin color, family income, or zip code. We took a standards-based approach in California because we believe that if we set expectations high, students will respond. Not every child will fully meet the challenge, but all will benefit from the effort.

A standards-and-assessments approach means that no matter which neighborhood or region of the state a child is from, that child should be held to the same high expectations. It means that we will not give up on some children by building a less rigorous program for them. To do so would mean holding them back and limiting their future.

To provide those children a lesser academic challenge would be to insult them and might well cheat them out of realizing their full potential. Instead, when children begin school, we should address whatever deficits in academic readiness they may bring to the classroom through early and effective remedial attention – not by creating an educational apartheid of lesser standards.

Standards also provide a way to measure progress and base decisions on objective evidence, not education fads. And they provide a measurable way to show how California's public schools are improving over time.

Remarkably, the need for change and the program to bring it about was supported by a bipartisan legislature and developed over the course of several Administrations. The essential ingredients-- the core content standards, curriculum frameworks, instructional materials, and tests aligned to the standards-- are now all in place and provide solid cornerstones for the work that must take place in schools across our great state. For a number of years now, students, parents, educators, and governors have known what is expected, what progress has been made, and what shortcomings still exist. These cornerstones provide much needed stability—more stability than California schools have known in decades.

Many in Sacramento have forgotten or are unfamiliar with this history. Some would suggest that California did better—and would do better again—if we returned to the "good old days" before uniform statewide standards. Although such a system may have served some of California's students, it clearly failed all too many of them. That system did not expect all students -- no matter their neighborhood -- to attain high levels of academic achievement. In all too many neighborhoods, it produced the "bad old days" of sham and illusion, of social promotion and functionally illiterate high school graduates.

Having rigorous academic standards does not by itself guarantee that the teaching, textbooks, and student effort needed to meet those standards will be there. But the absence of such standards would undoubtedly guarantee that far too many of our schools and our children would fail to fulfill their potential.

Further steps need to be taken to make sure that each student who enrolls in a California public school receives a high quality education which prepares that student for success in a rapidly changing world. But through California's rigorous academic standards and accountability system, the foundation is solidly in place.

Undoing California's present high academic standards would be a disastrous step backward. It would leave far too many children woefully unprepared for the challenges and demands they will face in today's ever more fiercely competitive global marketplace.

Sincerely,

Pete Wilson

PETE WILSON
GOVERNOR

Gray Davis

GRAY DAVIS
GOVERNOR

For the Record:
The History of Standards in California

Assertion: Some have attacked the standards by arguing that the process for creating them was politically manipulated or even hijacked by "ideologically based scholars" such as "fellows" at the conservative Hoover Institution. As a result, those critics would assert, California's standards do not enjoy broad-based support.

Fact: California's standards are widely acknowledged as the best and most specific standards in the country and have been adopted in whole or part for use by other states. Parents, teachers, taxpayers and all Californians can view these standards at http://www.cde.ca.gov/be/st/ss/.

To create these standards, the State Board of Education, working with the State Superintendent and the Legislature, appointed a Standards Commission made up of 36 teachers, principals, higher education representatives, and Californians from the broader community. Of course this work engendered robust debate: something as important standards for learning should not be decided easily. It also attracted national attention as many Nobel Prize wining scholars weighed in, offering the commission counsel. But when the commission brought forward it recommendations for History-Social Science, Science, and English Language Arts, those recommendations were approved essentially unchanged.

Prior to adopting the Mathematics standards, the State Board did make major revisions, but here we should be guided here by the results of their intervention. Groups such as Achieve, the American Federation of Teachers, and the Fordham Foundation have rated California's mathematics standards as among the best in the country. When Achieve, an organization made up of leading Governors and Business leaders, is asked to help other states develop or evaluate their standards for mathematics, they benchmark to California. The academic leaders who recommended the final revisions for California's standards were mathematicians from the University of California at Berkeley and Stanford University. Only one member of the 36-member Standards Commission was a Ph.D. scholar from the Hoover Institution.

Assertion: The English Language Arts/ Reading and English Language Development curriculum takes too much time. Too many elementary school students are being taught a curriculum devoid of all subjects except reading and math.

Facts: California's standards recognize reading fluency as the most critical skills students learn in the early grades. This only makes sense; if students cannot read, they cannot master much else. Hence, the State Board in California has adopted a reading/language-arts curriculum that not only follows the best scientific research on reading instruction, but also is a comprehensive program, encompassing phonics, reading comprehension, vocabulary, speaking and listening, writing (including spelling and grammar), and critical reasoning. So, yes, it is true that California's reading/language-arts curriculum requires a minimum of 2.5 hours of dedicated attention to these foundational, prerequisite skills in the primary grades. In adopting the reading/language-arts standards and textbooks, the State Board also recognized that English Learners

(ELs) may need more time than native language speakers; therefore ELs can spend as much as 3½ hours daily on reading/language-arts instruction in the early grades to develop this most essential skill.

But a crucial fact critics ignore is that to ensure that California's intensive focus on reading/language-arts in the early grades does not crowd out other topics, The State Board requires publishers to build reading/language-arts instructional materials in California that included content aligned to California's history and science standards as well as visual and performing arts standards. Put more simply, in the schools using state adopted curricular materials, students in grades K-3 read history and science content as well as literary text as a part of their reading/language-arts curriculum. California's focus on reading ought not "crowd out" other subjects; on the contrary, it uses these important subjects as the content that helps children develop their reading skills.

While many contributed to the creation of this aligned system of teaching and learning, we should especially acknowledge the countless hours spent by California teachers to create and review this system of instructional materials. In fact, the plurality of the advisory committees to the Curriculum Commission and the State Board are hard-working classroom teachers who played leading roles in developing the criteria and reviewing the materials that are used throughout California schools.

> *Assertion: California teachers are "prohibited from knowing what skills and knowledge the tests will test."*

Fact: The California Standards Tests (CSTs) are fully aligned to California standards, so if teachers are teaching to the standards, they are teaching the material that will be tested. CSTs include complex analytical tasks such as reading and comparing two short essays to identify common themes and require students to write essays in grades 4 and 7. (The California High School Exit Exam assesses high school writing.) Each year, a portion of the tests are released to the public, so that teachers and parent can see the types of questions and format of the tests. (Anyone can view these test questions online at http://www.cde.ca.gov/ta/tg/sr/css05rtq.asp). Teachers and parents receive annual customized reports of their results that highlight the strengths and challenges of their students. It is true that each year, the tests cover a portion of the standards; this is done to keep testing time reasonable. It is also important to note that the test blueprints are available for review and posted on the Internet. And it is true that no one knows *exactly* what is on the tests the students take before it is given; if they did, it wouldn't be a credible test.

> *Assertion: The State Board is made up of "narrow ideologues… The State Board of Education appoints to the Curriculum Commission only members who are loyal to this ideology also known as the 'one shining path'."*

Fact: California's widely respected program of standards, testing, and accountability has been built over the past ten years under the guidance of the State Board of Education, whose membership was appointed by three Governors of different political parties:

Wilson, Davis, and now Schwarzenegger. Every member of the State Board of Education must be confirmed by an extraordinary 2/3rds vote of the California State Senate. Over these years, this diverse, bipartisan group has included teachers, parents, administrators, local school board members, business leaders, retired educators, and general citizens who have been African American, Asian, Latino, and Caucasian. Some have been parents, others grandparents. They have represented different regions of this diverse state. Over the years, they have agreed and disagreed on many things, as their voting records will show. But, remarkably, despite changes in the parties that occupy the Governor's office, this group has shared a commitment to improve California's schools by rigorous academic content standards and a strong accountability system.

The curriculum commission, whose members are appointed by the Board, the legislature, and the Governor, has enjoyed a similar history of diverse membership and equally robust debate. That debate—and the passion behind it—is the reason California's standards are so exceptional. The meetings of the commission are open to the public and we encourage you to attend.

> *Assertion: California's low ranking NAEP scores "prove" the system has failed.*

Facts: California careened to the bottom long before the standards were introduced. California's NAEP rankings are a measure of a supposed Golden Age to which critics would like us to return; the state plummeted during the years that programs such as whole language and constructivist math were widespread. In fact, since 1994, California has seen slow and steady progress in the growth of those scores, especially when we look at the achievement gaps. For example, California closed the gap between English Learners and fluent English speakers by 5 points from 2002-2003. These differences are more profound when you consider that our results are compared to other states that do not include as many English Learners when they test. California tested 88% of its English Learners while states like Texas tested only 62% of its English Learners and New York tested 70%. Similar "who gets tested" gaps exist for Special Education students across the states.

> *Assertion: The move to standards and accountability is driving teachers out of the profession and into early retirement.*

Facts: There is no evidence to support the assertion that teachers are leaving the profession or retiring earlier than in previous decades. The numbers of teachers who remain committed to their jobs remains strong. According to the State Teachers Retirement System's most recent report, the average age of retirements has been remarkable steady. When these trends are considered in the context of labor statistics that show that more and more employees are likely to change careers multiple times over a lifetime, these data show there is considerable stability in the California teaching profession.

Assertion: Data show that California's Reading First program has failed.

Facts:
On the contrary, data show that California's Reading First program is succeeding. First, studies show a consistent trend of increasing percentages of Grade 2 Reading First students who are at the Proficient and Advanced performance-levels on the California Standards Test (CST). In the spring of 2005 this trend was apparent for all three Reading First cohorts (years 1, 2, and 3). Second, the evidence shows that the academic advantage of participating in the Reading First program increases with further time in the program. Third, substantial percentages of Reading First students who were at the lowest CST performance-levels (Below Basic and Far Below Basic) are improving their performance-levels. The same thing has been happening with Reading First students on the California English Language Development Test (CELDT).

Assertion: California Achievement Gaps are growing.

Facts: Like our NAEP scores, California's achievement gaps have been with us longer than the state's program of standards and assessments. That is why the achievement gap has become such a critical focus when we measure school performance. And, like NAEP, the measure of progress, not rank performance, should be our judge. For example, more than any other group, Latino students have made the most impressive gains during this time, growing faster year-to-year than any other ethnic group. The percent of English Learners scoring Early Advanced or above on the ELD assessment in primary grades rose from 21.7% in 2002 to 28.7% in 2004—a 32% increase. The percent of English Learners in grades 4 though 8 scoring Early Advanced or above rose from 39.8% to 55.6% in this same two year period. This is a stunning 39% increase. These gains should give us great hope. They certainly do not support calls for an abrupt course reversal.

Assertion: Performance of California's high schools, combined with the high levels of students required to take remedial coursework when they enter college, are further evidence that the standards have failed.

Facts: It is true that current high school students are our most daunting challenge — their difficulties provides the most concrete evidence of why we must stay the course. Today's high school students began elementary school before the standards were in place; they didn't benefit from improved instructional materials. (Here a brief history is helpful: Academic Contend Standards in the four core areas were adopted by 1998. English Language Development Standards were adopted in 1999. The curricular frameworks that provided the guidelines for instructional materials were brought forward in succession. K-8 adoption of instructional materials aligned to these standards was completed with the adoption of Reading/Language-Arts in 2002.) By California law K-8 schools were not required to have such materials until the 2004-05 school year. It is thus a bit of a stretch to blame standards and instructional materials for the very real problems of our high schools.

Assertion: The State Board of Education unfairly limits the choices that schools have for instructional materials

Facts: There are countless tools and resources available to help children learn, but only a few publishing companies that create materials to support comprehensive instructional programs aligned to grade-level core content standards. In the initial K-8 instructional-materials adoptions that were aligned to core content standards, many publishers did not submit materials to California -- Some because the timelines were too short and some because they were not certain the standards would be enforced. Only 3 to 5 publishing companies submit materials for adoption in each subject. In most grades the State Board of Education has adopted 2 to 5 programs, depending upon the number submitted and the number aligned to California's Academic Content Standards. The State provides categorical funding to cover much but not all of the districts' cost to obtain these basic instructional materials. State law and the Williams case settlement emphasize the absolute necessity for each student to have at least basic materials in each subject. State categorical funds for instructional materials by law are to be first used to ensure that this basic need is fulfilled.

Assertion: The instructional programs preferred by the State Board force teachers to follow a script that relies on "drills" to teach children or are too heavily reliant on books to guide instruction.

Facts:

The best research available tells us that while a very few students may learn to master subjects with little guidance, most students require direct teaching and guided skill development to learn the building blocks of reading. (Just as most children won't learn the piano unless they take lessons.) The instructional programs used in California to teach foundation skills such as reading and math provide teachers with teacher manuals, pacing plans, and other tools to help manage classrooms. This approach recognizes that students must build a robust foundation of skills and knowledge in order to seriously engage in critical and analytical thinking or experimentation and these instructional materials are critical tools in this process.

Assertion: California's instructional materials are not designed to support English language learners

Facts: When California adopted its materials for reading/language-arts, it required publishers to create materials in Spanish that are fully aligned to California standards, recognizing that students enrolled in bilingual classrooms deserve a program just as academically rigorous as the programs available in English.

California's English Language Learners may require more time for instruction to ensure that they develop a solid foundation in reading and language skills. Therefore, English Learners may require 60 minutes of focused instruction in addition to the 2.5 hours of reading instruction designated for all students in the early grades.

Assertion: California standards are too high and, by requiring topics such as Algebra, unfairly biased against students who have no interest in college.

Facts: While it was once true that Algebra was only needed for the college-bound, in today's economy, Algebra is the gateway to not only college but to most vocational and technical-training programs that prepare students to earn a professional wage. Of course, there are ample jobs that don't require Algebra, but few such jobs that would support a family. All students should be prepared for the full array of choices ahead of them; those choices should not be made for them by a school system that doesn't ensure that all students are prepared.

Rebuttal to Johnny Lott's "Stalkers"

Jan 26, 2004

I co-authored and collected endorsements of the 1999 open letter to former U.S. Secretary of Education Richard Riley (posted at http://mathematicallycorrect.com/nation.htm#doesham). Two and only two people have ever contacted me to ask me to remove their names from the letter, and I removed their names immediately after receiving their requests.

One of these two people expressed agreement with the letter, but indicated that because of his official position he would prefer to avoid controversy. I removed his name promptly before the letter was published. The other person is Leon Lederman. He initially endorsed the letter and his name appears in the published version in the Washington Post, Nov. 18, 1999. Subsequent to publication, on Nov. 26, 1999, I received an email note from him requesting that his name be removed from the letter. I removed his name that very day from the web version (it was too late for the Washington Post version), and requested an explanation from him. I received no explanation of any sort. After publication in the Washington Post, several leading mathematicians and physicists requested to add their names to the letter, and those names appear on the web version.

I have not received any other expressions of regret for signing the letter, or requests to "un-sign" it. A request to remove one's name after so long a time as this would be inappropriate and disingenuous. One cannot normally sign a document and then unsign it. As to accusations that it has been misused, I know of no such examples, but even if there were such incidents, what conclusions follow? Certainly documents of any sort can be misused, even celebrated and important documents such as the Bible or the U.S. Constitution. The open letter says what it says and there is nothing misleading about it. The signers of the letter read it before adding their names to it.

The NCTM officially endorsed all ten "exemplary" and "promising" K-12 mathematics programs criticized in the open letter to Riley (for a text of the NCTM endorsement, see the appendix to A Brief History of American K-12 Mathematics Education in the 20th Century at: http://www.csun.edu/~vcmth00m/AHistory.html), and at one time that NCTM endorsement was posted on the NCTM web site. Johnny Lott's condemnation of the open letter as a form of "stalking" (http://www.nctm.org/news/president/2004-01president.htm) appears to be an attempt to side step legitimate criticisms of the current direction of mathematics education in the United States.

David Klein

To: USD-268 Board of Education

From: Douglas A. Riepe

Date: July 12, 2002

Subject: A well referenced literature review of Interactive Math Program®.

CC: Mr. Neuenswander Mr. Traxson Concerned Patrons

Introduction:
This effort started with the author reading the freshman level Interactive Math Program® (IMP) book last December. It continued with the author contacting numerous university professors and departments concerning the use of calculators in freshman calculus classrooms. The author also contacted the registration offices at Kansas State University (KSU) and The University of Kansas (KU) to verify that IMP was accepted for entrance requirements to the state universities of Kansas. The author voiced concerns as to the nature of IMP to a board member and met with the high school principal, the director of curriculum and testing, and a high school math teacher March 14, 2002. The research continued in an attempt to verify the article in the *Cardinal Connection* that IMP was an exemplary math program. The sources of the statements made in the Conclusions and Recommendations area are given in the body of the report. All references used in this review are appended to the survey for the convenience of the reader. The conclusions are the opinion of the author, other opinions may differ.

The reader is encouraged to examine all of the references listed to verify 1) that the reference exists; 2) that the author has not electronically altered the reference; and finally 3) that the author has not taken the quote out of context. Since this is a literature review, the author makes frequent and lengthy use of quotations from the sources that have nothing to gain from the sale of textbooks. The intervening text is present to transition from one source to the next.

Conclusions and Recommendations:

1) The expert panel that granted the exemplary rating to IMP and other programs had the appearance of conflict of interest as well as other issues. While the exemplary rating is an historical fact, it is of little value.
2) IMP is a program designed to retain the attention of students who will either not attend college or will major in non math fields. It lacks the depth of study for students who will study math in college. It is not a college prep math curriculum.
3) SAT scores from high schools in California that use and are satisfied with IMP are lower than the California state average.
4) Some college professors do not allow the use of calculators and college professors in general warn of reliance on calculators being a handicap for students. Some KSU freshman calculus coordinators do not allow the use of calculators on calculus tests.
5) USD-268 should contract with Garden Plain or Norwich to provide college prep math because IMP is not a college prep math program.

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The author first learned of the exemplary rating in the district newsletter. No references were cited as to a web site or journal where this rating could be studied[1]. The director of curriculum and testing was contacted as to more information as to the source of the exemplary rating. The director of curriculum and testing forwarded reference material to the author[2]. The IMP newsletter did not contain references as to where to find the U.S. Department of Education report cited. The author located the report after searching the U.S. Department of Education web site[3]. The report gave no indication as to the source of the data used to arrive at the exemplary rating. The report gave no references or bibliography. The author located a list of Expert Panel members[4]. The author located a contacts list for the expert panel[5]. The contact person for the Mathematics and Science Education Expert Panel, Ms. Carol Sue Fromboluti, was contacted[6] to obtain a copy of the references used to prepare reference 3. Ms. Fromboluti told the author that no bibliography existed. No data other than that provided by IMP had been used to prepare the report that gave IMP an exemplary rating.

Following the endorsement of the exemplary and promising curriculums a letter was sent to the United States Secretary of Education, Richard Riley[7]. It was published in the *Washington Post*. The letter to Secretary Riley was authored and endorsed by[8]

> *Department heads at many universities, including Caltech, Stanford, Harvard, and ale, along with two former presidents of the Mathematical Association of America also added their names in support. Seven obel laureates and winners of the ields Medal, the highest award in mathematics, also endorsed. In addition, several prominent state and national education leaders co-signed our open letter*

The authors (4 of 6 were from California while 85 of 218 co-signers were from California) of the letter also stated,

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1 April 2002 issue of *The Cardinal Connection*, "Curriculum Corner" by Amy Wallace, Director of Curriculum and Testing, Cheney USD 268. Page 17 of report.
2 *IMPressions*, The Spring 1999 newsletter for IMP. Page 18 of report.
3 The Interactive Mathematics Program (IMP), pages 19-26 of report. website: http://www.enc.org/professional/federalres.../document.shtm?input=CDS-000496-496_1
4 U.S. Department of Education Math and Science Education Expert Panel, pages 27-29 of report website: http://www.enc.org/professional/federalresour.../document.shtm?input=CDS-000496-49
5 Expert Panels – Contact list, pages 30-31 of report web site: http://www.ed.gov/offices/OERI/ORAD/KAD/expert_panel/contact.html
6 Phone call, D. A. Riepe to C. S. Fromboluti, May 1, 2002
7 An Open Letter to United States Secretary of Education, Richard Riley, pages 32-57 of report. posted at: http/www.mathematicallycorrect.com/riley.htm
8 Testimony of David Klein Professor of Mathematics, California State University, Northridge, April 4, 2000, to U.S. House of Representatives Committee on Appropriations Subcommittee on Labor, Health and Human Services, Education and Related Activities. Page 58 of report.

We would li e to emphasi e that the standard algorithms of arithmetic are more than just ways to get the answer that is, they have theoretical as well as practical significance. or one thing, all the algorithms of arithmetic are preparatory for algebra, since there are (again, not by accident, by the virtue of the construction of the decimal system) strong analogies between arithmetic of ordinary numbers and arithmetic of polynomials. [9]

The letter also states:

It is not li ely that the mainstream views of practicing mathematicians and scientists were shared by those who designed the criteria for selection of exemplary and promising mathematics curricula. or example, the strong views about arithmetic algorithms expressed by one of the xpert Panel members, Steven Leinwand, are not widely held within the mathematics and scientific communities. [10]

Mr. Leinwand was on the advisory board for IMP[11].

The letter states[12],

The xpert Panel that made the final decisions did not include active research mathematicians. xpert Panel members originally included former S Assistant Director, Luther Williams

NSF provided funding for many of the programs submitted to the Expert Panel. Others voiced concerns[13]

ut some panel members were mystified and wondered whether having S officials on the expert panel opened the door to charges of vested interests. ot enough thought had gone into the ma eup of the panel, says James utherford, an advisor to the American Association for the Advancement of Science, who was on the panel, too. I really wondered if Luther should have been there at all. After all, at the S he was directly involved in funding the very programs that we were evaluating.

The output of the Expert Panel appeared to favor NSF funded programs,

ven after Williams left the panel, there was another S official on board. In the end, six of the 10 programs selected by the panel were S funded a stri ing success rate since only 13 were S funded in the 1990 s [14]

At least one of the members of the Expert Panel had no ties to the NSF or to book publishers, Dr. Manuel Berriozabal, a mathematician at the University of Texas at San

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[9] Ibid 7, page 2 of 26. Page 33 of report.
[10] Ibid 7, page 1 of 26. Page 32 of report.
[11] Mark Clayton, "How a new math program rose to the top", *Christian Science Monitor*, May 23, 2000, page 11 of 12, web site: http://www.csmonitor.com/sections/learning/mathmelt/p-2story052300.html. Page 71 of report.
[12] Ibid 7, page 1 of 26. Page 32 of report.
[13] Ibid 11, page 10 of 12. Page 70 of report.
[14] Ibid 11, page 10 of 12. Page 70 of report.

Antonio. Dr. Berriozabal was asked to join the Expert Panel by the U.S. Department of Education.[15]

Dr. Berriozabal is well aquainted with preparing high school students for college math programs as he is the coordinator for TexPREP.[16] TexPREP is a program that identifies the top math students in the state of Texas and prepares them for college engineering programs.[17] Dr. Berriozabal was also inducted into the Texas Math Hall of Fame.[18]

Dr Berriozabal commented on the Expert Panel

> *The panel was a good idea, Dr. errio abal says, but we made some bad judgements. rom the best I could tell, none of the programs we selected as promising or exemplary had any ind of long-term trac record of achievement.* [19]

Dr. Berriozabal abstained or voted against all 10 programs designated exemplary or promising .[20]

Dr. Berriozabal commented in a recent e-mail

> *Over a period of several days, the panel examined the reports and during this time we had hard copies of the program materials if we wanted to review them. I believe that evaluations for over 50 programs were submitted. Consequently, in my opinion, very little per program was given to an indepth study of a program.*[21]

If 50-60 programs were reviewed, let s say several days was one week (5 working days) that would allow for between 2/3 to 4/5 hour per program. 40 to 48 minutes is hardly enough time to read textbooks for a 4 year curriculum.

When asked to comment specifically on IMP, Dr. Berriozabal stated:

> *I am unable to provide you with any specific critiques on the IMP Program (or any other program) but I suggest that the Mathematically Correct group in California might be a good source. The e-address which I have for them is www.mathematicallycorrect.com/books.htm.* [22]

The reader is invited to surf to the web site and read through the material on IMP and other curriculums.

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[15] Ibid 11, page 4 of 12. Page 64 of report.
[16] Manuel Berriozabal – Home Page, web site: http://applied.math.utsa.edu/berriozabal/index.html. Pages 73-76 of report.
[17] TexPREP site, web site: http://www.math.utsa.edu/~prep/sa013.htm. Pages 77-78 of report.
[18] Ibid 16, Pages 73 - 76 of report.
[19] Ibid 11, page 4 of 12. Page 64 of report.
[20] Ibid 11, page 5 of 12. Page 65 of report.
[21] e-mail from Manuel Berriozabal to Douglas Riepe, Friday, May 10, 2002. Page 79 of report.
[22] Ibid 21. Page 79 of report.

In summary it can be said that reasonable people can have difficulty accepting the decisions of the Expert Panel due to the make-up of the panel, the criteria used by the panel, and the lack of time spent reviewing the actual textbooks by the panel.

Dr. Klein states in his testimony to Congress:

> *The ten so-called exemplary and promising math programs recommended by the Department of ducation for our children include some of the worst math boo s available. The programs I have examined radically de-emphasi e basic s ills in arithmetic and algebra. ncontrolled calculator use is rampant. One can draw a parallel between the philosophy that underlines the failed whole language learning approach to reading, and the Department of ducation s agenda for mathematics.* [23]

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The bulk of this section is material taken from two texts: Review of the Interactive Mathematics Program (IMP)[24] by H. Wu, Berkley and A Preliminary Analysis of SAT-I Mathematics Data for IMP schools in California[25] by R. James Milgram, Stanford. The authors and endorsers of the letter to Secretary Riley recommend these texts, so at least 200 college level math teaches agree with the authors of references 24 & 25.[26]

Dr. Wu first reviewed the IMP program (IMP was called CPM at the time) in March of 1992[27]. Dr. Wu has been a college math teacher for over 20 years. During that time, Dr. Wu has been a quality control inspector for high school math programs[28]. That is Dr. Wu has had to deal with students that were poorly prepared for college calculus. Just as a college coach knows what skills he wants to see in freshman recruits, Dr. Wu knows what skills must be present in college freshman entering calculus class.

Dr. Wu examines the audience for high school math programs. Dr. Wu comes to the conclusion that there are two target audiences or groups:

> ***Group 1***: *those who will not go to college as well as those who will, but do not plan to pursue the study of any of the exact sciences (mathematics, astronomy, physics, chemistry), engineering, economics, or biology, and*
> ***Group 2***: *those who plan to pursue the study of one of the exact sciences, engineering, economics, or biology, and those who entertain such a possibility.* [29]

Dr. Wu describes how the needs of Group 1 varies from Group 2. As students enter high school, not all the students can grasp or be comfortable with college prep mathematics

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23 Ibid 8 . Page 58 of report.
24 “Review of Interactive Mathematics Program (IMP) at Berkley High School”, posted at http://www.math.berkeley.edu/~wu. 35 pages, Pages 84-118 of report.
25 “A Preliminary Analysis of SAT-I Mathematics Data for IMP Schools in California”, posted at ftp://math/stanford.edu/pub/papers/milgram/. 10 pages. Pages 119-128 of report.
26 Ibid 7, page 2 and 3 of 26. Pages 33 & 34 of report.
27 Ibid 24, page 1. Page 84 of report.
28 Ibid 24, page 26. Page 109 of report.
29 Ibid 24, page 5. Page 88 of report.

courses.[30] Dr. Wu is not advocating the past practices where school officials "decided" who would take college prep classes. Dr. Wu states:

> *"It should be firmly stated at the outset that I am not advocating the "trac ing" of mathematics classes in the usual sense of having the school authorities dictate who should be assigned to which trac . What I have in mind is a system whereby the high school students get the free choice of enrolling in either trac , and are allowed to switch between trac s later on. In other words, they should be allowed to choose their mathematics classes the same way college students do. One may call this* ***"tracking by choice"****. The issue of when this choice should be first offered to the students may be left to another discussion"*[31]

At present Cheney USD 268 does not offer separate math curriculums for non college math versus college math. Thus it appears that all of the square pegs are pushed through the round hole or visa-versa. Dr. Wu s review goes to great detail to explain the difference between the two requirements and the reader is encouraged to read Dr. Wu s review in its entirety. If any of the mathematical concepts discussed by Dr. Wu are unfamiliar to the reader, the author will be happy to explain them.

Dr. Wu considers IMP s approach to Group I students to be superior to traditional approaches in many ways.[32] Dr. Wu further states that IMP was targeted at Group 1 students:

> *After tal ing to the designers of the curriculum, I slowly came to an understanding of their objectives and their accomplishments. They aim this curriculum squarely at the students of roup 1* [33]

But Dr. Wu cites the following as areas where IMP should improve:

> *(a) The almost total absence of drills.*
> *(b) The inability of the IMP text to follow through in its presentation of new ideas.*
> *(c) The misrepresentation of mathematics through the abuse of open-ended problems and the de-emphasis of correct answers.*
> *(d) The presentation of mathematical pu les (also nown as brain-teasers) as straight mathematics.*
> *(e) The refusal to ac nowledge that mathematics could be inspired by abstract considerations.* [34]

Rather than plagiarize Dr. Wu s explanations, the reader is encouraged to read them at pages 11-14 of Dr. Wu s review.

Dr. Wu explains that Group 2 students, college math prep, are:

> *already motivated to learn. In addition, one must ta e into account the fact that their technical s ill must be sufficiently developed in order to meet the*

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[30] Ibid 24, page 4, Page 87 of report.
[31] Ibid 24, page 5, Page 88 of report.
[32] Ibid 24, page 10, Page 93 of report.
[33] Ibid 24, page 18, Page 101 of report.
[34] Ibid 24, pages 11-14, Pages 94-97 of report.

challenge they face in college. Thus the mathematics curriculum for this group can minimi e the sweet-tal and at the same time be more exacting. When viewed from this perspective, the IMP curriculum falls far short of the ideal [35]

Dr. Wu s examination of IMP for the college bound math student are as follows:

(A) Lac of depth and breadth in the topics covered.

( ) Insufficient emphasis on technical drills(Compared to football on p.20)

(C) Insufficient emphasis on precision....

1) *The exposition overextends itself in the direction of chattiness and informality. This leads to sloppiness. Precise definitions are not always offered, and when they are it is often done with almost an apology...*
2) *The IMP curriculum does not ma e any serious concentrated attempts at teaching students what a mathematical proof is all about....*
3) *This particular aspect in which IMP curriculum contributes to an erosion of the standard of precision is more difficult to encapsulate in a single phrase or sentence. It is more of an attitude, pervasive and ever present, that encourages excessive discursiveness and informality,*

(D) Over emphasis on group activities...

If a camel is a horse designed by a committee, what then is the ind of mathematics learned exclusively from compulsory group activities? [36]

Dr. Wu s historical perspective on IMP is unchanged after being familiar with the IMP (originally named CPM) curriculum for a decade:

Other than the expected pedagogical and expository refinements, the virtues and defects such as I perceived them and discussed in the report of the preliminary 1991 version made available to me bac in 1992 have in the main survived in the published version. In particular, the reservations against IMP detailed in III and I below regarding its lac of precision and its inattention to mathematical closure apply equally well to the 1997 text. Thus I believe this review still serves a purpose. My recommendation against the use of IMP for future college students in science, engineering, and (of course) mathematics is in my view as valid now as before. [37]

IMP emphasizes the relation between the highest level of mathematics studied in high school and college degree completion. A portion of an article by Dr. N. Webb at the University of Wisconsin summarizes this:

One of the most powerful predictors of ultimate completion of college degrees is the highest level of mathematics one studies in high school (Adelman, 1999). The number of years of high school mathematics is a better predictor than high school grades or standardi ed tests [38]

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[35] Ibid 24, page 15, Page 98 of report.

[36] Ibid 24, pages 15-25, Pages 98 to 108 of report.

[37] Ibid 24, page 2, Page 85 of report.

[38] Ibid 25, page 6-7, Pages 89-90 of report.

The only issue is that Dr. Adelman s results:

> *refers explicitly to Algebra II, and the courses subsequent to it were trigonometry, pre-calculus, and calculus (see page 17 of C. Adelman, Answers in the Toolbox, Academic Intensity, Attendance Patterns, and atchelor s Degree Attainment , .S. Department of ducation, Office of ducational esearch and Development, 1999 for details). Thus, implicit in Dr. Webb s discussion above is the presumption that IMP 3 and IMP 4 are equivalent in terms of preparation for college mathematics courses to the more traditional curriculum* [39]

There apparently exists no or very limited evidence to evaluate if IMP 3 and IMP 4 are equivalent to Algebra II, Trig, and Pre-calculus. Especially since IMP substitutes statistics and probability for algebra in the high school curriculum.[40] The lack of evidence is not surprising since IMP is used by a relatively small number of schools.

College classes on statistics and probability are typically not taught until the junior year. This allows the students to complete classes in calculus (3 semesters) and differential equations(1 semester). Calculus and differential equations are required for a full and rigorous treatment of statistics and probability. The reader is encouraged to examine a class schedule for KU or KSU to verify when statistics classes for engineers and mathematics majors are taught and what classes are required.

Research at the University of Colorado at Boulder found:

> *Algebraic preparation from high school is the single best predictor of a student s success in Calculus I at C - oulder. In turn, success in Calculus I is the single best predictor of whether a student will graduate with a degree from C s College of ngineering and Applied Science and potentially pursue such technical careers* [41]

In summary it can be said that a college math teacher at Berkley who has followed and reviewed IMP (CMP) for 10 to 11 years and has written and periodically updated a review of IMP math has found it lacking for potential college math students. Also a college math teacher at Stanford has pointed out the assumptions made in the IMP logic that more of any math classes are good when the actual study was done based on Algebra II, trig, and pre-calculus. And lastly, the University of Colorado at Boulder has found that Algebraic preparation especially,

> *solid algebra, trig, and precalculus s ills are crucial for success in college-level calculus* [42]

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[39] Ibid 25, page 7 and 8. Pages 125-126 of report.
[40] Ibid 25, page 10. Page 128 of report.
[41] "Bridging the Mathematical Gap Between High School and the University"; Dougherty, Anne and Nelson, Mary; University of Colorado at Boulder Community Affairs, CU's Outreach and In-service Program Guide for Colorado's Teachers. Pages 129 & 130 of report. Posted at: …/search.cgi?id=13&sort=grade&resource=0&lang=0&curr=5&grade=3&x=41&y=21&pla
[42] E-mail from Anne Dougherty to Douglas Riepe, June 4, 2002. Pages 131 & 132 of report.

The percentage of students entering California State University students requiring remedial mathematics courses has increased from 23 in 1989 to 54 in 1998.[43] Professor Milgram attributes the increase:

As an indication of the effect of programs li e IMP in California since 1989 [44]

Based upon Dr. Wu s analysis of IMP and the historical data from California, discerning educators, but especially those who have taken college calculus could question if students in IMP will receive a set of solid algebra, trig and pre-calculus skills equivalent to those taught in a traditional setting.

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Two studies are quoted in this section. The primary one was completed by Dr. Milgram and a study conducted by N. Webb with rest of the story data analysis by Kim Mackey.

The 83 schools using IMP in California in the 1998-1999 school year were contacted to find out their level of satisfaction with the program. The schools were contacted to determine if IMP was the only math program in use and how long it had been in use (students taking the SAT had only taken IMP). These questions narrowed the list of schools to 33. The SAT scores for these schools were reported by the California Department of Education. The results for the schools that were satisfied with the IMP curriculum is given below:

> *In short, no discernible improvement in the overall performance of the students in these IMP schools against the measure of success in the SAT-I math test could be verified. It is worth noting that these schools consistently scored from 14 to 21 points below the overall state means, 14 points below the baseline years 1989 and 1990 and then more thereafter.* [45]

Dr. Milgram points out that between the school list for 1997 and the school list for 1999:

> *Thus, combining the two lists we find that at least 15 schools in the 1997 list have since changed to other curricula, and it is li ely that a further number de-emphasi ed the program* [46]

The test score data are given in tabular form in Dr. Milgram s report. The reader is invited to study the data. It is interesting that the IMP sites not satisfied (planning to change curriculum, limited use, for at risk students only)[47] with IMP had higher SAT-1 scores, though still below the California state average.[48] Dr. Milgram s data cover a larger number of schools than the IMP web site data. There are a number of possible explanations besides IMP for the below average test scores from the schools using IMP. It is not known by the author with what frequency schools typically change math curriculum, thus the number of schools quoted in footnote 45 may or may not be significant.

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[43] Ibid 25, page 6. Page 124 of report.
[44] Ibid 25, page 6. Page 124 of report.
[45] Ibid 25, page 3. Page 121 of report.
[46] Ibid 25, page 4. Page 122 of report.
[47] Ibid 25, page 2-3. Pages 120 & 121 of report.
[48] Ibid 25, page 4. Page 122 of report.

The following work was conducted by Kim Mackey and describes the student groups used in a well known IMP study:

> *One definitive study that can tell us more about IMP s effectiveness is a study quoted by IMP in their spring 1995 valuation update. This study, report number 95-4 is Impact of the Interactive Mathematics Program on the etention of nderrepresented Students: Class of 1993 Transcript eport for School 2: Hill High School by orman Webb and Marit a Dowling. xcerpt from this report have also appeared in the winter 1996-1997 edition of WC Highlights and is available on-line at the WC website. oth the IMP update and the WC highlights article ma e much of the fact that SAT scores for IMP students were nearly identical with those of non-IMP students in one carefully match group. ut what about other groups at this high school? When comparing student groups it is always important to consider the student population from which they are drawn.* [49]

The rest of the story is that based upon 7th grade Comprehensive Test of Basic Skills (CTBS) scores. The students taking IMP math, except the other category, were higher, statistically higher, than those not selected for the program:

> *Thus, IMP students in most categories had higher math achievement as 7th graders than non-IMP students who too algebra, and other math classes. (not included in the study were a 25 percent group of the population at the high school who too geometry or higher math classes as freshmen)... The conclusion seems to be that lower achieving students in the non-IMP groups made up a tremendous amount of ground on the IMP students in terms of math achievement between the 7th grade and the time the SAT was ta en, usually in 11th grade. rom this data it appears that IMP is a great leveler, not by pushing lower achieving students up but by holding higher achieving students bac .* [50]

There would appear to be data available that would suggest that test scores do not show improvement. The proof will be in the pudding as the USD-268 evaluates testing results to see if it can replicate report 95-4.

A most interesting comment comes from reviewing the effort of math reform in California:

> *Cohen and Hill point out that many teachers, including those who li ed much about the new approach, continued to practice conventional math instruction. Considering that the state decided in the late 90 s, to reemphasi e computational s ills and the li e, these teachers, it now appears, were more wise than stubborn in not buying into the new approach* [51]

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49 IMP: A Student's View, with Comments by Kim Mackey, page 5. Page 137 of report.
50 Ibid 49, pages 5 and 6. Pages 137 & 138 of report.
51 "Reviews: Surprise Endings", David Ruenzel, *Education Week,* March 1, 2002, web site: http://www.edweek.org/tm/tmstory.cfm?slug=06review.h13&keywords=math. Page 142 of report.

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The use of calculators in college math classes was once strictly forbidden. Math and statistics classes were theory classes and numerical methods classes were where computers were used to apply the theory to problem solving issues.

The author contacted area colleges and talked to freshman calculus coordinators (catch words to use if you want to talk to the professors who set up the testing for all of the introductory calculus classes at a particular university), department secretaries, and the professor who taught the author differential equations.

KSU, KU, Newman University, Wichita State University, and the University of Nebraska at Lincoln were contacted for information concerning calculator usage.

KSU and KU reported that to be admitted to a calculus class a student had to have an ACT math score of 28 or higher. If Cheney begins to teach calculus in the high school, the faculty should consider having a similar cut off. Based upon his high school class, the author estimates that Cheney will likely have 5 to 10 students each year that are qualified to take the class. If the class is taught to land-grant college standards some of those will have to switch classes or fail.

The KSU freshman calculus coordinator reported that the TI-83 is used only to check work . The students must show all steps in derivatives and integration by hand. The instructor stated that the ability to do algebra by hand is very important. The instructor also stated that calculator usage should be restricted especially for freshman and sophomore high school students. Not all freshman calculus coordinators allow the use of calculators on tests at KSU.[52] Time is provided during class for students to learn how to use the calculators, so prior experience apparently is not an advantage.

The department secretary at KU reported that the TI-86 was used in class.[53]

The Wichita State University Mathematics Department does not require calculator usage in their classes and the TI-83 is the calculator of choice.[54]

Newman University s freshman calculus coordinator reported that the calculus tests at Newman contained sections where no calculators were used and sections where calculators were allowed.[55]

The University of Nebraska-Lincoln Mathematics Department secretary reported that in college algebra and above TI-83 is allowed and TI-86 is used in calculus, but the students

[52] Phone call from Douglas Riepe to Dr. Todd Cochran, Freshman Calculus Coordinator, KSU, (785)532-6750 x-0565, January 2002,
[53] Phone call from Douglas Riepe to KU Mathematics Department (785)864-3651, January 2002.
[54] Phone call from Douglas Riepe to Wichita State University Mathematics Department, January 2002, (The phone number is in the Wichita phone book).
[55] Phone call from Douglas Riepe to Dr. Joel Burgeson, Freshman Calculus Coordinator, Newman University, (316) 942-4291 x-162, January 2002

must do differentials and integration by hand.[56] The use of calculators in college classes was briefly discussed with Professor Gordon Woodward. Professor Woodward reiterated what the department secretary had said, that the students are required to show derivative work by hand and the calculator is used to double check or do simple calculations. Dr. Woodward said that if the student is dependent on calculators then they are in trouble.[57]

KSU is not the only school in the U.S. where a student potentially may not be able to use a calculator. A student from a Core Plus curriculum made the following comment:

> *The reason I have not ta en any math courses in college is because the math I learned in high school does not apply to college math. I used the TI-82 for linear programming and colleges do math by hand, which is very tedious. Colleges all need to change ...* [58]

Core Plus was rated as Exemplary by the Expert Panel.[59]

Another student stated:

> *Core Plus taught me math well however, it did not teach me how to show my wor . ecause of that I failed college math.* [60]

Another comment on Core Plus:

> *The high school math program was good. The problems arose because colleges haven t re-structured their math programs accordingly.* [61]

Other countries around the world use calculators in the class room, the only problem is that most of them are in the bottom half of the math testing with the U.S. Ten of 11 nations with scores below the international average allowed the usage of calculators everyday.[62] This contrasted with the fact that 3 of the top 5 countries (Japan, Belguim, and Korea) do not allow calculators at all.[63]

A comment by a college freshman sums up the risks of heavy calculator usage in high school:

> *I feel as though three years of math at high school were lost, says Amir mami, a freshman at alama oo College. ven though he graduated with a 3.4 grade point average, he has a wea understanding of math. The*

56 Phone call from Douglas Riepe to the Mathematics Department, University of Nebraska-Lincoln (402)472-3731

57 Phone call from Dr. Gordon Woodward, (402) 472-3731 x-7239 to Douglas Riepe, January 15, 2002.

58 "Outcomes Analysis for Core Plus Students At Andover [Michigan] High School: One Year Later", R. James Milgram, Department of Mathematics, Stanford, University, website: ftp://math.stanford.edu/pub/papers/milgram/andover-report.html. page 10 of 32. Page 153 of report.

59 Ibid 11, page 2. Page 62 of report.

60 Ibid 58, page 11 of 32. Page 154 of report.

61 Ibid 58, page 11 of 32. Page 154 of report.

62 "Calculators in class: freedom from scratch paper or 'crutch'?, *Christian Science Monitor*, May 23, 2000, Mark Clayton, page 179 of report. website: http://www.csmonitor.com/sections/learning/mathmelt/p-7story052300.html.

63 Ibid 62, page 4 of 4. Page 179 of report.

answers in high school were written paragraphs, not equations or number crunching. ou learn to depend on our TI-82 calculator. [64]

The last reference in this section is for a study, which concerned the question of Could the use of technology increase test scores? Most of the schools in the study have spent large sums of money on technology, the answer?

> *They (authors) also point out that the one school in their study that has been reluctant to invest in technology, enaissance High in Detroit, also has the best academic trac record.* [65]

The initial paragraph of the review is particularly interesting:

> *Several years ago, three researchers at S I International s Center for Technology in Learning in Menlo Par , California, set out to discover what it ta es for urban high schools to effectively incorporate technology into the curriculum. The answer, as they tell it in this engaging boo , is a discouraging everything and more . Indeed, school administrators reading this volume may come to see computers and other new technologies as something a in to a pricey new boat that proverbial hole in the water into which the unfortunate owner pours great sums of money. This sense of foreboding is rather ironic given that the authors are boosters of educational technology and its potential to improve teaching and learning.* [66]

nteg ate at og a

USD-268 personnel make much of the fact that the rest of the world uses integrated math programs. But as Dr. Richard Askey points out in his article:

> *There are two types of integrated programs. There are those li e the ones in Singapore, which teach some arithmetic or algebra and some geometry each year with connections among them used. This is something that mathematicians have been proposing for a very long time (Mathematical Association of America 1923). Similar programs exist in many other countries. Singapore is mentioned because textboo s in that country are good, are written in nglish, and are accessible in the nited States. (See* www.singaporemath.com.)
>
> *The other is illustrated by some of the ational Science oundation (S)-funded programs from the S call for new programs in the early 1990s. These have a different focus, trying to teach mathematics in the context of*

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64 Ibid 62, page 4 of 4. Page 179 of report.

65 "Reviews: Vision Quests, THE CONNECTED SCHOOL" reviewed by David Ruenzel, *Education Week*, April 2002, page 2 of 5. Page 181 of report. website: http://www.edweek.org/tm/tmstory.cfm:slug=07books.h13&keywords=math

66 Ibid 65, page 1 of 5, Page 182 of report.

real-world problems. It is the second type of integrated program that has become the focus of controversy in the nited States. [67]

Professor Woodward mentioned that New York state has an Integrated math program[68], which Professor Woodward described at being good .

lo ing o g t

The author is left to ponder why USD-268 selected a largely unknown curriculum, a curriculum which had been reviewed for nearly 10 years (since 1991 or 1992) as unsuited for college prep, a curriculum which really didn t claim to improve standardized test scores, if the goal was to improve ACT scores or the odds of USD-268 students excelling in college, why not contact the area colleges and see if a consensus could be formed, ie., what high school math program is consistently mentioned as being the best? Another tack would be to survey the surrounding school systems and determine the curriculums used and the teaching methods used to teach them.

In talking with members of the community, it is mentioned that small groups have been used to teach math. If USD-268 wants to prepare students for college then, perhaps the classes should mimic college classes. In college there will be no small groups in mathematics classes, the professor will lecture & the students will take notes. The class will not work on problems for 4 to 5 weeks. Students will solve problems in minutes. The professor will assign grades, not classmates. There seems to be a number of disconnects between the math curriculum currently used in USD-268 and what a student can expect to face in college.

t o oti ation

The author has undertaken this literature survey after finding that the Board of Education of USD-268 and perhaps the administration and staff of USD-268 selected and supported a math curriculum based, (from comments of there was a nice presentation and the sole supporting literature was an IMP news letter) apparently on a sales presentation and quarterly newsletters from the vendor. Having been through calculus, the author realizes the importance of a firm mathematical foundation in college.

As stated by Dr. Klein (reference 23) the approach to teaching math used by IMP and other NSF funded curriculums is similar to whole language learning . Whole language learning has been taught in kindergarten at USD-268 during the years 2000-2001 and 2001-2002. My son Evan was enrolled in kindergarten during the year 2001-2002. Evan's reading skills are poor. He can t sound out words. He guesses at words. Among my three children he received the poorest kindergarten education. My other children had the advantage of phonics based reading programs. Apparently the whole language learning curriculum was forced on the grade school and teachers by the central administration of USD-268. This despite:

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[67] "There Are Integrated Programs, and There are Integrated Programs", National Council of Teachers of Mathematics, January, 2001, website: http://www.nctm.org/dialogues/2001-01/20010113.htm. Page 185 of report.

[68] Ibid 57.

Compounding the problem is the fact that the reading instruction to which your daughter was forced to submit is not phonics-based; rather, it is based on word-memorization, also known as whole language. The research indicates that phonics-based instruction significantly lowers the incidence of reading problems. I d strongly advise that for next year, you find and enroll your daughter in a school that respects the differences between children and embraces phonics, not fad. [69]

This coming year, USD-268 is going back to a phonics based curriculum in the kindergarten. The children that were taught with whole language learning are behind their peers at other schools, but with patient elementary teachers, they probably will catch up, eventually, probably without lasting effects. Interesting USD-268 is not trumpeting this change of curriculum in the newspaper nor is it offering remedial phonics for the students that were left behind by the whole language learning curriculum used over the last 2 years.

As a parent of a student that at present is enrolled to attend high school at USD-268, I fear that the same whole learning syndrome that has left my son Evan with poor reading skills will leave my daughter with poor math skills. This is simply unacceptable. The concerned parent is left with the following alternatives: 1)Move to another district, all of the area districts contacted have a standard curriculum, 2)Attend a neighboring school as an "out of district" student, 3)Attend a boarding school, 4)Home school, 5)Tutor children at home for the duration of high school. Option 5 would give the student adequate math skills, but science classes would still be dragged down by the lack of math taught to the remaining (IMP) students. The experiences of others (in California) indicate that the IMP curriculum will likely leave students without the math abilities required for science classes as the following quotations illustrates:

ST D TS: It is uncommon to find students attending a parent-teachers meeting at 7pm but such a meeting at TMASH(S SD s Thurgood Marshall Academic High School) on 3rd December 96 was attended by several students. The discussion was suitability of IMP as the only mathematics offering for an academic school. Quotes Science class has stopped (for the second year in a row) because students didn t now necessary mathematics [70]

The essay continues with feedback from parents:

Chemistry and Physics teachers are having to provide extra mathematics drills [71]

The experience of those in California was that IMP was unable to provide the basic skills needed in chemistry and physics classes. It would appear that the concerned parent would also have to tutor the physical sciences as well as math to compensate for the

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69 “What’s a Parent to Do?”, (Labeling children isn’t helpful), *The Wichita Eagle*, May 9, 2002, John Rosemond. Page 188-189 of report

70 “Interactive Mathematics Program Manifesto on an Experimental Concept Gone Awry”, Shaumen Datta, page 1 of 6, website: http://mathematicallycorrect.com/impsf.htm . Page 190 of report.

71 Ibid 70, page 5 of 6. Page 194 of report.

effects of IMP. Thus option 5 appears to be unacceptable. It is the author s sincere hope that the Board of Education for USD-268 will take a look at IMP and evaluate whether it merits continued use at USD-268. Those who don t learn from history are doomed to repeat it.

t o a e i a g o n

The author s background includes but is not limited to the following:

ig ool:

Member Honor Society
High School Class Valedictorian
UN-L Regents Scholarship (1-year)
ACT test score of 32 (the ACT score has since been recentered)
ACT math test score of 36 (a score of 32 was at the time a 99.9 percentile)
Winner of Inter-High scholastic contests
4 year football letterman, district champion-wrestling
Co-winner of conference academic award for high ACT test score.

ollege

Freshman chemistry student of the year at the University of Nebraska Lincoln
Graduated with 3.9 average while majoring in Chemical Engineering
Graduated with honors
Member Tau Beta Pi
Master of Science in Engineering Thesis Computer Control of Laboratory Equipment

Copies of transcripts available at author s residence.
Calculus books, 1 twenty year old book and 1 new book used by Friends University.

o e ional

Beginning and advanced SPC training
Understanding Industrial Experimentation
Completion of training for Six Sigma Statistical Black Belt

A PLAN for IMPROVING the QUALITY and EFFECTIVENESS of EXPOSITION in HIGH SCHOOL MATHEMATICS

Presented by Frank B. Allen
Professor Emeritus, Elmhurst College
National Advisor, Mathematically Correct

In order to raise the level of student achievement in secondary school mathematics, which everyone agrees is urgently necessary, there must be major improvements in the expository procedures employed by teachers. Accordingly, we specify the essential attributes of the teachers we need to bring about such improvements (Section I), offer some classroom-tested suggestions for their consideration (Section II), emphasize the current need to reinstate proof in secondary mathematics (Section III), outline the essential role of the mathematical community (Section IV) and describe certain external conditions that must exist in order for well-prepared teachers to be successful (Section V).

This document is contained in three .pdf files:

A PLAN for IMPROVING the QUALITY of EXPOSITION in HIGH SCHOOL MATHEMATICS

"It is by means of names and numbers that the human understanding gains power over the world" Oswald Spengler in Decline of the West.

In order to raise the level of student achievement in secondary school mathematics, which everyone agrees is urgently necessary, there must be major improvements in the expository procedures employed by teachers. Accordingly, we specify the essential attributes of the teachers we need to bring about such improvements (Section I), offer some classroom-tested suggestions for their consideration (Section II), emphasize the current need to reinstate proof in secondary mathematics (Section III), outline the essential role of the mathematical community (Section IV) and describe certain external conditions that must exist in order for well-prepared teachers to be successful (Section V).

Section I: Essential Attributes of the Kind of High School Mathematics Teacher We Need.

A) Knows his subject. Has completed a strong undergraduate major in mathematics and, hopefully, has a minor in Physics or some other subject where mathematics is applied. Was especially attracted to mathematics by such courses as Algebraic Structures, Linear Algebra, Topology and the other "proof" courses" that follow calculus. Since many college majors no longer include a course in geometry such as that formerly derived from Coxeter (R 1) or Atschiller-Court (R 2), it is now necessary to assert that one of the most important of these "proof courses" should be a course in College Geometry. This course should

(1) Present advanced topics in Euclidean Geometry.

(2) Provide an introduction to Non-Euclidean Geometry.

(3) Stress "The Importance of Certain Concepts and Laws of Logic for the Study and Teaching of Geometry" Lazar (R 3).

{Reading this volume, which was Lazar's Ph.D. thesis, revolutionized my teaching. I hope that Nathan Lazar will, one day, be recognized as one of the great seminal thinkers in mathematics education.}

(4) Review and extend the student's understanding of the axiomatic method, in which proofs are forged by logic on a postulational base.

(5) Consider "incidence relations"and the danger of mak-ing unwarranted inferences from a drawing. (R 4)

(6) Provide experience with the highly instructive, open-ended construction problems found in the older geometry texts. (See Appendix 3)

The inclusion of such a required course would gradually remedy the existing intolerable situation where, very often, the teacher's knowledge of geometry does not extend beyond the covers of the high school text he is using!

A teacher who has not successfully completed these "proof courses" has no grasp of its ESSENTIAL and CHARACTERIZING properties of mathematics and IS NOT QUALIFIED TO TEACH MATHEMATICS IN HIGH SCHOOL.

B) Has taken courses in the Teaching of Mathematics which described various methods of presentation.

C) Has some knowledge about and great interest in the history of mathematics, including the contributions of the great mathematicians, the influence of the Greeks and the discovery of non-Euclidean geometry. (" A Concise History of Mathematics" by Dirk J Struik (R 5) provides an excellent introduction.)

Has perspectives gained by study of the history of mathematics education since 1900 which

*** Explores the various and sundry theories about how to teach school mathematics which have been promoted by our Schools of Education during this century.

*** Assesses the theories that Columbia University promulgated in the twenties and notes that some of these are now coming back under the banner of "Reform". [see "Orthodoxy Masquerading as Reform", E. D. Hirsch, Jr (R 6)].

*** Compares the texts used in the 30's and 40's with those used today.

*** Examines the reasons for the rise and fall of the "New Math" in the sixties and seventies (R 7)

*** Questions "Radical Constructivism" as the basis for NCTM sponsored reforms.

D) Loves mathematics because it is EXACT, ABSTRACT and LOGICALLY STRUCTURED. Considers these to be the ESSENTIAL and CHARACTERIZING properties of mathematics which enable it, when properly taught, to make unique and indispensable contributions to the education of all youth. Is determined to cherish these properties and believes that it is time to lead his high school students to understand and appreciate them.

Exactness is to be sought more than ever in this unforgiving digital world where even a dot out of place can destroy you. Moreover the student should realize that there are many situations in "real life" where there is literally no room for error in mathematical thinking.

The abstract quality of mathematics produces power by developing valid generalizations which are derived from many specific examples but are stated without reference to them and, hence, are applicable to many other specific cases.

Thus, the observation that $\frac{4+9}{2} > \sqrt{4 \cdot 9}$ and $\frac{7+13}{2} > \sqrt{7 \cdot 13}$ leads eventually to the general abstract statement $\frac{a+b}{2} > \sqrt{a \cdot b}$, *"The arithmetic mean of two different positive real numbers is greater than their geometric mean."*

The structured character of mathematics enables us to derive new facts (conclusions) from previously established facts (hypotheses) by building logical arguments (proofs). This proof process establishes connections between existing facts and, builds structure by adding to our fund of known facts. Proof, properly introduced, DOES NOT make mathematics more austere, forbidding and difficult. On the contrary, it can be an exciting game which provides the only path to understanding.

E) Is resolved to make mathematics interesting for his students not by making it easy (mathematics properly taught is difficult. (R 8)

not by making it continuous fun (The learning of mathematics requires a lot of hard work.)

not by down-playing its essential and characterizing properties because they are deemed to be too austere,

but by providing clear explanations which build the student's confidence by making mathematics seem reasonable. A mathematical concept, once understood is no longer intimidating. For example, the student who can explain why the $2ab$ term appears in the expansion of $(a+b)^2$ has acquired a satisfying understanding which liberates him from the frustrating process of trying to rely on memory to supply facts that he doesn't understand. For the student who must so rely, algebra is just a bag of soon-forgotten tricks or, as Henry James once said "A low form of cunning".

F) Is resolved to give due emphasis to the fact that the manifold APPLICATIONS of mathematics to the solution of practical problems make it a prerequisite to the successful study of virtually all branches of science. Will search for and spend some class time on "real world" problems whose solutions illuminate the mathematics in the course syllabus and which appeal to all or most of his students. Realizes that such problems hold great interest for students who continue to study mathematics for career reasons, most of whom follow geometry with a course in pre-calculus, It is also important for these students to realize that it is their understanding of the basic mathematical theory and techniques that will serve them best when it comes to applying mathematics to "real world" situations. Sometimes problems that have no apparent practical significance make surprising contributions to this understanding.

G) Teacher as director of learning. The high school mathematics teacher that we need considers the teacher's role to be that of "an instructor, one who imparts knowledge, a director of learning" (Webster) and intends to be a teacher in this traditional and, until recently, generally accepted sense. Chose teaching as a career because he believes with Professor Sylvia Feinburg that high school students "desperately need leadership guidance and stimulation from adults"(R 9). Expects to provide this leadership in the field of mathematics and to accept the responsibility and accountability that go with it. Regards exposition,"the art of presenting, explaining or expanding facts or ideas" as the mathematics teacher's principal function. (See Section II below). Sharply questions the romantic notion that high school students learn best when they are allowed to "discover" facts for themselves in cooperative learning (CL) sessions without direct instruction by the teacher. Feels that this demotes the teacher to the role of "Facilitator", a role he does not relish. ("Let me know when you need new batteries for your TI92.")

Is aware the CL is often defended on the grounds that it parallels practices used in business and industry, but sees a vast difference between a group of well-motivated professionals pooling their knowledge to solve a problem and a group of high school students working together to acquire basic knowledge that might be more efficiently acquired from direct instruction, Feels that occasaional use of CL, *in conditions where it is appropriate*, is enough to inculcate the idea of cooperation. In order to profit from direct instruction a student must learn to listen and to follow directions. These attributes are also essential for success in the business world. Would, nevertheless defend the more extensive use of CL by teachers who are convinced that it leads to better test results.

Understands that, as director of learning, he has the right to use any of the various methods of presentation at his command (see B above) Does not want specific methods of presentation such as CL prescribed for him. Such prescriptions constitute a misguided effort to standardize something which should not be standardized--the TEACHING of mathematics. Teaching is a highly individualistic art where each teacher must be free to use the methods of presentation that work best for him, as indicated by his student's performance on standardized tests.

H) Is resolved to be fair and objective in assigning course grades. Realizes that students are strongly motivated when they understand the grading system and believe that grades are objectively assigned. Believes that the course grade should be a valid measure of only one thing, the student's degree of mastery of the course syllabus. Considerations such as race, "equity", social status or the teacher's perception of the student's "mathematical disposition" should never affect the course grade.

H1) Plans to continue the traditional uses of the assessment system to diagnose learning difficulties and to provide information that can be used to improve instruction. (R 10)

H2) Eschews group testing because it (i) destroys the integrity of individual course grades (ii) impairs their validity as predictors of future performance and (iii) makes the assignment of individually prescribed remedial work impossible.

H3) Confident of his own ability to write objective tests that are valid measures of the student's understanding of the syllabus, readily accepts the premise that expert test writers can do the same. Would agree that the success of teaching and learning must be defined in terms of test results.

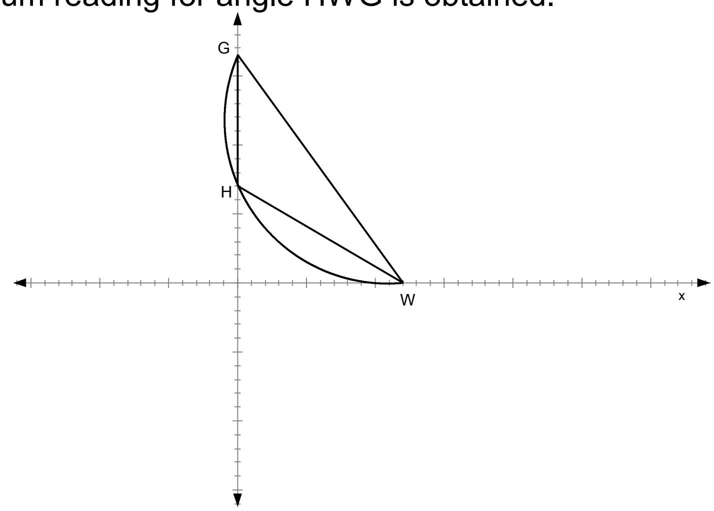
I) Recognizes the indispensable role of well-planned practice, repetition, review and drill in the development of the work and study habits that are necessary for the acquisition of mathematical knowledge. The student must not only understand routine algebraic operations, he must acquire facility in performing them--for himself without too early reliance on electronic devices. He must learn to perform routine tasks correctly and on time. This develops confidence, the feeling that "I can do that." It also frees the student's mind to deal with higher level challenges. After describing the "student skills" her sons acquired in Japanese schools, Carol Glick adds "Skills in the acquiring, may be uninspiring, but once possessed they are liberation itself." (R 11)

While drill can be overdone, there is no essential conflict between concept building and intensive practice on mathematical operations. Each reinforces and illuminates the other.

J) Is highly proficient in the use of graphing calculators, computers and dynamic geometries such as Geometer's Sketchpad and Cabri Geometry II, which is now incorporated in the TI92. Intends to use these electronic devices to teach mathematics -- not vice versa. He regards the high school years as a time for developing understanding of the classical fundamentals (theorems) of mathematics. Students who have acquired this understanding will have no difficulty learning the "Training Manual" techniques necessary to operate successfully in the rapidly changing electronic world. Thousands have done so without formal instruction. Very few have learned mathematics that way. While he fully realizes that students are fascinated by the gleaming computer labs that are appearing in our more affluent communities, he harbors certain concerns.

J-1) Does not want students to regard the graphing calculator as an "authority" when dealing with proof. For example $\sin 2x = 2\sin x \cos x$, NOT because the graphing calculator gives the same graph for $y = \sin 2x$ and $y = 2\sin x \cos x$, but because we get this result when we let $y = x$ in our PREVIOUSLY PROVED formula $\sin(x+y) = \sin x \cos y + \sin y \cos x$. The student's reaction to the same graph phenomenon should be "Look, the TI92 got it right." The student must realize that it is not a proof.

J-2) Does not want dynamic geometries to be used to trivialize problems which are highly instructive when solved by classical methods. Example: We are asked to find the point W on the x- axis where GH subtends the maximum angle (see diagram). Using Sketchpad we can solve this problem by simply moving W along the x-axis until the maximum reading for angle HWG is obtained.



We don't have to understand how the theorem "*If from a point outside a circle, a tangent and secant are drawn, then the product of the secant and its external segment is equal to the square of the tangent,*" is involved. In fact, we don't have to understand anything. There are no connections with or appeals to any previous theorems. Since the student must make such connections and appeals in order to gain understanding of the structure of geometry, this Sketchpad solution is not satisfactory. This is a misuse of a dynamic geometry. Properly used, dynamic geometries enable the student to make exciting explorations and to generate conjectures whose truth value must be determined by applying the laws that govern mathematical proof.

K) Problem solving. Recognizes the pedagogical value of taking students "behind the scenes" by showing them how problems are CONSTRUCTED. This takes away some of the mystery and sheds some light on what problems are for. Moreover, problem construction is a direct operation whose difficult inverse is problem solving. Therefore, experience with problem construction should precede or at least accompany the student's encounter with problem solving.

L) Mathematics teacher as EXPOSITOR. The mathematics teachers we need are able to provide students with

Intensive training in the precise use of language and symbolism. This includes gradual mastery of 'The Language of Mathematical Exposition" (See Appendix 2)

A systematic introduction to PROOF which evolves from considering WHY certain statements are true or false.

The ability to state CONJECTURES whose truth value must be determined. This fosters the spirit of inquiry.

A sensitivity to inconsistency developed by testing conjectures and sometimes invoking counterexamples.

The ability to devise and investigate open-ended problems that evolve from the mathematics in the syllabus.

Section II: A Classroom Teacher's Suggestions for Consideration by the High School Mathematics Teachers We Need.

Before I present these suggestions let me say that I do not expect to be alone in this project. I expect a great deal of support from the many experienced teachers and from mature mathematicians who have recently taken the time to study the present state of instruction in school mathematics, as it has been affected by the NCTM Standards (See Section IV below). I believe that these mathematicians and teachers will strongly endorse the need for producing mathematics teachers with, substantially, the attributes set forth in Section I. I believe, too, that informed parents demand such teachers and intend to have them, even if they have to resort to private schools. If this is true, we have general agreement on our objectives. We can rest assured that the mathematical community will find ways to achieve them. It would be inappropriate for me, or anyone, to tell them how to do it. It is with this in mind that the following ideas and procedures are OFFERED FOR CONSIDERATION, with the hope that it will be found that they make some contribution to the art of mathematical exposition. They are derived from my forty-six years of experience as a teacher of mathematics in high school and college, which is briefly described in Appendix 1.

The reader must be prepared to encounter some concepts and procedures which are unusual in the sense that they do not, so far as I know, appear in any current texts.

Introduction to Proof

We begin with the implication $p \overset{?}{\Rightarrow} q$ which focuses attention on why one statement follows from another and thus furnishes the essential building block of proof. If p is true and we can supply a reason for the question mark, we have a proof of the statement q. *(Appendix 2, 17ii)*. The student should be trained to consider the converse and contrapositive forms of an implication. For example, the student should consider why the implication $(a=b) \Rightarrow (a^2=b^2)$ and its contrapositive $(a^2 \neq b^2) \Rightarrow (a \neq b)$ are true while its converse $(a^2=b^2) \Rightarrow (a=b)$ is false.

Now implications enable us to progress from the known to the unknown, and they can sometimes be linked together linearly thus: $a \Rightarrow b \Rightarrow c \Rightarrow d \Rightarrow e \Rightarrow f$. If each of these five implications is valid we have proof that $a \Rightarrow f$. Example: Solve $5+\sqrt{x+7}=x$ for x :

$(5+\sqrt{x+7}=x) \overset{(1)}{\Rightarrow} (\sqrt{x+7}=x-5) \overset{(2)}{\Rightarrow}$

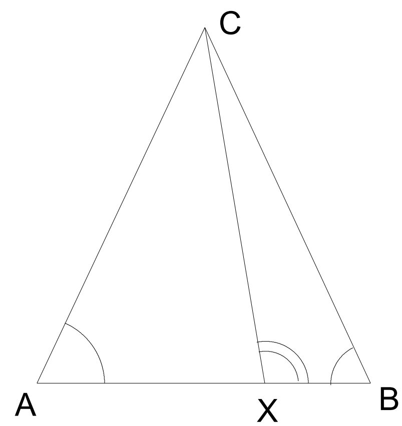
$(x+7=x^2-10x+25) \overset{(3)}{\Rightarrow} (x^2-11x+18=0) \overset{(4)}{\Rightarrow}$

$[(x-9)(x-2)=0] \overset{(5)}{\Rightarrow} (x=9 \vee x=2)$ or that $\{x|5+\sqrt{x+7}=x\} \subseteq \{9,2\}$.

We do not have a proof of the converse, $(x=9 \vee x=2) \Rightarrow (5+\sqrt{x+7}=x)$ because the argument is not reversible. In addition to supplying the numbered reasons the student should explain why it is not reversible and determine that the solution set is $\{9\}$.

This example suggests that the study of proof should begin in algebra. Such proofs require reasons and, as we proceed, it gradually becomes evident that the reasons are supplied by the properties of an ordered field and theorems based on these properties. So, at some point in the algebra course the properties of a field should be listed as they were in *Pearson-Allen, A Logical Approach, 1964.*

Sometimes flow proofs involve bracketed statements. Example:
Prove: *The length of a line segment which joins the vertex of an isosceles triangle to a point in the interior of the base is less than the length of either leg*.



In terms of our figure we must prove:

$$\left.\begin{array}{c}\text{ABC is a } \triangle\\ \overline{\text{AC}}\cong\overline{\text{BC}}\\ \text{X is an internal point of } \overline{\text{AB}}\ (\text{X}\in\overline{\text{AB}})\end{array}\right\}\Rightarrow \text{CX}<\text{CB}$$

$$\left.\begin{array}{l}\left.\begin{array}{c}\text{ABC is a }\triangle\\ \text{X}\in\overline{\text{AB}}\end{array}\right\}\overset{(1)}{\Rightarrow}\angle\text{CXB is an exterior angle of }\triangle\text{CXA}\overset{(2)}{\Rightarrow} m\angle\text{CXB} > m\angle\text{CAB}\\ \left.\begin{array}{c}\text{ABC is a }\triangle\\ \overline{\text{AC}}\cong\overline{\text{BC}}\end{array}\right\}\overset{(3)}{\Rightarrow}\angle\text{CAB}\cong\angle\text{CBA}\ (\angle\text{CBX})\overset{(4)}{\Rightarrow} m\angle\text{CAB} = m\angle\text{CBX}\end{array}\right\}$$

$$\overset{(5)}{\Rightarrow} m\angle\text{CXB} > m\angle\text{CBX}\overset{(6)}{\Rightarrow}\text{CX}<\text{CB}$$

Why the flow format? Well, it is my position that beginning students need some way to understand the structure of a proof, and therefore should not begin with the nebulous "essay" form where it is hard for the teacher to provide a diagnostic analysis. This has long been recognized and one result was the "ledger" or two-column format. Indeed these have come to represent and characterize proof for many teachers. This is unfortunate because two-column proof in geometry has become a target of ridicule and this has served to discredit, in some measure, the whole idea of proof. Also it tends to exclude the study of proof from ninth grade algebra where it should begin and where it is much more easily presented. Ledger proofs are better than no proofs at all (which seems to be the alternative now), but they have serious defects. Clearly our building block $p \Rightarrow q$ does not fit into the two-column proof format. Moreover, two-column proofs do not show the structure or flow of the argument. When you get to step 10, just which of the previous steps are you counting on? I used to insert numbers and draw lines all over the proof in order to explain this. Most unsatisfactory. Then I saw a flow proof somewhere and began to use linkages of statement connected by implication symbols -- sometimes linear, sometimes bracketed -- so that the conjunction of several statements could imply a new statement. The effect was startling! Once I began using flow proofs, my students would not allow me to use anything else until they finally gained enough confidence to translate flow proofs to essay proofs, which, of course, was my objective in the first place.

To return to the example above, students should seldom be confronted with such a proof, unless it is for the purpose of supplying reasons in an open-book quiz situation. Instead, the proofs should be constructed in class with teacher and students cooperating in the construction. Sometimes we work backward from the conclusion in search of sufficient conditions. Sometimes we work forward from the hypothesis in search of necessary conclusions. We try to make the two lines of reasoning meet to form a seamless sequence from our hypothesis to our conclusion, i. e. a valid proof. If we succeed we have considerably enhanced our own understanding and we have a proof that other people can understand and one that is easily translated to essay form. In making this translation the student is not writing about mathematics, he is learning to use language precisely to write mathematics. One does not learn mathematics by writing about it anymore than one learns to play the piano by writing an essay about the *Moonlight Sonata.*

Note on Indirect Proof

Such arguments as *"Jim must have survived that plane crash because, if he had not, his name would be on the victim list, and it is not there,"* are commonly used in every day conversation. They have the form

$$\text{(iii)}\left.\begin{array}{c}(\sim p \Rightarrow q)\\ \sim q\end{array}\right\} \Rightarrow p$$

(*Appendix 2, 17iii*). We verbalize this as *"If a contradiction of a statement p implies a false statement, then p is true."* The clarifying point that (iii) is the second contrapositive of the direct argument

$$\text{(ii)}\left.\begin{array}{c}(\sim p \Rightarrow q)\\ \sim p\end{array}\right\} \Rightarrow q$$

is not included in most texts. It means that we can always replace an indirect proof with a direct proof of a contrapositive.

Use of contrapositives to clarify meaning and enhance the student's ability to communicate.

Consider the postulate *"Two points determine a line."* What does it mean? First we put it in bracket form letting A and B represent points and l_1 and l_2 represent lines.

$$\left.\begin{array}{ll} h_1: & A \in l_1\\ h_2: & B \in l_1\\ h_3: & A \in l_2\\ h_4: & B \in l_2\\ h_5: & A \neq B\end{array}\right\} \Rightarrow l_1 = l_2$$

Verbalized:

S_1 : *"If each of two lines contains the same two distinct points, the lines are equal."*

According to Lazar's definition of the multi-contrapositive (*Appendix 2, 18a*) the implication above has five contrapositives. As an exercise in the precise use of language we ask the student to verbalize each of these. It turns out that only two are distinct when verbalized. By exchanging $l_1 \neq l_2$ with $A \notin l_1$ we have

$$\left.\begin{array}{l} h_1:\ l_1 \neq l_2 \\ h_2:\ B \in l_1 \\ h_3:\ A \in l_2 \\ h_4:\ B \in l_2 \\ h_5:\ A \neq B \end{array}\right\} \Rightarrow A \notin l_1$$

Possible verbalization:

S_2 : *"If two distinct lines have a point in common, then any other point in one line is not in the other line."*

If we exchange $l_1 \neq l_2$ with $A = B$ we have

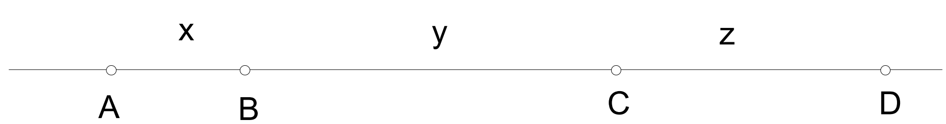
S_3 : *"If each of two points lie on both of two distinct lines then the points are equal."*

Now even though these statements are logically equivalent, they are significantly different: S_1 gives us a way to prove two lines equal, S_3 a way to prove two points equal, and S_2 a way to prove a point is not in a line. Taken together they completely exploit the meaning of the postulate.

We consider one more important example. The exterior angle theorem implies that: "If two lines are not parallel, and are cut by a transversal, then the alternate interior angles are not congruent." Contrapositive: "If the alternate interior angles are congruent, when two lines are cut by a transversal, then the lines are parallel." Since we know how to construct an angle congruent to another angle (Appendix 3 number 4 under Basic Constructions), it is easy to prove that through a point outside a line there exists a line parallel to it. But the student should be informed that it is impossible to prove Euclid's fifth postulate which asserts that there is only one such line. In fact centuries-long efforts to prove Euclid's fifth, using only the other postulates, led eventually to the discovery of non-Euclidean geometries, where Euclid's fifth does not hold. This is a spectacular triumph of logic over intuition.

Use of the converse to generate conjectures.

I first became interested in this idea when I encountered Euler's relation for four collinear points. Let A-B-C-D indicate that we have four collinear points in the order shown.



(|AB| $=$ x represents the distance between points A and B.)

Then, (A-B-C-D) $\Rightarrow$ (|AB| $\cdot$ |CD| $+$ |BC| $\cdot$ |AD| $=$ |AC| $\cdot$ |BD|).

(*In simpler notation:* $x \cdot z + y(x + y + z) = (x + y)(y + z)$).

Is the converse true? I thought it was true, but could not prove it. Then it occurred to me that the expression in the conclusion is satisfied when A, B, C, and D are the vertices of a square, or a rectangle, or an isosceles trapezoid, or indeed any cyclic quadrilateral (Ptolemy). At that point in my high school teaching career I began to use the multi-converse (*Appendix 2, 18ii*) to generate conjectures which must either be proved or disproved by citing a counter example.

Example 1: Given triangles ABC and $A'B'C'$ investigate the converses of

$$\text{(SAS)}\ \left.\begin{array}{l} h_1:\ \overline{AC} \cong \overline{A'C'} \\ h_2:\ \angle C \cong \angle C' \\ h_3:\ \overline{CB} \cong \overline{C'B'} \end{array}\right\} \Rightarrow \left\{\begin{array}{rl} \angle A \cong \angle A' & : c_1 \\ \overline{AB} \cong \overline{A'B'} & : c_2 \\ \angle B \cong \angle B' & : c_1 \end{array}\right.$$

(Standard notation. *Appendix 3*)
Making this investigation the student discovers that all the triangle congruence theorems belong to a multi-converse set which also includes two non-theorems, SSA and AAA.

Example 2: In $\triangle ABC$ we have:

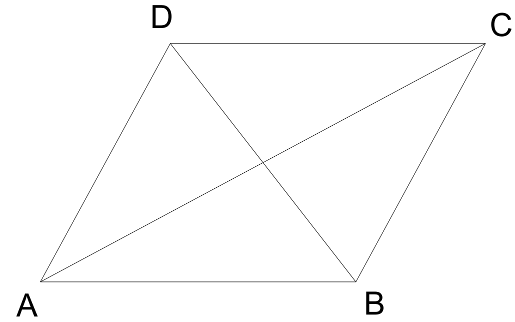
$$h:\ \overline{AC} \cong \overline{BC} \Rightarrow \left\{\begin{array}{rl} \angle A \cong \angle B & : c_1 \\ h_a \cong h_b & : c_2 \\ m_a \cong m_b & : c_3 \\ t_a \cong t_b & : c_4 \end{array}\right.$$

We ask the student to state each of the converses and determine its truth value. If we interchange h and c_2 the statement can be "*If two altitudes of a triangle are congruent, then the triangle is isosceles.*"

The fourth converse, whose hypothesis is $t_a \cong t_b$ is quite challenging. Teachers who use this procedure must expect to encounter many challenging problems not found in any text.

AT the end of the unit on parallelograms we use this example to lead the student on an explaration that will unify and extend his knowledge.

Example 3: In quadrilateral ABCD



$$\left.\begin{array}{l} h_1: \overline{AB} \parallel \overline{DC} \\ h_2: \overline{AD} \parallel \overline{BC} \end{array}\right\} \Rightarrow \left\{\begin{array}{rl} \overline{AB} \cong \overline{DC} & : c_1 \\ \overline{AD} \cong \overline{BC} & : c_2 \\ \angle A \cong \angle C & : c_3 \\ \angle B \cong \angle D & : c_4 \\ \overline{AC}\, bisects\, \overline{BD} & : c_5 \\ \overline{BD}\, bisects\, \overline{AC} & : c_6 \end{array}\right.$$

According to Lazar's definition (*Appendix 2, 18b*) there are $C_2^8 - 1 = 27$ converses. The student is asked to state each of these conjectures, and determine the truth value of those that are distinct when verbalized.

Consider the converse whose hypothesis is the conjunction of c_3 and c_6.

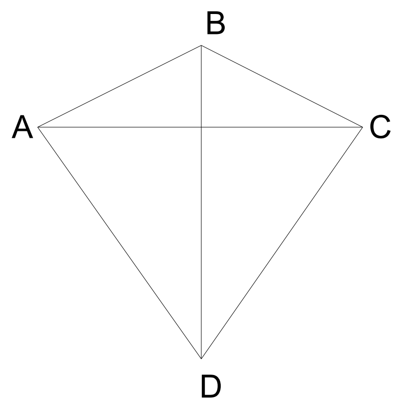
$$\left.\begin{array}{c} \angle A \cong \angle C \\ \overline{BD}\, bisects\, \overline{AC} \end{array}\right\} \overset{?}{\Rightarrow} ABCD \text{ is a parallelogram}$$

Possible statement: "*If two opposite angles of a quadrilateral are congruent and the diagonal joining their vertices is bisected by the other diagonal, then that quadrilateral is a parallelogram.*"

This conjecture is typical of those generated by Lazar's multi-converse definition in that:

1) its statement requires the careful use of language in a situation where the teacher knows what the student is trying to say, and
2) the decision regarding truth value is not immediately obvious.

In this case our hypothesis is satisfied by parallelogram $ABCD$, but a counter example is provided by a kite. So, the conjecture is false.



Problem formation (Problems from other Problems)

In order to describe this procedure I must again resort to specific examples. Our first example involves **allegation**: "The method of finding the value per pound of a mixture when the price per pound of each individual component is known." We deal with two components.

Example 1a: Find m, the value per pound of a mixture consisting of a pound of s-cent coffee (component 1) and b pounds of t-cent coffee (component 2). The solution develops the formula for the weighted mean, $m = \dfrac{as + bt}{a + b}$, which says that given a, b, s, and t, we can find m.

$$\left.\begin{matrix} a \\ s \\ b \\ t \end{matrix}\right\} \Rightarrow m$$

Example 1b: Let $s = 40,\ b = 30,\ c = 60,\ and\, m = 52$, and restate Example 1a to find a (converse 1).

Possible statement: "*How many pounds of coffee worth 40 cents per pound must be mixed with 30 pounds of 60-cent coffee to produce a mixture worth 52 cents per pound?*"

Example 1c: Restate example 1a to find s. (Converse 2)

$$\left.\begin{matrix} a \\ m \\ b \\ t \end{matrix}\right\} \Rightarrow s$$

Possible statement: "*A mixture contains 30 pounds of a brand worth 60 cents per pound and 20 pounds of another brand. If the mixture is worth 52 cents per pound, what is the per pound value of the other brand?*"

Example 1d: Prove: $(s < t) \Rightarrow (s < m < t)$

Example 1e: Generalize the formula in Example 1a for n components.

Question: What do we call a person who engages in allegations? Class?

Problems from formulas. The formula $A_n = P(1+r)^n$ represents the amount A_n which a principal of P dollars will accrue if invested at compound interest for n years at an annual interest rate of r.

Example 2: View the problem from the standpoint of a problem constructor.

Example 2a: Insert plausible values for three of the four variables and find the corresponding value for

a) A_n b) P c) r d) n

Example 2b: Prove the formula for A_n by mathematical induction.

The methods illustrated in this section have wide application at all levels of high school mathematics.

Section III. The need to reinstate proof as the culmination of exposition.

During roughly the first half of this century, this emphasis on proof-directed exposition would have been considered redundant by experienced teachers. In presenting proof they were seeking CLARITY IN EXPOSITION rather than pretentious rigor. Today, teachers trained in the last ten years in our Schools of Education may reject it as too rigorous, too austere, too "mathematical". Now *there is an urgent need to reinstate explanations that culminate in proof* at the secondary level. Let us review some of the evidence that supports this statement.

*** Proof has long been on the wane in secondary school mathematics. Older texts in geometry (circa 1920-60) placed great emphasis on proof by requiring students to prove many "originals" of all degrees of difficulty. Current texts seldom make such demands. This same trend toward downgrading proof to the point of elision is manifest in all sectors of the secondary curriculum. (Sadly, the NCTM has been too busy touting "Higher thinking skills" to notice.)

*** The employment of dynamic geometries such as Geometer's Sketchpad has been a boon, often replacing apathy with excitement, but, unless carefully directed by well-trained teachers, it does tend to blur the distinction between illustration and proof.

*** At a higher level the enormously complicated "proofs" generated by computers or, as in the case of the four-color problem, by vast linkages of computers, have served to make proofs less verifiable and to render the whole concept of proof more elusive. Some have seized on this as a reason for de-emphasizing axiomatic proof even at the research level. (R12), (R13)

*** NCTM's post-Standards policies no longer focus on improving exposition.

a) The NCTM has little to say about the need for improving the high school teacher's KNOWLEDGE OF MATHEMATICS without which improved exposition is impossible

b) The recent over-emphasis on cooperative learning, which seems to be based on a misinterpretation of "Constructivism" (R14) serves to deflect attention from what teachers do as expositors and focuses instead on whatever it is that "Facilitators" do.

*** The MAA's Task Force on the NCTM Standards expressed their views on this matter as follows: "One critical concern of the Task Force is the need for the Standards to more fully address issues of mathematical reasoning, the need for precision in mathematical discourse, and the role of proof." (R15)

*** In view of the miserable showing of US eighth grade math students on the TIMSS exams, (ranked 28th out of 41) the following quotes from the TIMSS videotape study, cited by Kim Mackey, are particularly significant. "It is likely that the kind of mathematics that students learn is related to the nature of the mathematics they are asked to study. Although constructing proofs and reasoning deductively are important aspects of mathematics, American students lacked opportunities to engage in these kinds of activities. None of the US lessons included proofs, whereas 10 percent of the German lessons and 53 percent of Japanese lessons included proofs." and "In a separate analysis of 30 lessons taken from each country by a group of experienced college teachers, 62 percent of Japanese lessons were found to contain deductive reasoning, compared to 21 percent in Germany and zero percent in the US."(R16)

Section IV. Essential Role of the Mathematical Community

Having established the need to improve exposition by reinstating proof we now turn to the essential role of the mathematical community in educating teachers who are able and eager to do this. While all members of CBMS should be involved, leadership should be provided by the college teachers of mathematics (MAA). Instruction designed to improve the exposition of high school mathematics must be closely geared to the subject matter of mathematics. This means that it cannot be left to our Schools of Education, but must involve the active participation of the mathematicians who teach at the college level. They must take some of the responsibility for improving the high school math instruction, whose failure has reached crisis proportions in some areas. (R17). They should help prepare the kind of teachers they want for their own children. Their own teaching should provide models of expository excellence, for the edification of the future teachers in their classes. (I was inspired by the presentations of Professor Allen T, Craig at the University of Iowa.) They alone have the deep understanding of mathematics which enables them to provide the lucid explanations that teacher trainees so urgently need. This requires effort, patience and a dedication to good teaching which some professors feel is often unrewarded. To open gates on the path to preferment one must publish research papers or obtain monetary grants. This often forces a concentration on non-teaching endeavors which may lead to neglect of exposition in the classroom.

In "A Mathematicians Apology", G.H. Hardy wrote "There is no scorn more profound, or on the whole, more justifiable, than that of the men who make for the men who explain. Exposition, criticism, appreciation is work for second-class minds". Considering the reward system now in place and their driving and commendable interest in research it is certainly understandable that many younger math department members may share Hardy's derogatory view of the expository process. They have reputations to build while still in their prime productive years, When it comes to improving exposition at the secondary level, we really can not expect much help from them. So we turn to their senior colleagues. These are mature mathematicians with well established reputations, who can now afford to focus their attention and indispensable expertise on the vitally important matter of improving exposition in secondary school mathematics by

1) Educating teachers who can accomplish this in the kind they want for their grandchildren (Section II) and

2) Establishing a situation in which these teachers can be successful

The first requires

(a) The reinstatement of a college course covering advanced topics in Euclidean geometry (see Section I above) and

(b) The development of a course in the teaching of secondary mathematics which considers the subject matter from an advanced standpoint, focuses on how to explain it and how to write tests that provide valid measures of the student's understanding.

The second requires

(a) Establishing national standards for measuring student competence in each high school course (algebra, geometry etc.) by providing standardized tests for each course, where individual scores can be compared with national and perhaps even world norms. These tests are to be externally graded, with individual grades widely publicized and made a part of the students permanent record. (See Bishop "The Power of External Standards". (R18)

(b) Providing a syllabus for each course which clearly delineates the mathematics that the student is expected to learn.

The establishment of national standards in school mathematics would be a long step toward maximizing the *common ground* held by our multiple and diverse cultures. This must be done if we want to live together in a harmonious and productive society. Also, it would help teachers deal with the "equity" problem, for it is the poor and disadvantaged who move frequently and, as a consequence, are victimized by the diverse standards generated by local control.

In dealing with these missions, committees of senior mathematicians, should seek input from high school math teachers who have a strong undergraduate major and some graduate work in mathematics and at least ten years of successful teaching experience. It would be highly desirable for the younger group of mathematicians to be represented on these committees, if this can be arranged.

Section V. External Conditions That Must Exist In Order for Well-Trained Teachers to be Successful These teachers must have

A) RESPECT. The well prepared high school mathematics teacher must be respected by parents, administrators and students, as one who is qualified to teach because he has successfully completed a RIGOROUS TRAINING PROGRAM, passed qualifying tests and keeps up to date by continuous study. His status and compensation should be on a par with that of an actuary or engineer. This will have the effect of making a career in the high school mathematics classroom attractive to our better students. Moreover, respected teachers, like respected coaches, are more effective instructors.

B) TIME. Mathematics teachers need TIME DURING THE SCHOOL DAY to plan presentations, confer with colleagues, write exams and diagnostic tests and grade student work as they have in other countries. In this regard the following statement by Linda Darling-Hammond is highly significant.

"School systems in Germany, France, Switzerland, the Netherlands and Japan invest in a greater number of well-prepared, well paid and well supported teachers, rather than in a large bureaucracy populated by non-teaching staff hired to manage, inspect and control the work of teachers. Teacher education and ongoing professional development are much more extensive in these countries than in the United States; substantial time for learning and collegial work is built into the school day; and teachers make most school and curriculum decisions. This is fiscally possible because classroom teachers comprise 80 percent of the education employees in these countries, as compared to under 50 percent in the United States. By investing in large administrative superstructures to control the work of teachers, rather than teach themselves, we have sucked resources out of classrooms where they could make a difference." (R 19)

Clearly, there must be a drastic reallocation of our financial resources in this country. If we want to give our well-trained math teachers a chance to succeed and are serious about meeting world standards, we must terminate this system where an employee's power, pay and prestige varies inversely with the number of classes taught and directly as the square of his distance from the classroom.

C) ADMINISTRATIVE SUPPORT. This involves

*** Providing teachers with swift and strong support in disciplinary matters. Even a Section I math teacher may occasionally need such support, and the knowledge that it is available will greatly enhance his effectiveness.

*** Insuring that each class assigned to a math teacher is a reasonably homogeneous group of students who have successfully completed all prerequisite courses. Yes, this means tracking and the end of "social promotion".

*** Creating a school atmosphere where academic achievement is respected; where it is "cool" to be smart; where class time is seldom interrupted and even the principal teaches a class or two in recognition of the fact that teaching is the most important thing and that the support of teaching is the administration's principal function.

*** Protecting math teachers from the pressures that generate grade inflation. This requires administrative endorsement of the external testing system described earlier. When students face a nationally administered standardized test, the teacher is no longer under pressure to inflate course grades. He becomes, instead, a coach or mentor who students need to help them meet an externally set and externally graded examination..

*** Maintaining the administrative arrangement where there is a separate MATHEMATICS DEPARTMENT, headed by a strong chairperson.

4) PARENTAL SUPPORT based on the understanding that learning mathematics requires hard work and nurtured by frequent parent-teacher contacts. *It is the parent's responsibility to send their children to school in a condition to learn. It is society's responsibility to insure that our schools are safe havens where students can concentrate on learning.* This must be

mentioned because improving exposition by providing well-trained (Section I) teachers will have little effect until these conditions are met.

In closing let me cite another statement in the "Mathematician's Apology" by G. H. Hardy. "My eyes were first opened by Professor Love who taught me for a few terms and gave me my first serious conception of analysis. But the great debt which I owe to him--was his advice to read Jordan's famous Cours d'analyze; and I shall never forget the astonishment with which I read this remarkable work--and learned for the first time--what mathematics really meant." Is this not a moving tribute to two great expositors? Does it not show that the"men who make" owe much to the "men who explain"? Good exposition can also "open eyes" and reveal what "mathematics really means", at the high school level. As the information age puts ever mounting pressure on our educational facilities, the work of the skilled expositor who can transmit knowledge and understanding to the next generation becomes increasingly important. There simply is not time to "reinvent the wheel" or to rediscover basic knowledge. If we want our math students to raise their level of performance and compete successfully with their peers in other industrialized countries, we must provide them with teachers who are skilled expositors and establish school situations where both teachers and students can succeed. Let's do it.

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(R10) Butler and Wren, "Teaching Secondary Mathematics" 1941. "To help provide more intelligent guidance of teaching and learning" and "To develop more effective curricula and educational experiences."

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(R14) John R. Anderson, "Applications and Misapplications of Cognitive Psychology to Mathematics".

(R15) Ken Ross, Chair of the MAA Committee on the NCTM Standards in his report to the MAA Board of Governors. dated March 7, 1997, point 5 under "Reservations and Concerns about the Standards."

(R16) Mackey (mackeys@alaska.net)

(R17) Kenneth R, Weiss, writing in the Los Angeles Times for March 20, 1997 reports that " More than half the freshmen who entered the California University System last fall were unprepared for college level math". He also notes that "The percentage of unpreparsd students was the highest since Cal State began tracking the data in 1969."

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Professional Diary of

Frank B. Allen
Professor of Mathematics Emeritus
Elmhurst College, Elmhurst, Ill

0. Born in MtVernon, Illinois Oct. 21, 1909 to Rev. and Mrs. Frank B Allen.
1. B.Ed. - Southern Illinois State Teachers College, August, 1929 . Majors in mathemstics and Physics.
2. Going summers I earned an M.S. in mathematics with minor in Physics from the University of Iowa,in August, 1934.
3. Graduate Study (mathematics)- University of Illinois. Summers 1936, 1937, 1938.
4. Thirty-five years as teacher of high school mathematics in the State of Illinois
5. Four years in the Army of the United States, 1942-1946. (Private-Captain)
6. President of Men's Mathematics Club of Chicago, 1952-53. Now called "The Metropolitan Mathematics Club of Chicago."
7. President of Illinois Council of Teachers of Mathematics, 1954-55.
8. Chairman of the Department of Mathematics, Lyons Township High School and Junior College, La Grange, Illinois 1956-68.
9. Chairman of the Secondary School Curriculum Committee of the National Council of Teachers of Mathematics, 1956-59.
10. Member of Board of Directors of the National Council of Teachers of Mathematics, 1958-61.
11. Director of the Regional Orientation Conferences on Mathematics, 1960. (National Science Foundation Support)
12. Member of Advisory Committee for the School Mathematics Study Group, 1958-63, 1965-68.
13. Member of Textbook Panel, SMSG, 1958-60.
14. Chairman of the Eleventh Grade Writing Team, SMSG, 1958-60.
15. Member of the Council of the Conference Board of the Mathematical Sciences, 1960-62.
16. President of the National Council of Teachers of Mathematics, 1962-64.
17. President of Mu Alpha Theta, 1965-68.
18. Distinguished Life Member Award from Illinois Council of Teachers of Mathematics, 1969.
19. Member of the NCTM Headquarters Facility Committee, 1968-70.
20. Elmhurst College, Department of Mathematics, Associate Professor 1968, Professor 1972, Chairman, 1971-1975.
21. Elmhurst College, Division of Natural Sciences and Mathematics, Chairman 1972-75.
22. Member District 205 Board of Education, 1977-83 Chairman Curriculum Committee
23. Elmhurst Evening Lions Club, 1977- President 1980-81.
24. Max Beberman Award for "Leadership and excellence in teaching mathematics education" from the Illinois Council of Teachers of Mathematics, October 1987.
25. Distinguished Service Award from the Metropolitan Mathematics Club of Chicago for "outstanding contributions to the advancement of education in mathematics", 1988.
26. Elmhurst United Way/Crusade of Mercy, Director-at-Large 1985-88 (fund raising-allocations).
27. Zoning and Planning Commission for City of Elmhurst, 1985-90.

28. Teacher Training Program - Northern Illinois University, 1989.
29. Elmhurst Jaycee Distinguished Service Award, 1990.
30. Grand Marshall for City of Elmhurst Fourth of July Parade (sponsored by Elmhurst Jaycees) 1990.
31. Melvin Jones Fellow Award by the Lions Clubs International Foundation 1990.
32. Elmhurst College establishes Frank B. Allen scholarship for senior math major who is planning to teach, 1991. (Award is made at end of junior year) The college is now attempting to fund this scholarship so that the annual interest will provide a substantial stipend for each winner. Those interested in this project should contact Larry Fricke, Office of College Advancement, Elmhurst College, 190 Prospect Ave., Elmhurst, IL. Phone: 630 617 3032, em ail{ Larryf@elmhurst.edu
33. Founders Medal Awarded by the Board of Trustees, October 1918 for Outstanding Service to the College.
34. Elmhurst Evening Lions Club establishes the
Frank B. Allen Community Service Award to be
presented annually to a graduating High Schoo senior,
May 1998.
35. INDUCTED INTO LYONS TOWNSHIP HIGH SCHOOL (LaGRANGE, ILLINOIS) "HALL OF FAME", NOV 9. 2001 . ONLY TEACHER SO HONORED. ALL OTHER INDUCTEES SINCE 1909 WERE GRADUATES OF LTHS.. I TAUGHT THERE FOR 23 YEARS.

Books:

Modern Algebra - A Logical Approach (Book One) (with Pearson), Ginn & Company, 1964.
Modern Algebra - A Logical Approach (Book Two) (with Pearson), Ginn & Company, 1966.
Basic Concepts of Geometry (with Guyer), Dickenson Publishing Company, 1973 college level)

Addendum to Diary:
I wish to convey the idea that, since my retirement in 1979, I have maintained my interest in both school and undergraduate mathematics and have kept myself informed about recent developments in these fields. Finding myself in strong disagreement with the policies of the National Council of Teachers of Matthematics as stated in the 1989 "Standards", I resigned frpm that organization. I purchased a computer in 1995, gained access to the Internet and now opose NCTM policies s National Advisor for Mathematically Correct on whose web site

www.mathematicallycorrect.com

several of my papers appear.

Teaching experience

1929-36 Sparta Twp. HS., Sparta, IL.
1936-37 Urbana HS., Urrbana.IL.
1937-41 Thornton Fractional HS., Calumet City, IL.
1841-42, 1946-68, Lyons Twp HS and JC, LaGrange, IL
1968-79 Elmhurst College, Elmhurst, IL

Here is the essay. This
Background from earlier years:

"Repairing School Mathematics"
http://mathematicallycorrect.com/report.htm

The following paper elicited considerable support from the Mathematical Community.

"A plan for Raising the Level of student Achievement in Secondary School Mathematics"

http://mathematicallycorrect.com/allen.htm

"Language and the Learning of Mathematics"

http://www.mathematicallycorrect.com/allen4.htm

"A Plan for Improving the Quality and Effectiveness of Exposition in High School Mathematics:

http://mathematicallycorrect/weneed.htm

Mathematics "Council Loses Hard-Earned Credibility

http://mathematicallycorrect.com/frankallen.htm

The Language of Mathematical Exposition
(Gradually Introduced Grades 7-12)

All teachers are concerned with vocabulary, sentence structure, and reading and writing skills. Mathematics teachers also employ a special symbolic language in which letters are used to represent statements, variables, etc. Essential components of this language are:

1. Set Language: Intersection $\cap$, union $\cup$, inclusions $\subseteq$, proper inclusion $\subset$, is a member of $\in$, is not a member of $\notin$, and empty set $\emptyset$. Set builder notation $\{x | \sqrt{x-3} > 5\}$.
2. Definitions and assumptions as a basis of reasoning.
3. Quantification involving "some", "all" ($\forall$) and "there exists" ($\exists$), etc. Single exception destroys an "all statement".
4. Statements. A statement is a positive declaration with no attempt at proof. Example: In any group of eight people at least two have birthdays on the same day of the week.
5. Equality. The statement "$a = b$" means that a and b are names for the same thing.
6. Conjuction. The statement "p and q", written "$p \wedge q$" is the conjunction of p and q. It says that both p and q are true.
7. Disjunction. The statement "p or q", written "$p \vee q$" is the disjunction of p and q. It says that at least one of the two statements is true.
8. Implication: The statement "p implies q", written "$p \Rightarrow q$" is an implication. It can also be written as "If p then q", where p is the hypothesis and q is the conclusion. An implication is true in all cases except when the hypothesis is true and the conclusion is false.
9. Equivalence. Two statements, a and b, are logically equivalent, written "$a \Leftrightarrow b$", if each implies the other.
10. Negation. The statement "p is false", written "$\sim p$", and read "not p"; is the negation of statement p.
11. Contradiction. A statement that is equivalent to the negation of p is a contradiction of p and can also be written as $\sim p$. Two statements are contradictory if exactly one of them is true; i. e. each is equivalent to the negation of the other. Example: *Some Scandinavians are vegetarians* $(S \cap V \neq \emptyset)$ and *o Scandinavians are vegetarians* $(S \cap V = \emptyset)$ are contradictory statements.
12. Double negative. $\sim(\sim p) \Leftrightarrow p$.
13. Contradicting conjunctions, disjunctions and implications.
    $\sim(p \wedge q) \Leftrightarrow (\sim p \vee \sim q)$
    $\sim(p \vee q) \Leftrightarrow (\sim p \wedge \sim q)$
    $(p \Rightarrow q) \Leftrightarrow (\sim p \vee q) \quad \therefore \sim(p \Rightarrow q) \Leftrightarrow (p \wedge \sim q)$
14. Converse, inverse and contrapositive. For any implication $p \Rightarrow q$ we have
    Converse: $q \Rightarrow p$
    Inverse: $\sim p \Rightarrow \sim q$
    Contrapositive: $\sim q \Rightarrow \sim p$
15. Equivalence of an implication and its contrapositive:
    $(p \Rightarrow q) \Leftrightarrow (\sim p \vee q) \Leftrightarrow [\sim p \vee \sim(\sim q)] \Leftrightarrow [\sim(\sim q) \vee \sim p] \Leftrightarrow (\sim q \Rightarrow \sim p)$

16. Distributive Laws: $[a \vee (b \wedge c)] \Leftrightarrow [(a \vee b) \wedge (a \vee c)]$;
$[a \wedge (b \vee c)] \Leftrightarrow [(a \wedge b) \vee (a \wedge c)]$.

17. Tautologies: A composite statement is a statement formed from other statements a, b, c, $\ldots$ by using some of the connectives $\wedge$, $\vee$ and $\Rightarrow$. The statements a, b, c, $\ldots$ are the components of the composite statement. A composite statement that is true for all possible truth values (T or F) of its components is called a tautology. Examples:

(i) $a \vee \sim a$ (ii) $\left.\begin{matrix} p \\ (p \Rightarrow q) \end{matrix}\right\} \Rightarrow q$ or $[p \wedge (p \Rightarrow q)] \Rightarrow q$ (iii) $\left.\begin{matrix} (\sim p \Rightarrow q) \\ \sim q \end{matrix}\right\} \Rightarrow p$

(iv) $[p \wedge q \wedge (p \Rightarrow x) \wedge (q \Rightarrow y) \wedge [(x \wedge y) \Rightarrow z]] \Rightarrow z$.

ote (ii), (iii) and (iv) are proof patterns.

18. a. Partial contrapositive and partial converse. In most theorems of the form $p \Rightarrow q$, the hypothesis p is the conjunction of several component statements. If there are four such component statements, the theorem takes this form:

$\left.\begin{matrix} a \\ b \\ c \\ d \end{matrix}\right\} \Rightarrow q$ This statement has four partial contrapositives, each obtained by exchanging a contradiction of q with a contradiction of one of the four statements in the hypothesis. **a one o t e e ont a o iti e o i e i alent to t e o iginal t eo e** . (Some of these contrapositives may also have the same verbal sense.) This theorem also has four converses, each obtained by exchanging q with one of the four statements in the hypothesis. **a o t e e one e e i a on e t e i a o a not e t e** (Some of these converses may be equivalent.)

b. Generalized definition of converses (Lazar) If a theorem has m statements in it hypothesis and n statments in its conclusion, a converse may be formed by exchanging any number of statements in the conclusion with the same number of statements in the hypothesis. Each of the $C_n^{m+n} - 1$ converses is a conjecture whose truth value must be determined, and some may have the same verbal expression.

19. An argument is an assertion that the conjunction of premises a, b, c, $\ldots$ implies a conclusion x. In bracket form it looks like this:

[I.] $\left.\begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix}\right\} \Rightarrow x$. An implication like [I.] can be a tautology (Compare with 17 (iv)). If so, it is a valid argument.

20. Proof: A valid argument with true premises is a proof of its conclusion.

Mathematically Correct

California Issues

Page Index

California Background

In the early 1990's, California bought into the fuzzy math reform in a big way. Mathematics content began to disappear from textbooks and was replaced by the hallmarks of the reform - color pictures, blocks, group work, activities, projects, calculators, and disdain for hard work and correct answers. Exposition in textbooks or by classroom teachers began to evaporate. The state, once known for good schools, now competes for the lowest test scores in the nation.

The fuzzy math reform in California was greatly promoted by the 1992 California Mathematics Framework. This document was based largely on the NCTM Standards of 1989. The state framework focuses on the pedagogy and philosophy of the so-called reform movement, and lacks rigorous mathematics objectives.

The 1992 Framework in turn impacted on the state adoption of textbooks in 1994. The weak programs that were approved then became much more common in schools. Some of the worst programs available became among the most popular in the state.

California faced an enormous struggle if it was to recover from declining mathematics achievement. Rigorous mathematics had all but disappeared from the state. The fuzzy-math forces were strong and entrenched. However, public pressure, the legislature, and the state board of education reacted in concert to oppose the so-called reform and the declines in achievement. The battle that resulted made California the front line in what has come to be known as the Math Wars.

Mathematics Standards

Perhaps the most significant development in California was the establishment of rigorous standards for K-12 Mathematics. Unlike what the NCTM calls *standards*, the California standards provide explicit expectations for learning in grade-by-grade detail.

The California Standards are not only significant to parents in California. These highly rated standards can be used by parents in other states to help them evaluate their own state or district standards or even the curriculum in their local schools. The California Standards were designed to reflect high-level achievement, equivalent to that of the most successful countries.

The California Mathematics Standards

- THE CALIFORNIA MATHEMATICS ACADEMIC CONTENT STANDARDS For Grades K - 12

About the California Standards

- State Mathematics Standards report rates California #1
- Statements About the California Mathematics Standards
- Number Sense in California

Reports about the California Standards and their history

- California Standards and Assessments, by R. James Milgram and Veronica Norris
- Some observations on the 1997 battle of the two Standards in the California Math War, by H. Wu

Some documents from the struggle

- The State's Invisible Math Standards, by David Klein in the LA Times, May 3, 1998
- California Mathematicians Respond
- Hearings on proposed California Standards
- And They Call It Algebra

The Luther Williams Saga

When the standards-setting process in California was underway and not moving in the direction of supporting fuzzy-math, the proponents of that so-called reform tried to stop the progress in many ways. In perhaps one of the most dramatic attempts, then head of the educational division of the National Science Foundation, Luther Williams, sent a letter to the California State Board of Education. Here are some documents that followed:

- Man of Science Has a Problem With Real Math, by Debra Saunders, San Francisco Chronicle, Dec. 19, 1997
- Comments on the Luther Williams Letter

The Mathematics Framework

The Mathematics Framework is the guiding curriculum document in the State of California. It defines the structure of the curriculum and stipulates the criteria used by the state for K-8 textbook adoptions. The 1992 Framework headed the state down the fuzzy-math road. The problem was so severe that work on the new framework began ahead of schedule. Replacing the 1992 version was a tremendous struggle that took years. Hearings began in December of 1995 and the final version was unanimously approved by the State Board of Education on December 10, 1998. The efforts were successful in producing a document that can guide the state on the road to recovery.

- Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve [requires Adobe Acrobat reader]
- Smoothing out the bumps in the '92 Framework
- Critique of the 1992 Framework, by Wayne Bishop
- Presentation to the State Board of Education, by Henry L. Alder

Textbooks

Following the new mathematics standards, California was still in a crisis state since textbooks had been selected on the basis of the 1992 Framework. Special funding was provided on an emergency basis to buy new materials and a special adoption was conducted June 10, 1999 to select materials most closely aligned with the standards.

- New California Math Adoptions

Subsequently, a regular adoption was held under the guidelines of the new Standards and Framework. This opens up regular state instructional materials funds for buying textbooks aligned to the standards.

- California Mathematics Program Adoptions for 2001

Related Documents

Before the Standards and Framework were adopted, California was in trouble. The State Board of Education faced difficulty in advocating solutions while earlier positions were still in force. In an early attempt to deal with the statewide difficulties, a Program Advisory was released.

- Mathematics Program Advisory from State Board calls for Basic Skills

The developments in California required legislative action as well. For example, the legislature mandated that standards be developed and that the Framework be aligned to the standards. Here is one piece of legislation and how it fit into the situation at that time:

- How AB170 and the Education Code Relate to Districts

At one point in the California struggle, E. D. Hirsch, Jr., was invited to address the State Board of Education. His talk addresses the use of research in guiding educational designs. Here is the text of his presentation:

- Hirsch addresses Calif State Board of Education

Testing

California stumbled in a prior attempt to produce statewide tests. As a result, the state went for many years without this important feature of accountability. To remedy this situation, California has adopted a new assessment program called STAR. The first step was to adopt an existing, nationally normed achievement test that was available off-the-shelf. The Stanford 9 (SAT9) was selected for this purpose. The test was first given in the spring of 1998. The second step was to augment this test with items written specifically to address the new standards. The augmented version of the test was first given in 1999. As might be expected, the mean augmented mathematics scores for most schools were very low. Despite the unpleasant implication, the 1999 data provide an important baseline so that California can track progress toward the standards.

STAR Information

- Sample Augmented STAR Mathematics Items
- STAR Augmentation Background
- California STAR test results
- STAR math test results for California, Los Angeles, and San Diego
- California STAR Math Scores by County
- API Scores from the state

Testing Background

- California Standards and Assessments, by R. James Milgram and Veronica Norris
- State OKs Standardized Tests for Kids

High School Exit Exam

California is also working toward a high-school exit examination (HSEE). Students will be required to pass this examination in order to graduate from high school. Despite the adoption of high-level standards, it is obvious that the majority of students will not be able to achieve them for several years. Consequently, the content of the mathematics portion of the HSEE has been the subject of considerable debate. Although there were efforts to include minimal aspects of algebra, there were negative reactions to this idea across the state. Now it appears that students will be expected to pass algebra questions when the examinations take effect. More advanced questions may be added later.

- Sept 28, 1999 -- Higher Goals for Exit Exams California cheats its students by expecting too little, by Pete Wilson and Bill Evers, SF Chronicle
- Dec. 16, 1999 -- High school graduates should at least know their algebra, by Joanne Jacobs, San Jose Mercury News

California State University Entry Level Mathematics Placement

Students entering the California State University (CSU) system are placed in mathematics courses by various means such as their SAT scores. However, many are required to take an examination to aid in placement. The number of students failing this examination has been increasing year after year. The examination was recently revised. The material below will clarify the nature of this examination.

- Evaluating Entry Level Mathematics Placement in the California State University System
- CSU Entry Level Mathematics Placement Examination Information [Requires Adobe .pdf file reader]

Other Testing Data

- School Wise Press provides many details for individual schools, including test scores!
- SAT test data for California High Schools
- California Community College and California High School Academic Progress Reports

Other Testing Information

- Math Gadfly Calls Math Faddists' Bluff, by Debra Saunders, SF Chronicle, Aug 7, 1998
- Mr. Educrat: Tear Down That Wall, by Debra J. Saunders, SF Chronicle, July 28, 1998
- State Kids Lagging in Math Skills, 4th-graders score 4th-worst in U.S.

Districts Around the State

Los Angeles

Recent information about LAUSD:

- Math Lessons: Beyond Rhetoric, Studies in High Achievement, LA Times, Feb 11, 2001
- Integrated Mathematics in LAUSD
- LA's Math Program Just Doesn't Add Up, by David Klein and R. James Milgram
- LA Times editorial on mathematics education

Details the struggle over mathematics standards in Los Angeles follow. The district (LAUSD) had adopted their own standards and not followed the lead provided by the state. To see how significant this issue was, it is necessary to compare the two documents.

- A Comparison of the LAUSD Math Standards and the California Math Standards

Related information provides some of the history of the struggle in LAUSD.

- LAUSD Board Meeting, July 14, 1998
- Freedom to Agree, by David Klein
- *A Too-Hasty Math Move*, Los Angeles Times, August 16, 1998 Editorial
  "Mathematically Correct and the Wilson administration have called on the district to incorporate the state's recently implemented traditional math standards into its integrated math curricula instead, and the district should accede to the request."
- LAUSD's Refusal to Adopt the California Mathematics Standards, by David Klein

Finally, LAUSD adopted the California Standards on June 8, 1999. LAUSD is now trying to design their district mathematics curriculum guidelines so that they will be in accord with those of the state, and to select textbooks that will fit with this design.

Other information about LAUSD:

- STAR math test results for California, Los Angeles, and San Diego
- LAUSD May 14 Curriculum Committee Hearing

San Diego

Some of the history of the Math Wars in California took place in San Diego when parents discovered that their students were being given a program called CPM instead of algebra.

- Middle School Math Forum
- Middle School Math Forum Re-run

Unfortunately, Mathematically Correct soon discovered that the problem was much more extensive. The earlier grades were slated for watered-down mathematics as well.

- *Fuzzy Math* comes to K-8 in San Diego

With the rising interest in standards of learning, the district began working on mathematics standards. At first these efforts were unsuccessful. This prompted Mathematically Correct to draft standards and submit them to the district.

- Mathematically Correct Submits Standards to District

The difficult path to the adoption of mathematics standards in San Diego is summarized in this historical document:

- The Rocky Road to Mathematics Standards in San Diego

After the first failed attempt, the district established a committee with broad representation to develop standards of learning. Finally, on Feb 10, 1998, the district adopted rigorous mathematics standards. As with the state standards, parents in other states and districts can consult the San Diego standards to make comparisons.

- San Diego Mathematics Standards

Meanwhile, some improvements were noted in special programs and curriculum selections in the district.

- District Adoption Committees Select Recommendations
- Cubberly Elementary granted a waiver to use Saxon Math
- *Charter school is approved* San Diego Unified approves Nubia Academy based on Core Knowledge from E.D. Hirsch (Sharon L. Jones, San Diego Union-Tribune)
- A Horizon without Calculators

During the struggles in San Diego, a significant radio debate took place in San Diego. Mike McKeown of Mathematically Correct, Jack Price - then president of NCTM, and the late John Saxon of Saxon Publishers participated. A transcription follows:

- Math Radio: The Debate in San Diego

Palo Alto

The reaction to the introduction of fuzzy math in Palo Alto is well documented by a group called Honest Open Logical Debate on math reform, or HOLD.

- HOLD

HOLD sprang up as a result of parents concern over issues that mirror those in San Diego and other districts. Briefly, the CPM version of fuzzy math was introduced in the schools there. Many parents found this approach inadequate. A group of concerned parents formed and local debate became heated. The HOLD group has sent a wake-up call to their district administrators.

HOLD provides some important evidence. In particular, their news release about math test scores shows that overall scores dropped from 91st percentile to 81st percentile. Computation scores dropped from 86th percentile to 58th percentile.

- News release

This is a huge decline. The CPM organization said it was because the kids were tired.

Another important document at HOLD is:

- The need for choice

This letter to the editor of the Palo Alto Weekly by Bill Evers of May 17, 1995, reviews the results of a poll taken by their school district. Evers notes that 63% of families of middle school students rely on outside math tutoring. HOLD calculated that $1 million a year was being spent on professional math tutoring in their district. These efforts, along with supplementation by teachers, helped bring test scores part way back.

For more information, see:

- A Chronicle from the San Francisco Chronicle

Other Districts

Similar *hot spots* in the Math Wars have developed up and down the state. Here are a few selections to illustrate the conflict:

- Los Altos - Mountain View Algebra And The New California High School Exit Exam: Will Our Children Be Prepared?
- Mountain View Achievement
- Orange Unified drops Integrated Math
- Visalia Update
- Charles L. Beavers writes to Old Adobe Union School District
- Atascadero picks Silver-Burdett over MathLand
- Simi Valley to offer both traditional math AND Integrated Math
- Sonoma bans school calculators
- Hemet Unified bans group grades
- Escondido offers choice of math programs in middle school
- Choice of Math Programs Supported in Palo Alto
- South Pasadena breaks with new-new math
- Math Gadfly Calls Math Faddists' Bluff, by Debra Saunders, SF Chronicle, Aug 7, 1998, comments on a school in Santa Ana Unified and one in LA

Other California Goodies

Here are some assorted materials from California that shouldn't be overlooked.

- Four Years of California Mathematics Progress
  Four years of history shows substantial success

    - 1999 Conference on Standards-Based K-12 Education
      Full reports of the conference

    - Where's the Beef in the Turkey Problem? Exposing a New-New Math Educrat
      Details one way fuzzy math has been promoted

    - Educrats, New-New Math and Eastin
      Participation of the state Superintendent

    - One Teacher's Plan for Improving Math Education
      Suggestions from a classroom teacher

    - Raimi's Words
      Professor Raimi comments on Mathematically Correct

    - The Northridge Chronicles and an Epilogue
      Don't miss this historic debate

Number Sense in California

Introduction

The folling material is taken from The California Mathematics Academic Content Standards for Grades K - 12. Only standards for grades K-7 appear here.

Individual standards relevant to number sense have been isolated and grouped into more specific topic areas. Within each topic area the standards are arranged by grade level. This arrangement makes it easier to see the development of topics across grade levels.

By showing topic development across grade levels, it becomes easier to evaluate any particular piece of mathematics content relative to the California standards. The California grade level of content from textbooks or tests or other sets of mathematics standards can thus be more easily identified. The groupings can also be useful for the study of the California standards themselves.

Users of this document should be aware of the fact that the California standards set high-level achievement objectives, roughly on par with the academic development in some of the most successful countries in international comparisons.

This document focuses on the number sense strand in the California standards. A few of the standards listed come from other strands. The strands are identified by the following codes:

NS Number Sense
AF Algebra and Functions
MG Measurement and Geometry
ST Statistics, Data Analysis, and Probability
MR Mathematical Reasoning

For more detail about the mathematics curriculum in California see:

Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve
[requires Adobe Acrobat reader]

For test items that illustrate the mathematics standards in California see:

Mathematics Assessmentsfrom NCITE/LACOE

Topic Areas

Basic Number Sense - Count, Read, Write, Compare Whole Numbers

- Grade K
  - NS1.0 Students understand the relationship between numbers and quantities (i.e., that a set of objects has the same number of objects in different situations regardless of its position or arrangement)
  - NS1.1 Compare two or more sets (up to ten objects in each group) and identify which set is equal to, more than, or less than the other
  - NS1.2 Count, recognize, represent, name, and order a number of objects (up to 30)
  - NS1.3 Know that the larger numbers describe sets with more objects in them than the smaller numbers have

- Grade 1
  - NS1.0 Students understand and use numbers up to 100
  - NS1.1 Count, read, and write whole numbers to 100
  - NS1.2 Compare and order whole numbers to 100 by using the symbols for less than, equal to, or greater than (<, =, >)
  - NS1.3 Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions (to 20) (e.g., 8 can be represented as 4+4, 5+3, 2+2+2+2, 10-2, 11-3)
  - NS1.4 Count and group objects in ones and tens (e.g., three groups of 10 and 4 equals 34 or 30+4)

- Grade 2
  - NS1.0 Students understand the relationship among numbers, quantities, and place value in whole numbers up to 1,000
  - NS1.1 Count, read, and write whole numbers to 1,000 and identify the place value for each digit
  - NS1.2 Use words, models, and expanded forms (e.g., 45 = 4 tens + 5) to represent numbers (to 1,000)
  - NS1.3 Order and compare whole numbers to 1,000 by using the symbols <, =, >

- Grade 3
    - NS1.0 Students understand place value of whole numbers
    - NS1.1 Count, read, and write whole numbers to 10,000
    - NS1.2 Compare and order whole numbers to 10,000
    - NS1.3 Identify the place value for each digit in numbers to 10,000
    - NS1.5 Use expanded notation to represent numbers (e.g., 3,206 = 3,000 + 200 + 6)

- Grade 4
    - NS1.1 Read and write whole numbers in the millions
    - NS1.2 Order and compare whole numbers and decimals to two decimal places

- Grade 5
    - NS1.0 Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions and percents. They understand the relative magnitudes of numbers
    - NS1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers

Counting

- Grade K
    - NS1.2 Count, recognize, represent, name, and order a number of objects (up to 30)

- Grade 1
    - NS1.1 Count, read, and write whole numbers to 100
    - NS1.4 Count and group objects in ones and tens (e.g., three groups of 10 and 4 equals 34 or 30+4)
    - NS2.4 Count by 2s, 5s, and 10s to 100

- Grade 2
    - NS1.1 Count, read, and write whole numbers to 1,000 and identify the place value for each digit
    - NS3.1 Use repeated addition, arrays, counting by multiples to do multiplication
    - ST1.1 Record numerical data in systematic ways, keeping track of what has been counted

- Grade 3
    - NS1.1 Count, read, and write whole numbers to 10,000
    - AF2.2 Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses can be calculated by counting by 4's or by multiplying the number or horses by 4)

- Grade 4
    - NS1.8 Use simple concepts of negative numbers (e.g., on a number line, in counting, in temperature, "owing")

Place Value of Whole Numbers

- Grade K
  - NS3.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones and tens places
- Grade 1
  - NS1.4 Count and group objects in ones and tens (e.g., three groups of 10 and 4 equals 34 or 30+4)
  - NS3.0 Students use estimation strategies in computation and problem solving that involve numbers that use the one, tens, and hundreds places
- Grade 2
  - NS1.0 Students understand the relationship among numbers, quantities, and place value in whole numbers up to 1,000
  - NS1.1 Count, read, and write whole numbers to 1,000 and identify the place value for each digit
  - NS6.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, hundreds, and thousands places
- Grade 3
  - NS1.0 Students understand place value of whole numbers
  - NS1.3 Identify the place value for each digit in numbers to 10,000
- Grade 4
  - NS1.0 Students understand place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions. Students use the concepts negative numbers

Operations with Money

- Grade 1
  - NS1.5 Identify and know the value of coins and show different combinations of coins that equal the same value
- Grade 2
  - NS5.0 Students model and solve problems by representing, adding, and subtracting amounts of money
  - NS5.1 Solve problems using combinations of coins and bills
  - NS5.2 Know and use the decimal notation and the dollar and cents symbols for money
- Grade 3
  - NS2.7 Determine the unit cost when given the total cost and number of units
  - NS3.3 Solve problems involving addition, subtraction, multiplication, and division of money amounts in decimal notation and multiply and divide money amounts in decimal notation by using whole number multipliers and divisors
  - NS3.4 Know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents 1/2 of a dollar, 75 cents is 3/4 of a dollar)

Addition and Subtraction of Whole Numbers

- Grade K
  - NS2.0 Students understand and describe simple additions and subtractions
  - NS2.1 Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10)

- Grade 1
  - NS2.0 Students demonstrate the meaning of addition and subtraction and use these operations to solve problems
  - NS2.1 Know the addition facts (sums to 20) and the corresponding subtraction facts and commit them to memory
  - NS2.2 Use the inverse relationship between addition and subtraction to solve problems
  - NS2.3 Identify one more than, one less than, 10 more than, and 10 less than a given number
  - NS2.5 Show the meaning of addition (putting together, increasing) and subtraction (taking away, comparing, finding the difference)
  - NS2.6 Solve addition and subtraction problems with one- and two-digit numbers (e.g., 5+58=__)
  - NS2.7 Find the sum of three one-digit numbers
  - AF1.1 Write and solve number sentences from problem situations that express relationships involving addition and subtraction
  - AF1.2 Understand the meaning of the symbols +, -, =
  - AF1.3 Create problem situations that could lead to given number sentences involving addition and subtraction

- Grade 2
  - NS2.0 Students estimate, calculate, and solve problems involving addition and subtraction of two- and three-digit numbers
  - NS2.1 Understand and use the inverse relationship between addition and subtraction (e.g., an opposite number sentence for 8+6=14 is 14-6=8) to solve problems and check solutions
  - NS2.2 Find the sum or difference of two whole numbers up to three digits long
  - NS2.3 Use mental arithmetic to find the sum or difference of two two-digit numbers
  - AF1.0 Students model, represent, and interpret number relationships to create and solve problems involving addition and subtraction
  - AF1.1 Use the commutative and associative rules to simplify mental calculations and to check results
  - AF1.2 Relate problem situations and number sentences involving addition and subtraction
  - AF1.3 Solve addition and subtraction problems using data from simple charts, picture graphs, and number sentences

- Grade 3
  - NS2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division
  - NS2.1 Find the sum or difference of two whole numbers between 0 and 10,000

- Grade 4
  - NS2.1 Estimate and compute the sum or difference of whole numbers and positive decimals to two places
  - NS3.0 Students solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among the operations
  - NS3.1 Demonstrate an understanding of, and the ability to use, standard algorithms for addition and subtraction of multidigit numbers

Computational Targets for Addition and Subtraction with Whole Numbers

- Grade K
  - NS2.1 Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10)

- Grade 1
  - NS2.1 Know the addition facts (sums to 20) and the corresponding subtraction facts and commit them to memory
  - NS2.6 Solve addition and subtraction problems with one- and two-digit numbers (e.g., 5+58=__)
  - NS2.7 Find the sum of three one-digit numbers

- Grade 2
  - NS2.2 Find the sum or difference of two whole numbers up to three digits long
  - NS2.3 Use mental arithmetic to find the sum or difference of two two-digit numbers

- Grade 3
  - NS2.1 Find the sum or difference of two whole numbers between 0 and 10,000

- Grade 4
  - NS3.1 Demonstrate an understanding of, and the ability to use, standard algorithms for addition and subtraction of multidigit numbers

Multiplication and Division of Whole Numbers

- Grade 2
  - NS3.0 Students model and solve simple problems involving multiplication and division
  - NS3.1 Use repeated addition, arrays, counting by multiples to do multiplication
  - NS3.2 Use repeated subtraction, equal sharing, and forming equal groups with remainders to do division
  - NS3.3 Know the multiplication tables of 2s, 5s and 10s (to "times 10") and commit them to memory

- Grade 3
  - NS2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division
  - NS2.2 Memorize to automatcity the multiplication table for numbers between 1 and 10
  - NS2.3 Use the inverse relationship of multiplication and division to compute and check results
  - NS2.4 Solve simple problems involving multiplication of multidigit numbers by one-digit numbers (3,671 x 3 = _)
  - NS2.5 Solve division problems in which a multidigit number is evenly divided by a one-digit number (135÷5)
  - NS2.6 Understand the special properties of 0 and 1 in multiplication and division
  - NS2.7 Determine the unit cost when given the total cost and number of units
  - NS3.3 Solve problems involving addition, subtraction, multiplication, and division of money amounts in decimal notation and multiply and divide money amounts in decimal notation by using whole number multipliers and divisors

- Grade 4
  - NS3.0 Students solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among the operations
  - NS3.2 Demonstrate understanding of, and ability to use, standard algorithms for multiplying a multidigit number by a two-digit number and for dividing a multidigit number by a one-digit number; use relationships between them to simplify computations and to check results
  - NS3.3 Solve problems involving multiplication of multidigit numbers by two-digit numbers
  - NS3.4 Solve problems involving division of multidigit numbers by one-digit numbers
  - AF1.4 Use and interpret formulas (e.g., area = length X width or A=lw) to answer questions about quantities and their relationships

- Grade 5
  - NS1.4 Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 x3$)
  - NS2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors

- Grade 6
  - NS2.3 Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative numbers and combinations of these operations

Computational Targets for Multiplication and Division of Whole Numbers

- Grade 2
  - NS3.3 Know the multiplication tables of 2s, 5s and 10s (to "times 10") and commit them to memory

- Grade 3
  - NS2.5 Solve division problems in which a multidigit number is evenly divided by a one-digit number (135÷5)

- Grade 4
  - NS3.2 Demonstrate understanding of, and ability to use, standard algorithms for multiplying a multidigit number by a two-digit number and for dividing a multidigit number by a one-digit number; use relationships between them to simplify computations and to check results
  - NS3.3 Solve problems involving multiplication of multidigit numbers by two-digit numbers
  - NS3.4 Solve problems involving division of multidigit numbers by one-digit numbers

- Grade 5
  - NS2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors

Decimals

- Grade 2
  - NS4.0 Students understand that fractions and decimals may refer to parts of a set and parts of a whole
  - NS5.2 Know and use the decimal notation and the dollar and cents symbols for money

- Grade 3
  - NS3.0 Students understand the relationship between whole numbers, simple fractions, and decimals
  - NS3.3 Solve problems involving addition, subtraction, multiplication, and division of money amounts in decimal notation and multiply and divide money amounts in decimal notation by using whole number multipliers and divisors
  - NS3.4 Know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents 1/2 of a dollar, 75 cents is 3/4 of a dollar)

- Grade 4
  - NS1.0 Students understand place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions. Students use the concepts negative numbers
  - NS1.2 Order and compare whole numbers and decimals to two decimal places
  - NS1.6 Write tenths and hundredths in decimal and fraction notation and know the fraction and decimal equivalents for halves and fourths (e.g., 1/2 = 0.5 or 0.50; 7/4 = 1 3/4 = 1.75)
  - NS1.9 Identify on a number line the relative position of positive fractions, positive mixed numbers, and positive decimals to two decimal places

- NS2.0 Students extend their use and understanding of whole numbers to addition and subtraction of simple decimals
- NS2.1 Estimate and compute the sum or difference of whole numbers and positive decimals to two places
- NS2.2 Round two-place decimals to one decimal or the nearest whole number and judge the reasonableness of the rounded answer

- Grade 5
  - NS1.0 Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions and percents. They understand the relative magnitudes of numbers
  - NS1.2 Interpret percents as part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number
  - NS1.5 Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers
  - NS2.0 Students perform calculations and solve problems involving addition, subtraction and simple multiplication and division of fractions and decimals
  - NS2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results
  - NS2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors

- Grade 6
  - NS1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages.
  - NS1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line
  - ST3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that probabilities computed are reasonable; know that if P is related the probability of an event, 1-P is the probability of an event not occurring

- Grade 7
  - NS1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions and terminating decimals) and take positive rational numbers to whole number powers
  - NS1.3 Convert fractions to decimals and percents and use these representations in estimation, computations, and applications
  - NS1.5 Know that every fraction is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions

Decimal Arithmetic

- Grade 3
  - NS3.3 Solve problems involving addition, subtraction, multiplication, and division of money amounts in decimal notation and multiply and divide money amounts in decimal notation by using whole number multipliers and divisors

- Grade 4
  - NS2.0 Students extend their use and understanding of whole numbers to addition and subtraction of simple decimals
  - NS2.1 Estimate and compute the sum or difference of whole numbers and positive decimals to two places

- Grade 5
  - NS1.0 Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions and percents. They understand the relative magnitudes of numbers
  - NS2.0 Students perform calculations and solve problems involving addition, subtraction and simple multiplication and division of fractions and decimals
  - NS2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results
  - NS2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors

- Grade 6

- NS1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages.

- Grade 7
  - NS1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions and terminating decimals) and take positive rational numbers to whole number powers

Relationship between Fractions and Decimals

- Grade 2
  - NS4.0 Students understand that fractions and decimals may refer to parts of a set and parts of a whole

- Grade 3
  - NS3.0 Students understand the relationship between whole numbers, simple fractions, and decimals
  - NS3.4 Know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents 1/2 of a dollar, 75 cents is 3/4 of a dollar)

- Grade 4
  - NS1.0 Students understand place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions. Students use the concepts negative numbers
  - NS1.6 Write tenths and hundredths in decimal and fraction notation and know the fraction and decimal equivalents for halves and fourths (e.g., 1/2 = 0.5 or 0.50; 7/4 = 1 3/4 = 1.75)
  - NS1.9 Identify on a number line the relative position of positive fractions, positive mixed numbers, and positive decimals to two decimal places

- Grade 5
  - NS1.0 Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions and percents. They understand the relative magnitudes of numbers
  - NS1.2 Interpret percents as part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number
  - NS1.5 Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers

- Grade 6
  - NS1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line
  - ST3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that probabilities computed are reasonable; know that if P is related the probability of an event, 1-P is the probability of an event not occurring

- Grade 7
  - NS1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions and terminating decimals) and take positive rational numbers to whole number powers
  - NS1.3 Convert fractions to decimals and percents and use these representations in estimation, computations, and applications
  - NS1.5 Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions

Fractions

- Grade 2
  - NS4.0 Students understand that fractions and decimals may refer to parts of a set and parts of a whole
  - NS4.1 Recognize, name, and compare unit fractions from 1/12 to 1/2
  - NS4.2 Recognize fractions of a whole and parts of a group (e.g., one-fourth of a pie, two-thirds of 15 balls)
  - NS4.3 Know that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one

- Grade 3
  - NS3.0 Students understand the relationship between whole numbers, simple fractions, and decimals
  - NS3.1 Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., 1/2 of a pizza is the same amount as 2/4 of another pizza that is the same size; show that 3/8 is larger than 1/4)
  - NS3.2 Add and subtract simple fractions (e.g. determine that 1/8 + 3/8 is the same as 1/2)
  - NS3.4 Know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents 1/2 of a dollar, 75 cents is 3/4 of a dollar)

- Grade 4
  - NS1.0 Students understand place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions. Students use the concepts negative numbers
  - NS1.5 Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; explain equivalents of fractions
  - NS1.6 Write tenths and hundredths in decimal and fraction notation and know the fraction and decimal equivalents for halves and fourths (e.g., 1/2 = 0.5 or 0.50; 7/4 = 1 3/4 = 1.75)
  - NS1.7 Write the fraction represented by a drawing of parts of a figure; represent a given fraction by using drawings; and relate a fraction to a simple decimal on a number line
  - NS1.9 Identify on a number line the relative position of positive fractions, positive mixed numbers, and positive decimals to two decimal places

- Grade 5
  - NS1.0 Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions and percents. They understand the relative magnitudes of numbers
  - NS1.2 Interpret percents as part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number
  - NS1.5 Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers
  - NS2.0 Students perform calculations and solve problems involving addition, subtraction and simple multiplication and division of fractions and decimals
  - NS2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less) and express answers in the simplest form
  - NS2.4 Understand the concept of multiplication and division of fractions
  - NS2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems
  - ST1.3 Use fractions and percentages to compare data sets of different sizes

- Grade 6
  - NS1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages.
  - NS1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line
  - NS1.3 Use proportions to solve problems (e.g., determine the value of N if 4/7 = N/21, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as multiplication of both sides of an equation by a multiplicative inverse.
  - NS2.1 Solve problems involving addition, subtraction, multiplication, and division of fractions and explain why a particular operation was used for a given situation
  - NS2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., 5/8 ÷15/16 = 5/8 x 16/15 = 2/3)
  - NS2.4 determine the least common multiple and greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator in order to add two fractions or to find the reduced form for a fraction)

- Grade 7
  - NS1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions and terminating decimals) and take positive rational numbers to whole number powers
  - NS1.3 Convert fractions to decimals and percents and use these representations in estimation, computations, and applications
  - NS1.5 Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions
  - NS2.0 Students use exponents, powers, and roots and use exponents in working with fractions.
  - NS2.2 Add and subtract fractions using factoring to find common denominators
  - NS2.3 Multiply, divide, and simplify rational numbers by using exponent rules

Arithmetic with Fractions

- Grade 3
  - NS3.1 Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., 1/2 of a pizza is the same amount as 2/4 of another pizza that is the same size; show that 3/8 is larger than 1/4)
  - NS3.2 Add and subtract simple fractions (e.g. determine that 1/8 + 3/8 is the same as 1/2)

- Grade 5
  - NS1.0 Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions and percents. They understand the relative magnitudes of numbers
  - NS1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers
  - NS2.0 Students perform calculations and solve problems involving addition, subtraction and simple multiplication and division of fractions and decimals
  - NS2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less) and express answers in the simplest form
  - NS2.4 Understand the concept of multiplication and division of fractions
  - NS2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems

- Grade 6
  - NS2.1 Solve problems involving addition, subtraction, multiplication, and division of fractions and explain why a particular operation was used for a given situation
  - NS2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., 5/8÷15/16 = 5/8 x 16/15 = 2/3)
  - NS2.4 determine the least common multiple and greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator in order to add two fractions or to find the reduced form for a fraction)

- Grade 7
  - NS1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions and terminating decimals) and take positive rational numbers to whole number powers
  - NS2.0 Students use exponents, powers, and roots and use exponents in working with fractions.
  - NS2.2 Add and subtract fractions using factoring to find common denominators
  - NS2.3 Multiply, divide, and simplify rational numbers by using exponent rules

Estimation and Rounding

- Grade K
  - NS3.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones and tens places
  - NS3.1 Recognize when an estimate is reasonable

- Grade 1
  - NS3.0 Students use estimation strategies in computation and problem solving that involve numbers that involve numbers that use the one, tens, and hundreds places
  - NS3.1 Make reasonable estimates when comparing larger or smaller numbers

- Grade 2
  - NS2.0 Students estimate, calculate, and solve problems involving addition and subtraction of two- and three-digit numbers
  - NS6.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, hundreds, and thousands places
  - NS6.1 recognize when an estimate is reasonable in measurements (e.g., closest inch)

- Grade 3
  - NS1.4 Round off numbers to 10,000 to the nearest ten, hundred, and thousand
  - MG1.1 Choose appropriate tools and units (metric and U.S.) and estimate and measure length, liquid volume, and weight/mass of given objects
  - MG1.2 Estimate or determine the area and volume of solid figures by covering them with squares or by counting the number of cubes that would fill them
  - MR2.1 Use estimation to verify the reasonableness of calculated results

- Grade 4
  - NS1.3 Round whole numbers through the millions to the nearest ten, hundred, thousand, ten thousand, or hundred thousand
  - NS1.4 Decide when a rounded solution is called for and explain why such a solution may be appropriate
  - NS2.1 Estimate and compute the sum or difference of whole numbers and positive decimals to two places
  - NS2.2 Round two-place decimals to one decimal or the nearest whole number and judge the reasonableness of the rounded answer
  - MR2.1 Use estimation to verify the reasonableness of calculated results

- Grade 5
  - NS1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers
  - MR2.1 Use estimation to verify the reasonableness of calculated results

- Grade 6
  - MG1.2 Know common estimates of (3.14; 22/7) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements
  - ST3.2 Use data to estimate the probability for future events (e.g., batting averages or number of accidents per mile driven)
  - MR2.1 Use estimation to verify the reasonableness of calculated results
  - MR2.3 Estimate unknown quantities graphically and solve for them using logical reasoning and arithmetic and algebraic techniques

- Grade 7
  - NS1.3 Convert fractions to decimals and percents and use these representations in estimation, computations, and applications
  - MG2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects
  - MR2.1 Use estimation to verify the reasonableness of calculated results
  - MR2.3 Estimate unknown quantities graphically and solve for them using logical reasoning and arithmetic and algebraic techniques

Negative Numbers

- Grade 4
  - NS1.0 Students understand place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions. Students use the concepts negative numbers
  - NS1.8 Use simple concepts of negative numbers (e.g., on a number line, in counting, in temperature, in "owing")
- Grade 5
  - NS1.5 Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers
  - NS2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results
- Grade 6
  - NS1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line
  - NS2.3 Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations that use positive and negative numbers and combinations of these operations

Integers

- Grade 3
  - MG1.3 Find the perimeter of a polygon with integer sides
- Grade 5
  - NS1.3 Understand and compute positive integer powers of nonnegative integers; compute examples as repeated multiplication
  - NS1.5 Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers

Rational Numbers

- Grade 7
  - NS1.0 Students know the properties of, and compute with, rational numbers expressed in a variety of forms
  - NS1.1 Read, write and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation
  - NS1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions and terminating decimals) and take positive rational numbers to whole number powers
  - NS1.4 Differentiate between rational and irrational numbers

Absolute Value

- Grade 7
  - NS2.5 Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers

Multiples, Factors, Primes, Composites

- Grade 4
  - NS4.0 Students know how to factor small whole numbers
  - NS4.1 Understand that many whole numbers break down in different ways (e.g., 12 = 4x3 = 2x6 = 2x2x3)
  - NS4.2 Know that numbers such as 2, 3, 5, 7, 11 do not have any factors except 1 and themselves and that such numbers are called prime numbers

- Grade 5
  - NS1.4 Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$)

- Grade 6
  - NS2.4 determine the least common multiple and greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator in order to add two fractions or to find the reduced form for a fraction)

- Grade 7
  - NS2.2 Add and subtract fractions using factoring to find common denominators

Powers, Exponents, and Roots

- Grade 5
  - NS1.3 Understand and compute positive integer powers of nonnegative integers; compute examples as repeated multiplication
  - NS1.4 Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$)

- Grade 7
  - NS1.1 Read, write and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation
  - NS1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions and terminating decimals) and take positive rational numbers to whole number powers
  - NS2.0 Students use exponents, powers, and roots and use exponents in working with fractions.
  - NS2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base
  - NS2.3 Multiply, divide, and simplify rational numbers by using exponent rules
  - NS2.4 Use the inverse relationship between raising to a power and extracting a root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why
  - AF2.0 Students interpret and evaluate expressions involving integer powers and simple roots.
  - AF2.2 Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent

Percentages

- Grade 5
  - NS1.0 Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions and percents. They understand the relative magnitudes of numbers
  - NS1.2 Interpret percents as part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number
  - ST1.3 Use fractions and percentages to compare data sets of different sizes

- Grade 6
  - NS1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages.
  - NS1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips
  - ST3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that probabilities computed are reasonable; know that if P is related the probability of an event, 1-P is the probability of an event not occurring

- Grade 7
  - NS1.3 Convert fractions to decimals and percents and use these representations in estimation, computations, and applications
  - NS1.6 Calculate percentage of increases and decreases of a quantity
  - NS1.7 Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest

Ratios, Rates, and Proportions

- Grade 6
  - NS1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages.
  - NS1.2 Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations (a/b, a to b, a:b)
  - NS1.3 Use proportions to solve problems (e.g., determine the value of N if 4/7 = N/21, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as multiplication of both sides of an equation by a multiplicative inverse.
  - AF2.0 Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions.
  - AF2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches)
  - AF2.2 Demonstrate understanding that rate is a measure of one quantity per unit value of another quantity
  - AF2.3 Solve problems involving rates, average speed, distance, and time
  - ST3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that probabilities computed are reasonable; know that if P is related the probability of an event, 1-P is the probability of an event not occurring

- Grade 7
  - AF4.2 Solve multistep problems involving rate, average speed, distance, and time or a direct variation
  - MG1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer

Some observations on the 1997 battle of the two Standards in the California Math War

H. Wu

1.

In October of 1997, the Standards Commission of California submitted to the State Board of Education a set of Mathematics Content Standards [1] which took the Commission more than a year to complete. But within ten weeks, the State Board released a revised version of its own [2], first the portion on grades K--7 and then that on 8-12. The reaction to the revision was swift and violent:

> "*[The Board's Standards] is 'dumbed down' and is unlikely to elicit higher order thinking from the state's 5.5 million public school students.*"
>
> Delaine Eastin, as reported in the NY Times
>
> "*I will fight to see that California Math Standards are not implemented in the classrooms.*"
>
> Judy Codding, as quoted at an NCEE conference
>
> "*The critics claimed the Board's `back-to-basics' approach marked a return to 1950s-style methods. ... Opponents characterized the [Board's] Standards as a 'return to the Dark Ages*'"
>
> as reported in the San Diego Union

The interest engendered by these two sets of Standards has remained unabated in the intervening months. For example, in the February issue of its *News Bulletin*, NCTM has weighed in with unflattering comments about the Board's revised version [3]. Because education is a very political issue, there is no need to bemoan the fact that opinions are often delivered without relation to facts. However, a set of *mathematics* standards for schools also deserves a critical inspection from the mathematical and educational perspectives, one that is based on facts and not on hype. With this in mind, this article takes a close look at both sets of standards from a scholar's perspective. Section 2 details some of the mathematical flaws in the Commission's Standards, and Section 3 contrasts these flaws with the clarity and the overall mathematical soundness of the Board's revision. In Section 4 there is a discussion of possible additions to the California Mathematics Framework Draft [4] that would enhance and provide balance for the Board's Standards.

The Commission's Standards is a thoughtful document. In both the Interim Report from the Commission Chair to the State Board and the Introduction to Mathematics Standards, one sees clearly the care that went into the enunciation of the goals, the work that had been done to achieve them, and the work that is still needed in the days ahead for their implementation. Even if one disagrees with some of the details, one can applaud the overall soundness of purpose and the conscientious effort that went into the writing. Yet there are also grievous defects in the document that made its revision inevitable. This is a classic example of how good intentions are shipwrecked by questionable execution. Parts of the document are extremely controversial, such as the omission of the division algorithm in the lower grades[1], the omission of the Fundamental Theorem of Algebra in the upper grades, or the mixing of pedagogical statements with statements on content. There is also a pervasive ambiguity of language that makes the document less than readable in many places, e.g., were the authors aware that the word "classify" has a precise meaning in a mathematical context? Or, what is a 7th grader to make of "identify, describe, represent, extend and create linear and nonlinear number patterns? " However, this article chooses to focus attention on the numerous mathematical defects because they are more susceptible to a dispassionate discussion.

But first, are mathematical defects really that important in a set of mathematics standards? Such a question, were it posed thirty years ago, would have been met with howls of derision. Times have changed, however, and there are those who claim that it is not what is taught, but *how* it is taught that matters (cf. e.g., [10], especially pp. 203-8). Risking some sneers from my colleagues, let me therefore affirm that indeed I believe getting the mathematics right is very important in any mathematics standards, in the same way that correct pronunciation is critical to being a good teacher of a foreign language. Are there people who feel comfortable about sending their children to a French class taught by a teacher who mispronounces a word every other sentence? While the general public cannot conceive of a set of mathematics standards not being mathematically correct, the fact remains that mathematically correct public documents on mathematics education are more rare than people realize. For example, the mathematics standards of most of the states from around the nation exhibit mathematical ignorance (cf. [11]). Even the NCTM Standards [5] is no exception to this rule: there is an outright mathematical error at the top of p. 136, and many discussions show a lack of understanding of the underlying mathematics (e.g., p. 149 and bottom of p. 165; cf. also [12]). This is why when this article comes around to affirming the mathematical correctness of the Board's Standards later on, such an affirmation must be taken as strong endorsement.

2.

The mathematical flaws in the Commission's Standards [1] are of two kinds. First there are the *local* ones, *i.e.*, those which contain obvious errors which can be corrected without causing damage elsewhere. A colleague has estimated that there are over a hundred of these, and that is a conservative estimate. Since it is impossible to be exhaustive, we will only exhibit a few that are easily understood even when taken out of context. Starting with the Glossary at the end, we find, for example:

> Asymptote: *a straight line to which a curve gets closer and closer but never meets, as the distance from the origin increases*

Since this definition of an asymptote does not specify that the distance between the curve and the straight line has to decrease to zero, it would make the line y=-1 an asymptote of y=1/x for x>0.

> Axiomatic system: *system that includes self-evident truths: truths without proof and from which further statements, or theorem, can be derived*

By dictating that the "axioms" of an axiomatic system must be "self-evident truths", this definition excludes the axioms for non-Euclidean geometry from being an axiomatic system. After all, the statement that given a line and a point not on the line there are infinitely many lines from the point not intersecting the given line is certainly not a `self-evident truth".[2]

> Recursive function: *in discrete mathematics, a series of numbers in which values are derived by applying a formula to the previous value*

This term has a precise technical meaning in symbolic logic, and its definition is nothing this simple. Perhaps the authors had in mind "recurrence relations" instead. Assuming this to be the case, then the correct definition would change "the previous value" to "previous values". Otherwise, even the Fibonacci numbers would not fit this description.

Next, we turn to the Standards proper and look at some representative local flaws. It may be noted that the following examples do not include any that might have been the result of carelessness, such as that about the asymptotes of a polynomial (*Clarification and Examples*:[3] for 1.1 and 1.2 in Algebra and Functions of grades 11/12).

> **Grades K-8***Problem Solving and mathematical Reasoning*
> ***2.1*** *Predict outcomes and make reasonable estimates.*

It is not common to equate "predict outcomes" with mathematical reasoning. One would gladly overlook this as an inadvertent error but for the fact that the same sentence appears nine times all through grades K through 8

> **Grade 3***Problem Solving and Mathematical Reasoning*
> Clarifications and Examples *Your friend in another classroom says that her classroom is "bigger" than yours. Find the answer, and prove that your solution is correct using mathematics you have learned this year. (Note: Students should be able to approach this task using concepts of perimeter and area.)*

This passage is supposed to *clarify* the content of the Standards, but it has achieved the opposite effect of obfuscating it. It would take many pages to write an analysis that does this passage justice, so here is a very abbreviated account (but see [13]). First of all, mathematics deals with precise statements, and if we are going to educate our children at all, we would do well to teach them the necessity of eliminating the inherent fuzziness in many everyday utterances before transcribing them into mathematical terms. "Her classroom is bigger" is clearly a case in point. Faced with such a statement, a set of mathematics standards has the responsibility to instruct children of grade 3 to make sense of the word "bigger" before proceeding any further. If they *interpret* "bigger" to mean "more area", then they should measure the respective areas. If they *interpret* "bigger" to mean "longer perimeter", then measure the perimeters. The basic message is therefore that each answer would be correct according to whichever interpretation is used. Furnishing such an explanation would seem to be the minimum requirement of a mathematics education for the young. Now look at the passage above: it tells teachers and students alike to accept an instruction that has no precise meaning ("bigger") and immediately proceed to "find the answer", and worse, "prove that your solution is correct using mathematics". If a teacher in an English class shows students a black box without telling them what is inside other than that it is an expensive piece of jewelry, and asks them to write an essay to describe the latter and *justify why their description fits the object*, there would be an uproar. Yet when the *same* thing happens in a set of mathematics standards, we have people leaping to its defense and calling it "world class". Why is that?

> **Grade 4***Measurement and Geometry*
> **1.***Students understand and use the relationship between the concepts of perimeter and area, and relate these to their respective formulas.*

> **Grade 5** *Measurement and Geometry*
> **1.** *Students understand the relationship between the concepts of volume and surface area and use this understanding to solve problems.*

The trouble with both standards is that there is no relationship whatever between perimeter and area, or between volume and surface area, unless it be the isoperimetric inequality. However, the latter would be quite inappropriate for students at this level. What could the authors have in mind?

> **Grade 5***Number Sense*
> Clarifications and Examples *What is the fractional value of each of the tangram pieces to the whole set of tangrams? Determine equivalences between one or more pieces and other pieces, based on the fractional values that you have determined.*

What does "fractional value" refer to in this case? Does each piece count as one unit, or is the area of each piece being sought in proportion to the whole area? What kind of "equivalence" between the pieces is intended here and why has it not been clearly defined?

> **Grade 6***Number Sense*
> Clarifications and Examples *Emphasize how fractions and ratios as well as operations involving them are similar and how they can differ.*

Since a fraction is a ratio of integers, how can there be any difference between them with respect to their mathematical operations? Some educators, it is said, have begun to advocate that fractions are not ratios. If so, then we must redouble our efforts not to allow such ideas to creep into any mathematics standards.

> **Grade 6***Measurement and Geometry*
> **1.2***Determine estimates of pi (3.14; 22/7) and use these values to estimate and calculate the circumference and the area of circles.*

There is no explanation of how a 6th grade student could "determine estimates of pi" with this kind of accuracy, 3.14 or 22/7, especially the latter value. Is such a precise estimate even remotely conceivable?

> **Grade 7***Algebra and Functions*
> Clarifications and Examples *Order of operations may be helpful when evaluating expressions such as $3(2x+5)^2$, recognizing the structure of the algebraic notation may be more helpful when evaluating $3(2x+5)$; both should be included as techniques.*

The order of operations to evaluate algebraic expressions is a matter of *definition*, and is not a technique. Moreover, to say in a mathematics standards document that knowing the simple definition of the notation is more helpful in the situation of $3(2x+5)$ than in $3(2x+5)^2$ is to undercut its own credibility.

> **Grade 8***Algebra and Functions*
> Clarifications and Examples *Record and graph the relationship between time and the height of water in a cylindrical container when a drain on the bottom of the container is open and determine an equation which generalizes the situation.*

One would *guess* (although that is asking a lot of the general reader of the Standards) that the "relationship between the time and the height of water" is that the height is a well defined function of time. This function happens to be quadratic, but what could it mean to find an equation that would *generalize the situation*?

> **Grades 9/10** *Algebra and Functions*
> Clarifications and Examples *Students should understand the fact that equations in one variable (e.g., (x-3)(x+1)(x-1)=0, 3^{x-2} -81=0) have related two-variable counterparts (e.g., (x-3)(x+1)(x-1)=y, 3^{x-2} -81=y) and use this fact to solve or check the original equation and analyze the graph.*

If the intended message is that the zeros of a given polynomial can be approximated by examining the intersection of its graph and the x-axis, then this statement is very poorly phrased. If the intended message is something else, then obviously this statement needs to be completely re-written.

Next we examine a different kind of mathematical flaw: the global ones. Their corrections would involve changes in several related parts. The first such example occurs in grade 7:

> **Grade 7** *Measurement and Geometry*
> **3.2** *understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under simple transformations in the plane*

Now Grade 7 is not the usual place to find references to *simple transformations in the plane* and their *images*. What is meant by a "simple transformations"? Has it been defined? Has the image of a transformation been discussed? It turns out that "simple transformations" are defined nowhere in the Standards, but one could guess from related comments that the authors had in mind *reflections* and *translations*. It is difficult to decide whether the authors were unaware of the need to fulfill the minimum mathematical requirement of clarity, or simply considered such matters unimportant. Could such negligence be nothing more than a momentary lapse? Not likely, because one also finds in grades 9/10 another reference to "transformations" in the plane with no explanation:

> **Grades 9/10** *Algebra and Functions*
> **1.** *Students classify and identify attributes of basic families of functions (linear, quadratic, power, exponential, absolute value, simple polynomial, rational, and radical).*
> **1.4** *demonstrate and explain the effect that transformations have on both the equation and graph of a function*

A pertinent related issue in connection with the above standard in grade 7 is how much coordinate geometry has been developed up to that point so that students may appreciate such a discussion. The answer appears to be "not enough". The first introduction of coordinates in the plane takes place in the 4th grade under, of all places, Algebra and Functions:

> **Grade 4** *Algebra and Functions*
> **1.** *Students use and interpret variables, mathematical symbols and properties to write and simplify expressions and sentences.*
> **1.4** *understand and use two-dimensional coordinate grids to find locations, and represent points and simple figures*

Special attention should be called to the fact that the important idea of "algebraicizing" the geometric plane occurs here almost as an after-thought in a discussion on variables and mathematical symbols. Since it is now fashionable to talk about "conceptual understanding", one can say unequivocally that such a set of mathematical standards displays a lack of conceptual understanding of mathematics. But to continue with the present discussion, in the standards of grade 5, one finds "write the [linear] equation and graph the resulting ordered pairs of whole numbers on a grid" (*Algebra and Functions*, 2.2) and in grade 6, more graphing of linear functions and "single variable data" is called for in *Algebra and Functions* and *Statistics, Data Analysis and Probability*. This would seem to be the extent to which students have been exposed to coordinate geometry before they are asked to contemplate the image of a transformation in the plane.

Consider now a second example, which is the way the Commission's Standards approaches the Pythagorean theorems, a fundamental result in school mathematics. The first mention of this theorem is in grade 7:

> **Grade 7** *Measurement and Geometry*
> **3.3** *use the Pythagorean Theorem to find the length of the missing side of a right triangle and lengths of other line segments, and check the reasonableness of answers found in other ways.*
> Clarifications and Examples *Help students understand the relationship between the Pythagorean Theorem and direct measurement. Experience with both measurement tools and measurement on a coordinate grid should be included.*

(One's first reaction to the last sentence---"*Experience* with ..."---is: "How very Californian!") This standard certainly makes it sound as though the Pythagorean theorem is a tool already familiar to the students, and it took some time to find out that in fact this is the first time the theorem is discussed. One could bend over backward to give a benign interpretation of this standard as: "State the Pythagorean theorem and verify it empirically by direct measurements". Few readers, however, would recognize that this is the intended message. Because this theorem is so surprising to a beginner, one would expect a demonstration of its truth early on. For example, the so-called "tangram proof" using four congruent right triangles nestled in a square is so elementary that it could be presented to 4th or 5th graders. One finds instead that when the theorem is mentioned again in grade 8 and for the first time in grades 9/10, no proof is mentioned:

> **Grade 8***Measurement and Geometry*
> **1.3***use the Pythagorean Theorem to determine distance and compare lengths of segments on a coordinate plane.*
> *Clarifications and Examples Include using the Pythagorean Theorem to confirm accuracy of scale drawings, and contexts involving coordinate graphing.*
>
> **Grades 9/10***Measurement and Geometry*
> **2.4***use the Pythagorean Theorem, its converse, properties of special right triangles (e.g., sides in the ratio 3-4-5) and right triangle trigonometry to find missing information about triangles.*

It should be obvious that this standard in grade 8 merely repeats what is already in grade 7. What purpose does this serve? In addition, is it good education to ask students to believe in the converse of this theorem in grades 9/10, as indicated above, without first giving them a proof of the theorem itself? It remains to point out that only later in sub-Standard 4.4 of Measurement and Geometry, grades 9/10, do we find: "prove the Pythagorean Theorem using algebraic and geometric arguments".

It was mentioned earlier that the Commission's Standards omits the long division algorithm in the early grades except for the case of a single digit divisor (grade 4). With that in mind, let us look at what happens in grade 7.

> **Grade 7***Number Sense*
> **1.3***describe the equivalent relationship among representations of rational numbers (fractions, decimals and percents) and use these representations in estimation, computation and applications.*
> Clarifications and Examples *Students should understand the relationship between terminating and repeating decimals and fractions.*

Yet the mere fact that a fraction yields a repeating decimal depends on the understanding of the sequence of remainders in the division algorithm. How are students going to *understand* that terminating and repeating decimals represent fractions without first knowing this algorithm by heart? Furthermore, in grades 11/12, we have:

> *Algebra and Functions*
> Clarifications and Examples *Graphing calculators, long and synthetic division may be used to factor polynomials and rational equations to verify attributes of the equation and graph.*

Perhaps not enough thought was given to the fact that, without learning the division algorithm for integers, it may not be possible to teach synthetic division for polynomials.

Incidentally, the preceding two examples from the Commission's Standards show an all-too-common sloppiness of language: "equivalent relationship among ...", "relationship between terminating and repeating decimals ...", and "attributes of the equation and graph" are too vague for a set of mathematics standards.

As a final example, let us look at how the Commission's Standards handles the concept of a function. Although the term "functional relationship" is used already in grades 4 and 5 ("the functional relationships within linear patterns" in grade 4, and "solve problems involving functional relationships" in grade 5), a knowledgeable reader could conceivably deal with such missteps by ignoring them. (The Board's Standards in fact simply deletes all such references.) However, in grade 6 of Commission's Standards, one finds:

> **Grade 6***Algebra and Functions*
> **2.***Students analyze tables, graphs and rules to determine functional relationships and interpret, and solve problems involving rates.*
> **2.1***identify and express functional relationships in verbal, numeric, graphical and symbolic form.*

Since it calls for a direct confrontation with the concept of a function itself, this standard is less likely to be ignored and the potential damage is consequently greater than before. Are students to learn about the definition of a function, or are they not? That is the question. The hazy conception of mathematics itself as exemplified in this instance (and elsewhere too, of course) is unnerving to the mathematically informed. If one cannot resolve this issue here, what about the next one in grade 8?

> **Grade 8***Algebra and Functions*
> **1.1***identify the input and output in a relationship between two variables and determine whether the relationship is a function.*
> Clarifications and Examples *Students should be able to identify key ideas when a relationship is expressed through a table, with symbols, or through a graph.*

Because this explicitly asks students to distinguish between a *relation* and a *function*, nothing short of a full-scale investigation of the functional concept would suffice. But should one do this in grade 8, and is this really what the authors had in mind? The answer seems to be supplied, however indirectly, by the following standard in grade 9:

> **Grades 9/10***Algebra and Functions*
> **2.***Students demonstrate understanding of the concept of a function, identify its attributes, and determine the results of operations performed on functions.*

It would appear that here is the first time that students learn what a function is. If this is to be believed, then what is one to make of all the rumblings on this topics in grades 4 through 8? But if not, *i.e.,* if a function is supposed to have been defined earlier, then what is such a standard doing in grades 9/10?

I hope the foregoing gives some idea of the magnitude of the problems besetting the Commission's Standards. At the same time, it should be pointed out that these problems are probably not detectable by someone who is not mathematically knowledgeable. The criticisms of the Board's Standards coming from educators and politicians are therefore understandable to a certain degree. By the same token, this gap in mathematical knowledge then imposes on those of us in mathematics the obligation to serve as intermediaries between the Standards and the public. Regardless of our philosophical orientations in matters pertaining to education, we should have spoken as a single voice in detailing the glaring mathematical failings of the Commission's Standards in order to furnish a valid platform for the ensuing debate. We should have been the voice of reason to inform and to mediate. The mathematical community in California may well ask itself at this point if it has indeed met its obligation, and met it well.[4]

3.

Now a brief look at the Board's Standards [2]. The overriding fact is that no document of this nature can be expected to be without blemish, and it would be foolhardy to look for perfection or to argue that this set of Standards is close to perfection. What is important is to ask whether it has fatal flaws, and whether in the main it points in the right direction of a sound mathematics education. The answers to both are easily *no* and *yes*, respectively, and their justifications will emerge in the succeeding discussion.

An important point regarding the Board's Standards is that, in reading this document, one does not wince in embarrassment over mathematical errors. Let us first start with the portion on grades K--7. This portion is *very* close to the Commission's Standards, and the only difference between the two is that the Board's version eliminates the ambiguous and superfluous, corrects the erroneous, and deletes the *Clarifications and Examples* in the right column of the original. I will have more to say about the latter presently, but let us sample some of the differences. It was mentioned above that in grade 4, the Commission's Standards incorrectly asks for "the relationship between the concepts of perimeter and area". By comparison, the Board's version now reads:

> **Grade 4***Measurement and Geometry*
> **1.***Students understand perimeter and area.*
> > **1.1***measure the area of rectangular shapes, using appropriate units (cm^2, m^2, kmD^2, yd^2, square mile)*
> > **1.2***recognize that rectangles having the same area can have different perimeters*
> > **1.3***understand that the same number can be the perimeter of different rectangles, each having a different area*
> > **1.4***understand and use formulas to solve problems involving perimeters and areas of rectangles and squares. Use these formulas to find areas of more complex figures by dividing them into parts with these basic shapes*

It is clear, and it is correct. More than that, 1.2 and 1.3 anticipate students' possible confusion, and 1.4 emphasizes the importance of applications and the general principle of progressing from the simple to the complex.

Another example is the Board's correction of the error committed in the Commission's version regarding the introduction of coordinates in the plane in grade 4. Now it is accorded a standard all its own and is placed correctly in the strand on Measurement and Geometry.

> **Grade 4** *Measurement and Geometry*
> **2.** *Students use two-dimensional coordinate grids to represent points and graph lines and simple figures*
>
> > **2.1** *draw the points corresponding to linear relationships on graph paper (e.g., draw the first ten points*[5] *for the equation* $y=3x$ *and connect them using a straight line)*
> > **2.2** *understand that the length of a horizontal line segment equals the difference of the x-coordinates*
> > **2.3** *understand that the length of a vertical line segment equals the difference of the y-coordinates*

Note that sub-standard 2.1 pays special attention to the *tactile* aspect of learning mathematics: use graph papers and draw the first ten points (by hand). We should be grateful that it does not say: enter these data in a graphing calculator and watch the graph emerge on the screen. Moreover, sub-standards 2.2 and 2.3 again anticipate students' confusion by singling out two key points for discussion. There is no question that this is an education document that truly tries to educate.

As a final example, let us look at how the Board's version discusses in one instance the issue of mathematical reasoning:

> **Grade 4** *Mathematical Reasoning*
> **3.** *Students move beyond a particular problem by generalizing to other situations.*
>
> > **3.1** *evaluate the reasonableness of the solution in the context of the original situation.*
> > **3.2** *note method of deriving the solution and demonstrate conceptual understanding of the derivation by solving similar problems.*
> > **3.3** *develop generalization of the results obtained and extend them to other circumstances.*

In plain English---readable English---this standard lays out a step-by-step method of doing mathematics. Educational writing can be no better than this.

It is improvements of this nature that make the Board's Standards [2] a superior document over the Commission's Standards [1] in grades K--7. Yet, intense criticisms were already pouring in as soon as the K--7 portion of the Board's Standards appeared. Looking at the facts, how does one presume to claim that this set of standards is "basics only", or that it "almost cuts out almost everything that is not related to computation and the memorization of formulas" ? Obviously not on account of the standards themselves, but one explanation is that some people reacted strongly to the deletion of the *Clarifications and Examples* that are in the Commission's Standards.

It is time to point out that whereas in other states the Mathematics Standards must stand alone as the sole guide-post for mathematics education, we in California have two documents: the Standards *and* the Framework [4]. In this arrangement, the curricular comments on the Standards, including examples, properly belong to the Framework, which is yet to be approved by the Board. It serves no purpose to criticize the absence of examples in the Board's Standards when they have merely been moved to a companion document. If one's goal is to improve the Board's Standards rather than stir up controversy, the natural thing to do would be to make concrete suggestions for changes in the existing Framework Draft. This article will make several such suggestions.

Let us complete our brief survey of the Board's Standards by looking at grades 8--12. There is a basic change of format here, in that the grade-by-grade account in the Commission's version is replaced by a listing of topics in the traditional strands across the grades: Algebra I, Geometry, Algebra II, etc. The justification is that since at present an overwhelming majority of the schools teach mathematics in the traditional manner while others do so in an "integrated"[7] manner, listing only the content of each subject would provide maximum flexibility. Instead of prescribing one particular approach to the curriculum, it throws the door open to many approaches. Such a change is a defensible one, and is in any case not one to make a lot of fuss about. With this understood, one can immediately appreciate the clear and uncompromising demand that the Board's Standards places on students' all-around mathematical competence---not the formula-laden, rote-learning variety, but the genuine one. Students must be technically proficient, and they must also know what they are doing. For example, consider the discussion of the quadratic formula in Algebra I (which contains twenty-five standards):

> **Algebra I** *(Grades 8-12)*
> **14.** *Students solve a quadratic equation by factoring or completing the square.*

19. *Students know the quadratic formula and are familiar with its proof by completing the square*
20. *Students use the quadratic formula to find the roots of a second degree polynomial and to solve quadratic equations.*

It does not say: derive the quadratic formula and use it to solve all quadratic equations. Instead, it makes students learn the important technique of completing the square first. Five standards later---which presumably indicates that one gives students time to digest it before proceeding---it asks for a derivation of the formula. Then, and only then, does it mention *using* the formula to solve equations. Does a document that handles the learning of a formula in this sensitive manner strike anyone as a "back-to-basics" document that emphasizes memorization and computation? Next, a similar example in a different subject:

Geometry *(Grades 8-12)*
2. *Students write geometric proofs, including proofs by contradiction.*
3. *Students construct and judge the validity of a logical argument. This includes giving counterexamples to disprove a statement.*
4. *Students prove basic theorems involving congruence and similarity.*

The unequivocal demand on students' ability to write down proofs and counterexamples is important in this day and age of diminished standards when proofs produce allergic reactions in many education circles. One can quibble with the precise meaning of standard 4---and more of this later---but that is not the same as insinuating that these Standards ax the development of mathematical understanding in the students. My personal opinion is that these are thoughtful standards, but their virtues are by no means apparent to the general public. Perhaps for this reason, the torrent of abuse heaped on these Standards took over the front pages of many newspapers for several weeks. Here are some reminders:

"I think [the Board's Standards are] half a loaf. We went from a world-class set of standards to one that cannot be characterized as world-class"

"The reality is one set of standards had basics and *problem-solving and conceptual understanding but what the Board adopted was the basics only."*

Delaine Eastin

"When the State Board took a knife to the Commission's Standards, it cut out almost everything that was not related to computation and the memorization of formulas. What was gained? Nothing.... What the State Board deleted or weakened were Standards intended to make sure students understand the key concepts underlying mathematics."

Judy Codding

"While emphasizing important basics and memorization, [the Board's set of Standards] axes development of understanding, applications and critical thinking skills students will need to live in the 21th century.

In one stroke, the Board discards the last three years' hard work and reasoned consensus among math professors and teachers, college professors who use math in their teaching (science and business) and public representatives."

James Highsmith, Chair, Academic Senate, CSU
Letter to the editor,LA Times

"The wistful or nostalgic `back-to-basics' approach that characterizes the Board Standards overlooks the fact that the approach has chronically and dismally failed. It has excluded youngsters from engaging in genuine mathematical thinking and therefore true mathematical learning."

Luther Williams, *NSF Director for Education and Human Resources*
Letter to the Board

"The Commission's Standards are the best set of mathematics standards in the U.S. ... The Board's Standards are most disappointing, [and are nothing more than] a `back-to-basics' document that emphasizes memorization and computations."

William Schmidt
Executive Director for the U.S. Center for TIMSS

It may be noted that the NCTM editorial [3] endorsed the preceding statement by Luther Williams.

One may ask, in light of all the flaws in the Commission's Standards and the obvious emphasis on mathematical understanding in the Board's version, how people could bring themselves to make indefensible statements about the high quality of the former and the unworthiness of the latter. There are probably political and psychological reasons that are beyond my power to probe, but as an educator, I would like to offer a speculation on how this has happened. I believe there is a fundamental misconception about mathematics education that has sprung up more or less in the past decade, which is that there are *conceptual understanding* and *problem solving ability* on the one hand and *basic skills* on the other. Furthermore, this misconception postulates that it is possible to acquire the former without the latter. It is likely that the explicit requirement of fluency in basic skills in the Board's Standards was seen by some as an artificial obstacle intentionally set up by elitists to thwart students' "mathematical empowerment". Hence the resulting furor. One *can* acquire some appreciation of mathematics without mastering technical skills, in much the same way that one can instantly recognize an opera in recordings of "operas without the human voice"[8] and even enjoy it to some extent. But if we wish to educate students properly about the art of the opera, using such recordings "without the human voice" is not recommended. In the same way, a correctly written set of mathematical standards has to be more like the Board's version rather than the Commission's. In mathematics, understanding *goes through technique*, and technique is built on understanding. That is the way it is.

4.

It is time to take a critical look at the Board's Standards [2] and make explicit some of the concerns adumbrated earlier. There is no pretension to being comprehensive in this critique, however. Almost all the recommendations below are concerned with what to add to the Framework Draft [4] in order to round off the Board's Standards. Since this Draft is still in a state of flux, it is quite possible that these recommendations have already been anticipated by those in charge of [4]. If so, then nobody would be happier than I to have been rendered irrelevant in this undertaking.

First, a minor concern. By the end of the 5th grade, the Board's Standards mandates that "students increase their facility with the four basic arithmetic operations applied to positive and negative numbers, fractions and decimals." In principle there should be no problem with this goal. In practice, students in other countries usually achieve this level of competence in the 7th grade. For this reason, we may wish to watch carefully how this stipulation would play out in the classroom.

It is gratifying to know that examples which would clarify the terse statements of the Board's Standards will be incorporated into the final version of the Framework [4], but let me explicitly lobby for more clarifications along this line. There is no doubt that in order to help educators across the state understand the Standards, especially in grades 8--12, more guidance in the Framework is needed both in the details and in the overall planning. Regarding the former, a statement such as standard 4 in Geometry (Grades 8--12), "Students prove basic theorems involving congruence and similarity", means many things to many people. Should the AAA theorem for similar triangles be proved, for example? Depending on how this statement is approached, it can be a difficult theorem. Or take standard 2 of Algebra II: "Students solve systems of linear equations and inequalities (in two or three variables) simultaneously, by substitution, graphically, or with matrices". This may or may not be calling for some discussion of linear programming, and since matrices appear here for the first time, the question naturally arises as to how best to handle it. We must remember that these Standards are pioneering something new in California, and pioneers have to be transcendentally clear at each step or they run the risk of having no followers on their trail. I wish to drive home this point by comparing with what I consider a very admirable set of mathematics standards, the 1990 Mathematics Standards of Japan [6]. There the statement about similarity (in grade 8!) is equally terse:

> *To enable students to clarify the concepts of similarity of figures, and develop their abilities to find the properties of figures by using the conditions of congruence or similarity of triangles and confirm them.*
>
> a. *The meaning of similarity and the conditions for similarity of triangles.*
> b. *The properties of ratio of segments of parallel lines.*
> c. *The applications of similarity.*

There is a big difference, however. The Japanese change their standards every ten years and, because they already have a well established tradition, the changes are gradual and minor by comparison with the kind of sea change we have over here. Moreover, they have excellent textbooks (cf. [7]--[9][9]) already in place, so there is no great need to spell out everything. By contrast, we are almost starting anew in California, especially in these turbulent times in education. There is therefore very great need for the Board's Standards to be absolutely clear.

On the matter of overall planning, the Standards intentionally eschew any prescription on how to teach students in grades 8--12, whether in the traditional way or the "integrated" way.[10] The intention for greater flexibility is admirable, except that in the absence of a tradition, the added flexibility may turn out to be a curse. For example, the Standards specify that each discipline (Algebra I, Geometry, etc.) need not "be initiated and completed in a single grade". It would appear that this specification makes it possible to describe the desirable content of each discipline without undue regard to the time limitation of fitting everything into exactly one year. Perhaps for this reason, there are more topics in Algebra II than can be reasonably completed in a single year. How to teach this material in more than two semesters then becomes a challenge which few schools could meet. Also Algebra I asks that "Students [be] able to find the equation of a line perpendicular to a given line that passes through a given point." No matter how this is done, it would involve theorems about similar triangles. Does it then imply---contrary to the traditional curriculum---that Geometry may be taught simultaneously with Algebra I? The Framework would have to give more explicit instructions on how to bring this off. Finally, it appears that the forthcoming 10th or 11th grade statewide mathematics test would include some statistics. Is the Framework going to suggest ways of teaching statistics in the early part of secondary school if the traditional curriculum is followed?

Considerations of this nature bring out the fact that the traditional method of offering year long sequences on algebra and geometry is too rigid to be educationally optimal. While none of the current "integrated" models in this country seems to be entirely successful, the argument cannot be ignored that we should pursue the kind of integrated mathematics education that has been in use in Japan or Hong Kong for a long time. The Framework would be fulfilling its basic function if it could nudge California in this direction in a forceful manner.

An idea that has undoubtedly occurred to many people is how much the standards of grades 8--12 in the Board's Standards read like a "Manual for Pure Mathematics". One almost gets the feeling that this document could not bring itself to face the relationship between school mathematics and practical problems. It is now incumbent on the Framework to restore the balance between the pure and applied sides of school mathematics. While it is true that the reform exaggerates the role of "real-world" problems in mathematics, ignoring them altogether is for sure not a cure either. We would do well to remember that the overwhelming majority of school students will be *users* of mathematics, and that as future citizens they need to be shown the power of mathematics in the context of daily affairs. But all through grades 8--12, I seem to see only three explicit references to applications:

> **Algebra I**
> **15.***Students apply algebraic techniques to rate problems, work problems, and percent mixture problems.*
> **23.***Students apply quadratic equations to physical problems such as the motion of an object under the force of gravity.*
>
> **Trigonometry**
> **19.** *Students are adept at using trigonometry in a variety of applications and word problems.*

I hope I am not over-using the Japanese model if I look at the corresponding situation in [6]. The description of the *Content* of the Standards in [6] is every bit as abstract and "pure" as the Board's Standards, but *The Construction of Teaching Plans and Remarks Concerning Content* after each of grades K, 1--6, 7--9, and 10--12 pays careful attention to the bearing of "daily affairs" on the curriculum. For example, here is what is said after grades 7--9:

> *In the [8th and 9th] grades, problem situation learning should be included in a total teaching plan with an appropriate allotment and [implementation] for the purpose of stimulating students' spontaneous learning activities and of fostering their views and ways of thinking mathematically. Here, `problem situation learning' means the learning to cope with a problem situation, appropriately provided by the teacher so that the content of each domain may be integrated or related to daily affairs.*

The tone makes it abundantly clear that this is no mere lip service to applications, but that the applied component is central to the whole curriculum.

I hope that the Framework will be equally emphatic on this point in order to make clear that the relation of mathematics to daily affairs is also central to the Mathematics Standards of California.

A final comment is on the contentious subject of technology. From K to 12 in the Board's Standards, I could detect only the following two references to technology:

> **Grade 6***Algebra and Functions*
> **1.4***solve problems using correct order of operations manually and by using a scientific calculators*
>
> **Grade 7** *Statistics, Data Analysis and Probability*

> ***1.** Students collect, organize and represent data sets ... both manually and by using an electronic spreadsheet program.*

I fully expect to be shown to be wrong, that there is another place where technology is mentioned. Nevertheless, I hope my point is clear. If this reticence is not complemented by a strong message in the Framework on how to confront technology, then we would be conceding that we, as educators, do not know how to deal with the technology around us. But the computer and the graphing calculators are here to stay, and the younger generation is besieged on all sides by them. It would not be an effective education policy to retreat and abdicate responsibility exactly when we are supposed to come forward to provide guidance. We do not want any kind of technological debauchery in the mathematics classroom, but neither do we want to make technological prudes out of our students. What we want are students who are technologically informed, especially about the role of technology in mathematics, but we won't get them if we continue to pretend that technology does not exist. I am being intentionally suggestive in my use of language in order to force the comparison with sex education. In both situations, it is better to keep our students informed than to let them pick up the wrong information in a state of prevailing ignorance.

Allow me to cite for the last time the Japanese Standards [6]. Part of *The Construction of Teaching Plans and Remarks Concerning Content* also deals with the technological issue after each of grades K, 1--6, 7--9, and 10--12. Here is what is said after grades 1--6 and 10--12, respectively.

> *At the 5th Grade or later, the teacher should help children adequately use "soroban" or hand-held calculators, for the purpose of lightening their burden to compute and of improving the effectiveness of teaching in situations where many large numbers to be processed are involved for statistically considering or representing, or where they confirm whether the laws of computation still hold in multiplication and division of decimal fractions. At the same time, the teacher should pay attention to provide adequate situations in which the results of computation may be estimated and computation may be checked through rough estimation.*
>
> *In teaching the content, the following points should be considered. The teacher should make active use of educational media such as computers, so as to improve the effectiveness of teaching.*
> *In the teaching of computation, the teacher should have students use hand-held calculators and computers as the occasion demands, so as to improve the effectiveness of learning.*

The Board has already wisely decided that no state test in grades K--6 would use calculators. This general policy on technology, sensible as it is, needs to be supplemented by a more comprehensive one which gives guidance not only on when *not* to use it but also on *when* to use it. For example, encouraging teachers in K--6 to use problems with more natural---and therefore more unwieldy---numerical data by enlisting the help of calculators is a beginning. In the presence of the no-calculator-in-tests rule, students would get a clear perspective on what they need to know regardless of technology, and on how they can use technology to their benefit when the need arises. Encouraging students in calculus to use calculator to estimate the limits of sequences while also holding them responsible for proofs of convergence is another example. Doubtlessly, thoughtful educators will be able to formulate similar specific recommendations in other situations. As the preceding passages from [6] indicate, we must make *active* use of calculators and computers to improve the effectiveness of teaching and learning, and what better place to launch this idea than in the Framework?

5.

It is very likely that another person who is willing to read the Board's Standards carefully would come to slightly different conclusions about its strengths and weaknesses. It is even more likely that, in that case, the differences can be calmly discussed and the resulting discussions would benefit the next generation in the long run. One can either avail oneself of this opportunity to improve education in California, or one can turn one's back to the welfare of the young and act irresponsibly.

This then brings me to the news release about U.S. 12th-grade performance on TIMSS on February 24. Gail Burrill, the President of NCTM, made the following comment on the TIMSS result: "What's important is that we are working together toward a common goal of excellence in mathematics. The recent math wars have done nothing to improve mathematics education." These are sobering statements. On the one hand, Ms. Burrill's optimistic view that we are already working together toward a common goal in mathematics education could not have been based on the reckless public condemnations of the Board's Standards that have just transpired. NCTM's editorial [3] has not exactly contributed to producing harmony either. On the other hand, the math war in California did manage to reverse the disastrous trend initiated by the *1992 Mathematics Framework for California Public Schools*. While much work remains to be done to achieve a balanced mathematics education in California, this achievement of the math war alone would give the lie to the assertion that math wars have done nothing to improve mathematics education. Nevertheless, educational reconstruction should be our common goal at this juncture, and the battle over the Standards is in this light nothing but a distraction. In his address before the Annual Meeting of AMS-MAA on January 8, 1998, Secretary Richard W. Riley had sounded the same theme of reconciliation: "This leads me back to the need to bring an end to the shortsighted, politicized, and harmful bickering over the teaching and learning of mathematics. I will tell you that if we continue down this road of infighting, we will only negate the gains we have already made -- and the real losers will be the students of America." In all our education activities we should think of our children first. No, we *must*. If there is any lesson to be learned from the battle of the Standards, it is that it serves very well as an object lesson on how not to behave in the future.

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Department of Mathematics #3840,
University of California,
Berkeley, CA 94720-3840
wu@math.berkeley.edu
April 11, 1998

An expanded version of a colloquium lecture at the California State University at Sacramento, February 12, 1998

Footnotes

[1]According to Commissioner Williamson Evers, "the omission of long division with two or more digit divisors was a conscious decision" by the Commission. See [14].
[2]Dictionaries usually define "axiom" as "self-evident truths", but since dictionaries aim merely to inform the laymen, such lapses are marginally excusable. However, in a set of mathematics standards which must address the professionals---mathematics teachers and mathematics educators,---there is no place for this kind of error.
[3]The Commission's Standards is published in a two column format which displays the mathematics standards on the left and the "Clarifications and Examples' on the right.
[4]On February 2, an open letter to CSU Chancellor Reed signed by over 100 mathematicians was released to the public; it expresses sentiments in support of the Board's Standards. See [14].

[5]There is an unfortunate linguistic slip here: "draw ten points" is undoubtedly what is meant.
[6]As of February, 1998.
[7]The meaning of this word has to be carefully qualified because there are several "integrated" approaches to mathematics in secondary schools.
[8]A popular undertaking by conductors such as Carmen Dragon and Andre Kostelanetz in the 50's and 60's.
[9]Their original publication date is 1984.
[10]See footnote 7.

Los Angeles Times
May 3, 1998

Perspective on Education
The State's Invisible Math Standards

California adopts a set of world-class guidelines for public schools -- and the powers that be promptly try to make them disappear.

by

David Klein

> **Question:**What would happen if California adopted the best, grade-by-grade mathematics achievement standards in the nation for its public schools?
>
> **Answer:** The education establishment would do everything in its power to make them disappear.

In December 1997, the State Board of Education surprised the world by not accepting extremely bad, "fuzzy" math standards written by one of its advisory committees, the Academic Standards Commission. Instead,in a few short weeks and with the help of four Stanford University math professors, the state board developed and adopted a set of world-class mathematics standards of unprecedented quality for California's public schools.

The prestigious Fordham Foundation recently conducted an independent review of the mathematics standards for 46 states and the District of Columbia, as well as Japan. California's new board-approved mathematics standards received the highest score, outranking even those of Japan, an educational superpower.

In sharp contrast, high-ranking school administrators, bureaucrats and legions of experts with doctorates in education have denounced California's new math standards. State Supt. of Public Instruction Delaine Eastin has given speeches throughout the state criticizing the standards and calling on local educators not to implement them. Judy Codding, a member of the Academic Standards Commission and the powerful National Center on Education and the Economy has given similar explicit advice. Luther Williams, the National Science Foundation's assistant director for education and human resources, also joined the chorus of denunciation.

Why the opposition to world-class math standards? California's new standards require a deep understanding of mathematical principles, but also a heavy dose of the requisite basic skills. Unlike the rejected Academic Standards Commission version, the new math standards require students to master long division, they do not include the use of calculators in elementary school and they make no pronouncements about teaching methods so long as grade-level benchmarks are achieved. Teachers are not compelled to follow the failed methods promoted by the nation's colleges of education. This lack of coercion enraged the education bureaucrats to the point of making their threats.

In February I wrote an open letter to Charles Reed, the chancellor of the Cal State University system and a former professor of education. The open letter, which defended and praised the recently adopted State Board of Education standards for K-12 mathematics, was endorsed by more than 100 California mathematicians, including the chairs of the math departments at Stanford University, Caltech, UC Irvine, UC Riverside, Cal State Los

Angeles, the vice president of the American Mathematical Society and a former president of the Mathematical Assn. of America. The nation's most famous math teacher, Jaime Escalante, portrayed in the movie "Stand and Deliver," also endorsed my letter.

I sent copies of this open letter to Los Angeles Unified School District board members and Supt. Ruben Zacarias. Zacarias had previously instructed subordinates to ignore the California math standards. Turning reality on its head, he insisted that the pathetic LAUSD math standards are better than California's. The LAUSD standards allow students to use calculators in third grade, they under-emphasize algebra in high school and they are vague and arbitrary. They are consistent with the weakest curricula, such as 'Mathland," which all but eliminates standard arithmetic in elementary school. LAUSD is deeply committed to mediocrity.

The CSU system admits the top one-third of graduating high school seniors in California. But more than half of the entering CSU freshmen statewide must take remedial math courses during their first year of college, often at the seventh-grade level. In Los Angeles it is worse. Fully 67% of entering freshmen at Cal State Northridge require remedial work in math during their first year, and the percentages have been steadily increasing since 1989. With Zacarias' anti-math policies in force, this is unlikely to improve.

One might expect support from the CSU for California's math standards. Instead, the CSUN elementary schoolteacher training courses in arithmetic integrate calculators throughout, and projects are underway to promote the weak LAUSD math standards. James Highsmith, the chair of the CSU Academic Senate, publicly denounced the new math standards in The Times. The CSU is currently revising its Entry Level Mathematics Exam to be easier. Students will be allowed to use calculators on the exam, and unpleasant topics such as logarithms will be eliminated. In this way, the CSU can sweep away the embarrassing mathematics performance of its entering freshmen the easy way, and eliminate the need for remediation by definition. CSU Chancellor Reed never responded to the open letter, endorsed by Jaime Escalante and more than 100 mathematicians, including some of the most creative mathematical thinkers in the world.

The only real hope for K-12 education reform lies with parents and citizens.

Voting Eastin out of office along with a clean sweep of her education bureaucracy would go a long way toward improving mathematics education. If LAUSD continues blindly to reject the California math standards, breaking up the district may be the best thing we could do on a local level for our schoolchildren.

David Klein Is a Mathematical Physicist and Professor of Mathematics at Cal State Northridge.
His E-mail Address Is: David.klein@csun.edu

California Mathematicians Respond

Introduction

In December of 1997, the California State Board of Education was moving toward final adoption of the mathematics standards for California. In an unsuccessful attempt to influence the final 11-0 vote by the Board, the faculty leaders of the academic senates of the University of California, the California State University system, and the California Community College system issued a joint statement condemning the Board's actions. Although not one of these three academic leaders is a mathematician, they implied in their statement that "*the consensus position of the mathematical community*" was in opposition to the Board approved mathematics standards.

In response, over 100 mathematicians -- and the list is growing -- from a broad spectrum of California's colleges and universities have endorsed an open letter sharply disagreeing with the academic senate officials. They urge recognition of "*the important and positive role California's recently adopted mathematics standards can play in the education of future teachers of mathematics in the state of California*."

- - - -

> *I overwhelmingly support this open letter to CSU Chancellor Reed. In fact, you can take out "mathematician" and make one statement "Most math teachers are not familiar with any of the proposed K-12 mathematics standards and they've certainly never been polled." For more than two decades, I have been trying to instill "ganas", the desire to succeed, in my students. I was successful in part because of the clear and unequivocal standard that I chose as my goal for those students willing to accept the challenge, a passing score on the AP Calculus exam. Having the clear standards as approved by the SBE in December will help many more teachers and their students develop this sense of ganas to succeed in mathematics in school, and in life.*
>
> Jaime Escalante
> "Stand and Deliver"
> Hiram Johnson High School
> Sacramento. CA

- - - -

The open letter, along with supporting documents follows.

AN OPEN LETTER TO CSU CHANCELLOR REED

Dear Chancellor Reed;

Welcome to your new position as Chancellor of the California State University system. The education and certification of California's K-12 mathematics teachers is one of the many important functions of the CSU. For this reason, we believe that it is important that you have accurate information about matters relating to mathematics education in California.

We are in disagreement with the letter below signed by James Highsmith, Chair of the CSU Academic Senate, with William Scroggins, President of the CCC Academic Senate, and Sandra J. Weiss, Chair of the UC Academic Senate listed as co-authors.

The December 8, 1997 letter from these Chairs of the Academic Senates of the UC, CSU, and CCC suggests that "*the consensus position of*

***the mathematical community*" is in opposition to the mathematics standards for K-12 adopted by the California Board of Education and is generally in support of the rejected draft standards written by the Academic Content and Performance Standards Commission.**

It is our opinion that no such consensus exists within the mathematics community of the state of California. Most mathematicians are not familiar with any of the proposed K-12 mathematics standards and they've certainly never been polled.

If their views were solicited, we believe that most mathematicians would find serious shortcomings in the Commission's draft standards which the California Board of Education rejected. For example, the Commission standards which Drs. Highsmith, Scroggins, and Weiss described as "*aligned with the Roundtable's standards*" which "*incorporate the best advice from the most respected faculty in our systems and in the country*" fail to require K-12 students ever to master long division when the divisor has more than a single digit. According to the letter below by Academic Standards Commissioner Bill Evers, this was not merely an oversight, but rather a conscious decision on the part of the Commission.

The California Board of Education's decision to avoid stipulating teaching methods, instead focusing on appropriate mathematical content, was a wise decision considering the disparate views on pedagogy.

Good mathematics standards require mastery of both basic skills and broader mathematical concepts. We urge you to recognize the important and positive role California's recently adopted mathematics standards can play in the education of future teachers of mathematics in the state of California.

- **David Klein**
  Professor of Mathematics
  California State University, Northridge

- **Hung-Hsi Wu**
  Professor of Mathematics
  University of California, Berkeley

- **Wayne Bishop**
  Professor of Mathematics
  California State University, Los Angeles

- **William L. Armacost**
  Professor of Mathematics
  California State University, Dominguez Hills

- **Sheldon Kamienny**
  Professor of Mathematics
  University of Southern California

- **Ralph Cohen**
  Professor of Mathematics
  Stanford University

- **Jerry Rosen**
  Profesor of Mathematics
  California State University, Northridge

- **Marshall Cates**
  Department of Mathematics
  California State University, Los Angeles

- **Jacek Polewczak**

Associate Professor of Mathematics
California State University, Northridge

- **John Dye**
  Professor of Mathematics
  California State University, Northridge

- **Richard Katz**
  Professor of Mathematics
  California State University, Los Angeles

- **David Protas**
  Professor of Mathematics
  California State University, Northridge

- **Ali Zakeri**
  Professor of Mathematics
  California State University, Northridge

- **Martin Scharlemann**
  Professor of Mathematics
  University of California, Santa Barbara

- **Prof. James D. Stein Jr.**
  Department of Mathematics
  California State University, Long Beach

- **Gunnar Carlsson, Chair**
  Professor of Mathematics
  Stanford University

- **Kurt Kreith**
  Professor Emeritus of Mathematics
  University of California, Davis

- **George Biriuk**
  Professor Emeritus of Mathematics
  California State University, Northridge

- **Don Kiel**
  Professor Emeritus of Mathematics and Computer Science
  California State University, Los Angeles

- **Charles Akemann**
  Professor of Mathematics
  University of California, Santa Barbara

- **Dr. Richard Ferrier**
  Tutor
  Thomas Aquinas College

- **R. James Milgram**
  Professor of Mathematics

Stanford University

- Lorraine L. Foster
Professor of Mathematics
California State Univeristy, Northridge

- Geoffrey Mess
Associate Professor of Mathematics
UCLA

- Carolyn Facer
faculty member
Department of Mathematics
El Camino College

- Michael J. Hoffman, Chair
Professor of Mathematics
Dept. of Mathematics and Computer Science
California State University, Los Angeles

- Kent G. Merryfield
Associate Professor of Mathematics
California State University, Long Beach

- Peter Petersen
Professor of Mathematics
UCLA

- Howard Swann
Professor of Mathematics
San Jose State University

- Reinhard Schultz, Chairman
Department of Mathematics
University of California, Riverside

- Mary Rosen
Professor of Mathematics
California State University, Northridge

- Abigail Thompson
Associate Professor of Mathematics
University of California, Davis

- Paul Chabot
Professor of Mathematics
California State University, Los Angeles

- Gordon L. Nipp
Professor of Mathematics
California State University, Los Angeles

- Steve Kerckhoff

Professor of Mathematics
Stanford University

- **Brian White**
  Professor of Mathematics
  Stanford University

- **Rick Schoen**
  Professor of Mathematics
  Stanford University

- **Tracy Gustafson**
  Instructor of Mathematics
  College of the Sequoias

- **Dr. Raj Pamula**
  Undergraduate Advisor
  Dept. Of Mathematics & Computer Science
  Calfiornia State University, Los Angeles.

- **Jun Li**
  Associate Professor of Mathematics
  Stanford University

- **Leon Simon**
  Professor of Mathematics
  Stanford University

- **Abel Klein**
  Professor and Chair
  Department of Mathematics
  University of California, Irvine

- **Peter Li**
  Professor of Mathematics
  University of California, Irvine

- **George Woodbury**
  Mathematics Instructor
  College of the Sequoias

- **Svetlana Jitomirskaya**
  Associate Professor of Mathematics
  University of California, Irvine

- **Daryl Cooper**
  Professor of Mathematics
  University of California, Santa Barbara

- **Ross Rueger**
  Instructor of Mathematics
  College of the Sequoias

- **Mark Tom**
  Instructor of Mathematics
  College of the Sequoias

- **John Redden**
  Instructor of Mathematics
  College of the Sequoias

- **Christine DeFlitch**
  Instructor of Mathematics
  College of the Sequoias

- **Nora Wheeler**
  Instructor of Mathematics
  College of the Sequoias

- **Gary Howland**
  Instructor of Mathematics
  College of the Sequoias

- **Steve Houk**
  Instructor of Mathematics
  College of the Sequoias

- **Dennis Morley**
  Instructor of Mathematics
  College of the Sequoias

- **Darren Long**
  Professor of Mathematics
  University of California, Santa Barbara

- **Fred Wilhelm,**
  Assistant Professor of Mathematics
  University of California, Riverside

- **Maria Noronha**
  Professor of Mathematics
  California State University, Northridge

- **Mary Bologna**
  Lecturer in Mathematics
  California State University, Los Angeles

- **Andrei Verona**
  Professor of Mathematics
  California State University, Los Angeles

- **Brant Wassall**
  Instructor in Computer Science
  California State University, Los Angeles

- **Mary L. Browne**

Secondary Mathematics Instructor, LAUSD
Teaching Associate in Mathematics
California State University, Los Angeles

- **Peter Doyle**
  Professor of Mathematics
  University of California, San Diego

- **Tudor Ratiu**
  Professor of Mathematics
  University of California, Santa Cruz

- **V. S. Varadarajan**
  Professor of Mathematics
  UCLA

- **Geoffrey Mason**
  Professor of Mathematics
  University of California, Santa Cruz

- **Don Rose**
  Instructor of Mathematics
  College of the Sequoias

- **Mark Laurel**
  Instructor of Mathematics
  College of the Sequoias

- **Kamel Haddad**
  Associate Professor, Mathematics
  California State University, Bakersfield

- **Chetan Prakash**
  Professor of Mathematics
  California State University, San Bernardino

- **Roberto Schonmann**
  Professor of Mathematics
  UCLA

- **Peter Williams**
  Professor of Mathematics
  California State University, San Bernardino

- **Lisa Anderson**
  Assistant Professor of Mathematics
  Ventura Community College

- **Radu Toma**
  Mathematics Instructor
  Canada College

- **Michelle Erickson**

Mathematics Instructor
College of the Canyons

- **Steve Breen**
  Associate Professor of Mathematics
  California State University, Northridge

- **Rena Petrello**
  Instructor of Mathematics
  Moorpark College

- **Edward Effros**
  Professor of Mathematics
  UCLA

- **Murray Schacher**
  Professor of Mathematics
  UCLA

- **Richard Quint**
  Professor of Mathematics
  Ventura Community College

- **Hans Wenzl**
  Professor of Mathematics
  University of California, San Diego

- **Scott Wilson**
  Instructor of Mathematics
  College of the Sequoias

- **Yoshi Inoue**
  Instructor of Mathematics
  College of the Sequoias

- **George Jennings**
  Associate Professor of Mathematics
  California State University, Dominguez Hills

- **Merrill Eastcott**
  Math Instructor &
  Math Department Faculty Coordinator
  El Camino College

- **Mei-Chu Chang**
  Professor of Mathematics
  University of California, Riverside

- **Ziv Ran**
  Professor of Mathematics
  University of California, Riverside

- **Matthew G. Robertson**

Instructor of Mathematics
Cuesta College

- **Haruzo Hida**
  Professor
  Department of Mathematics
  UCLA

- **Joel Hass**
  Professor of Mathematics
  University of Califorrnia, Davis

- **P. K. Subramanian**
  Professor of Math & Comp Sci
  California State University, Los Angeles

- **Peter Basmaji**
  Professor of Mathematics
  California State Uiversity, Los Angeles

- **Albert Schwarz**
  Professor of Mathematics
  University of California, Davis

- **Dmitry Fuchs**
  Professor of Mathematics
  University of California, Davis

- **Sherman Stein**
  Professor Emeritus of Mathematics
  University of California at Davis
  Author of Strength in Numbers

- **Bruce K. Driver**
  Professor of Mathematics
  University of California, San Diego

- **Gregory Kuperberg**
  Assistant Professor of Mathematics
  University of California, Davis

- **John M. Bachar, Jr.**
  Professor of Mathematics
  California State Univeristy, Long Beach

- **John de Pillis**
  Professor of Mathematics
  University of California, Riverside

- **Barry Simon**
  IBM Professor of Mathematics and Theoretical Physics
  Mathematics Department Chair
  California Institute of Technology

- **Henry L. Alder**
  Professor Emeritus of Mathematics
  University of California, Davis
  Former President of the Mathematical Association of America
  Former Member of the State Board of Education

- **Jennifer Chayes**
  Vice-President, American Mathematical Society
  Professor of Mathematics
  UCLA

Please EMAIL your comment or endorsement.

Background Information

December 8, 1996
Mrs. Yvonne Larsen, President
State Board of Education
721 Capitol Mall, Room 532
Sacramento, CA 95814

Dear President Larsen,

On behalf of the Intersegmental Committee of Academic Senates (ICAS) of California's institutions of public higher education, we are writing to urge the State Board of Education to reconsider the directions it is taking with its mathematics standards. You have an opportunity to select standards that will demand the basic skills, conceptual understanding, and problem solving skills that California's students need.

Since ICAS represents the faculty of UC, CSU and CCC, we are very interested in the academic preparation of students for college. ICAS recently developed the Statement on Competencies in Mathematics Expected of Entering College Students, which is a cogent description of the mathematics that students should understand and be able to do in order to be successful in college. This document has been endorsed by the faculty senates in our three systems, which would not have been possible had the document chosen an extreme or unusual position on the controversial issues of math education. Although the faculty of our three systems hold a wide range of opinion on these matters, the majority support a moderate approach to math competencies.

ICAS also participated in the development of the California Education Roundtable's (CERT) recommended high school graduation standards. These standards are consistent with the ICAS statement on math competencies, and strike an appropriate balance in approaches to math expectations. The standards incorporate the best advice from the most respected faculty in our systems and in the country. We were quite pleased when the Commission recommended to your board standards that were aligned with the Roundtable's standards. It appeared that the teachers of math in California were going to get coherent directions from both higher education and from their board of education. We were then disappointed to learn that our state board of education was selecting a vastly different direction for the state, in clear contrast to the position offered by the joint expertise of our institutions of higher education.

We are concerned about the Board's lack of open, broad based consensus-building in developing the math standards. Your choice of experts to help you write the standards reflect [sic] a traditional view of mathematics which is not broadly representative. In contrast, the ICAS competencies and the CERT standards carry the collective endorsement of the faculty and the consensus position of the mathematical community.

The world has learned much about effective mathematics standards in recent years. Other States have built upon available research, and experiments in other countries, as they have written standards that are in stark contrast to the directions that your State Board is taking. We urge you to reconsider your decision so that our State can also have math standards which truly prepare our children for a contemporary world.

[the letter is hand signed by Highsmith; the other names are listed and presumably agreed to the letter, but the letter does not have their signatures]

James Highsmith
Chair, CSU Academic Senate

William Scroggins
President, CCC Academic Senate

Sandra J. Weiss
Chair, UC Academic Senate

Text of James Highsmith's letter to Yvonne Larsen, 12/9/97

Your board committee's proposals for K-7 and 8-12 mathematics standards are at great variation from the consensus developed in California regarding mathematics education for public schools. Moreover, the truncated and closed process you have used to ignore the advice of the vast majority of math professionals and experts is antithetical to your public mission.

The faculties of California higher education endorsed mathematical standards for high school graduation that were developed under the aegis of the California Education Round Table. Broad agreement was reached with teachers, school professionals, and public members. It is troubling, to put it mildly, to think that our public representatives are now prepared to chuck that work and the work of the AB 265 Commission which aligned its standards with the Round Table's.

It is time for us to send a clear, consistent message to schools and teachers--a message that allows teachers to incorporate the best about what we have learned from around the world regarding rigorous and achievable math preparation.

I most strongly urge you to continue working on the math standards proposal with faculty appointed by the California State University, University of California, and California Community Colleges who have managed to reach agreement on what our children will need in the next millennium. The broad range of opinion has been distilled to a rigorous middle course that would allow teachers to prepare students well for the math skills and understanding [sic]. The State Board of Education should not adopt proposals that stray from that strong moderate course.

James M. Highsmith, Chair
Academic Senate, CSU

Stanford University
Palo Alto, California
December 26, 1997

For the record, the omission of long division with 2 or more digit divisors was a conscious decision. I pointed out in writing and in oral comments before the Commission that this was missing.

I cannot pretend to read the minds of all Commissioners and consultants. Undoubtedly some thought long division obsolete; others probably thought teachers would teach long division and that it need not be specifically mentioned. The problem with this latter view is that since long division was not an explicit expectation under the Commission's standards, teachers and testmakers could and would in good conscience omit it. Since it would not have appeared on a state standards-based test (had one been created under the Commission's standards), it would have widely been dropped from textbooks and curricula used around the state.

Bill Evers
Commissioner, Academic Standards Commission

Proposed California Standards Hearings

State Board of Education
Hearings on Draft Standards

The State Board of Education held a series of public hearings on the academic and content standards in reading, writing and mathematics.

The hearing locations and dates were:

- Sacramento County, Monday, October 20
- Santa Clara County, Monday, October 27
- Fresno County, Wednesday, October 29
- Los Angeles County, Thursday, October 30
- San Diego County, Monday, November 3

New California Mathematics Adoptions

This is an "unofficial" report of the California State Board of Education action on June 10, 1999 in connection with AB2519.

Approved Programs

Publisher	Title	Grades and comments	Mathematically Correct Rating
Addison Wesley Longman (Now from Prentice Hall)	UCSMP Transition Math	Grade 7 with changes	
Addison Wesley Longman (Now from Prentice Hall)	UCSMP Algebra	Grade 8	"C"
Creative Publications	Hot Words, Hot Topics (partial program)	Grade 5 with changes	
CSL Associates	Math Coach (partial program)	Grades 1-4 with changes	
Glencoe/McGraw Hill	Mathematics: Applications & Connections	Grades 5-7 with changes	grade 7: "B"
Glencoe/McGraw Hill	Pre-Algebra: An Integrated Transition to Algebra and Geometry	Grade 7	grade 7: "A"
Glencoe/McGraw Hill	Algebra 1: Integration Applications Connections	Grade 8 with changes	"B"
Globe Fearon	Access to Math (partial program)	Grade 6	
Harcourt Brace	Math Advantage	Grades K-2	grade 2: "B"
Houghton Mifflin	MathSteps (partial program)	Grades K-7, with changes	
Mastery Learning Systems	Count, Notice, & Remember (partial program)	Grades K-3	
McDougal Littell	Passport to Mathematics	Books 2 & 3 only, with changes	grade 7: "C"
McDougal Littell	An Integrated Approach	Grades 6-8	
McDougal Littell	Algebra: Structure and Method	Grade 8	"A"
Metropolitan	Problem Solving Step-by-Step (partial program)	Grades 1-5	
Prentice Hall	Middle Grades Math: Tools for Success	Grade 6 and 7 only, with changes	"B"
Prentice Hall	Algebra: Tooles for a Changing World	Grade 8 with changes	"C"
Sadlier	Progress in Mathematics	Grades K-6 with changes	
Saxon Publishers	Saxon Mathematics (K-3)	Grades K-3, changes in 3	grade 2: "B"
Saxon Publishers	Saxon Mathematics (3-6)	Grades 3-6, changes in 6	grade 5: "B+" But see footnote 1 below
SRA/McGraw Hill	Connecting Math Concepts (partial program)	Grades K-5 with changes	

SRA/McGraw Hill	Math Explorations and Applications	Grades K-6 with changes	grade 2: "A" grade 5: "A-" But see footnote 2 below
William K. Bradford	Math Trek Series (probability and statistics partial program)	Grade 8 with changes	

Footnote 1: The review for the actual textbook adoptions for these books are off by one grade level. Thus, the Mathematically Correct review of the grade 5 text is actually the text adopted for grade 4. This clearly pushes the content up to a higher level in the California adoption. Had Mathematically Correct taken this into account, the Saxon would probably have received a higher mark. See also A Comparision of Three K-6 Mathematics Programs: Sadlier, Saxon, and SRA McGraw-Hill

Footnote 2: This particular program contains methods, such as extensive calculator use, that may be counter indicated but were not taken into account in the review. For an alternative perspective, see A Comparision of Three K-6 Mathematics Programs: Sadlier, Saxon, and SRA McGraw-Hill

Programs Not Approved

- 21st Century Company
  - Math Live Book
- Addison Wesley Longman
  - Scott Foresman-Addison Wesley Math
  - Connected Mathematics
- Advantage Learning Systems
  - Accelerated Math
  - Star Math
- AMATH Systems
  - AMATH Pre-Algebra
- American Guidance Service
  - AGS Basic Math Skills
  - AGS Consumer Mathematics
  - AGS Life Skills Math
  - AGS Pre-Algebra
  - AGS Algebra
- Bellwork Enterprises
  - Bellword: A Daily Practice Program
- Clearwater Publishing
  - Algebra Unplugged
- Continental Press
  - Math: Skills, Concepts, Problem Solving
  - Focus on Problem Solving
  - Number Sense
  - Get Ahead in Math
- Creative Publications
  - Mathscape: Seeing and Thinking Mathematically
  - Algebra: Puzzles and Problems
  - Groundworks: Developting Algebraic Thinking
  - Skill Power
  - El Poder De Las Matematicas
- Cuisenaire Company
  - The Super Source
- Dale Seymour
  - Developing Number Concepts
  - Nimble with Numbers
- Developmental Studies Center
  - Number Power

- Everyday Learning Corporation
    - Contemporary Mathematics in Context
- Globe Fearon
    - Practical Mathematics for Consumers
    - Success in Math
    - Basic Geometry
    - Consumer Math
    - Pre-Algebra
    - Basic Algebra
    - Math for Proficiency
    - Globe Fearon Algebra 1
    - Globe Fearon Basic Mathematics
    - Globe Fearon Pre-Algebra
- Great Source Education Group
    - Everyday Counts
- Grow Publications
    - ADD - Arithmetic Developed Daily
- Holt, Rinehart and Winston
    - HRW Algebra One Interactions
    - Practical Mathematics
- Kendall/Hunt
    - Math Trailblazers: A Mathematical Journey
- Kid Success Learning Tools
    - Efficient Easy Memorizing of Math Tables
- Math Teachers Press
    - Moving with Math
- McDougal Littell
    - Explorations and Applications
    - Integrated Mathematics
    - Basic Algebra
- Newbridge Educational Publishing
    - Early Math
    - Math Manipulatives
- Prentice Hall
    - Prentice Hall Mathematics: Explorations and Applications, A Pre-Algebra Approach
    - Prentice Hall Geometry: Tools for a Changing World
- Riverdeep
    - Destination Math
- Sadlier
    - New Progress in Mathematics
- Saxon Publishers
    - Saxon Mathematics: Algebra 1/2
    - Saxon Mathematics: Algebra 1
- Silver Burdett Ginn
    - Exploring Your World
    - Exploremos tu Mundo
- SRA/McGraw Hill
    - Spectrum Math
    - SRA Mathematics Laboratory
    - Total Class Math

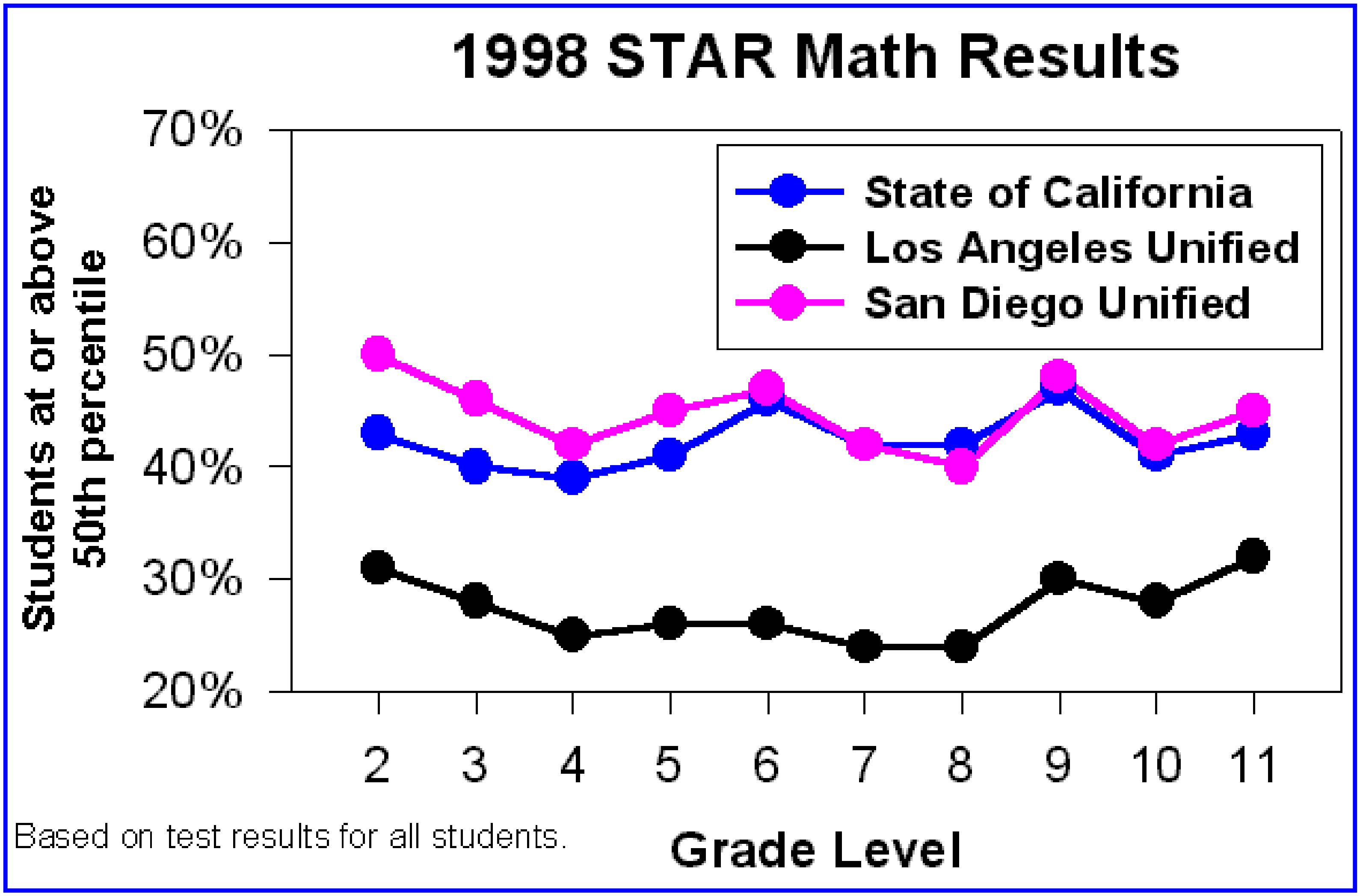
Votes Postponed

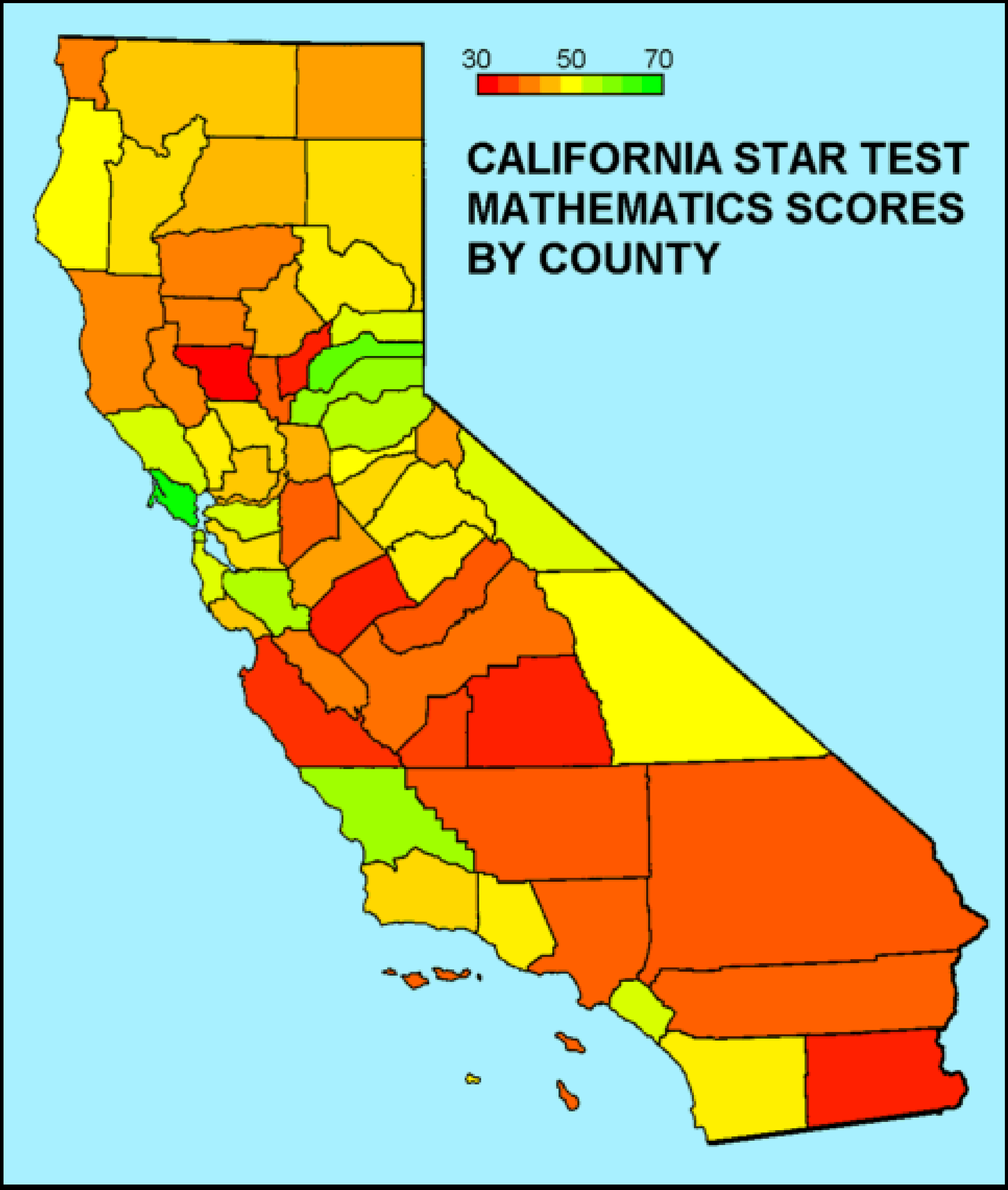
- Barrett-Kendall
    - Algebra: A First Course
- Everyday Learning
    - Everyday Mathematics (Required changes were not made, program was withdrawn).
- Metropolitan
    - Metro Math Readers (partial program, subsequently approved)

Notes

This is an unofficial report. Some of the entries not approved may have been withdrawn. Some programs that are approved have other grade levels that were not approved. Official reports should come from the state. Mathematically Correct ratings are not part of the state review, but are listed above for reference. Programs reviewed by Mathematically Correct may have been revised prior to submission to California. See also AB 2519 Adopted Mathematics Instructional Materials from the California Department of Education

STAR Sample 1999 Augmented STAR Math Items


1998 STAR Math Results
70%
60%
50%
40%
30%
20%
Students at or above 50th percentile
State of California
Los Angeles Unified
San Diego Unified
2
3
4
5
6
7
8
9
10
11
Based on test results for all students.
Grade Level



STAR Percentile rank for average student, averaged across grades

Evaluating Entry Level Mathematics Placement in the California State University System

Paul Clopton and R. James Milgram
April, 1999

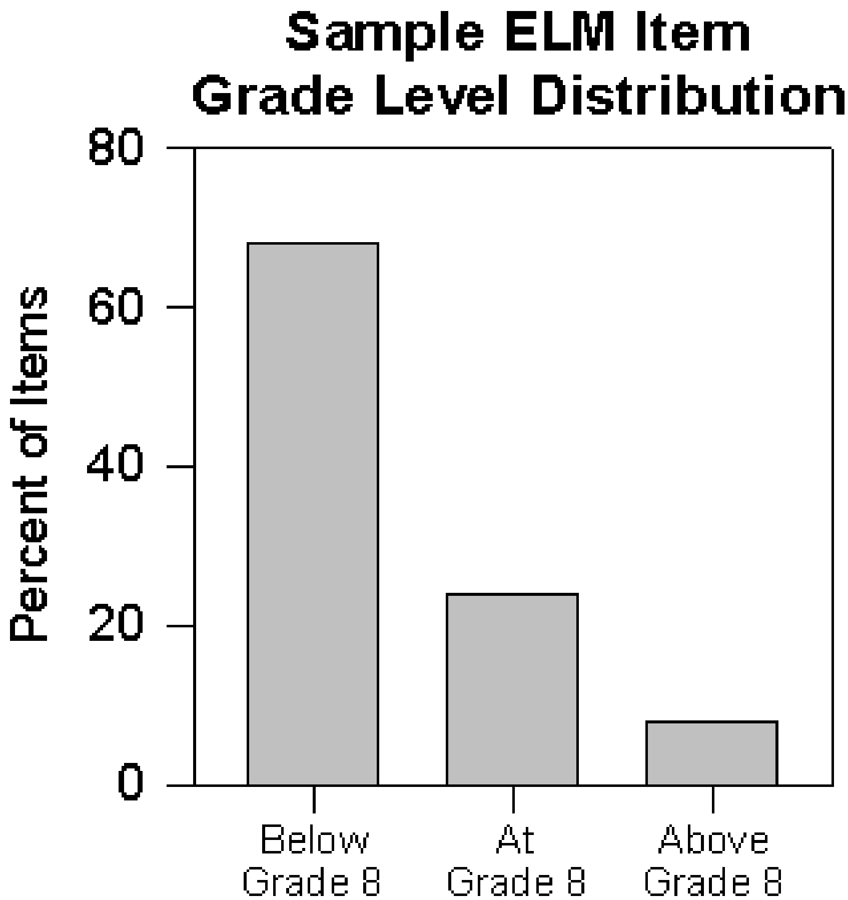
All students entering the California State University (CSU) system are expected to have completed a rigorous sequence of college preparatory subjects. In mathematics, this means three years of college preparatory course-work, while a fourth year of pre-calculus is recommended. Students who are not exempt on the basis of other test scores must take an Entry Level Mathematics Placement Examination (ELM).[1] If they do not pass this examination, students are required to take remedial course-work. The failure rate has been steadily increasing over the past several years, and now well above 50% of entering students require remediation.[2] The CSU system has recently revised the ELM.[1] This report summarizes some of the characteristics of the revised ELM.

The new version of the ELM retains the multiple choice format. A passing grade varies somewhat from exam to exam, but should fall in the range from 58% to 65% correct. Items are 5-alternative, multiple choice format, so 20% correct is expected by chance. Although calculators are now permitted, they are not of great benefit. The examination is divided into three general areas - Algebra (roughly 60% of test items), Geometry (roughly 20% of test items), and Data Interpretation, Counting, Probability and Statistics (roughly 20% of test items). Various subtopics are defined within each of these content areas.[1]

To assess the target grade level of the ELM against external criteria, individual sample items[1] were evaluated for their grade level based on the newly established California Mathematics Standards.[3] These standards provide a desirable benchmark for several reasons. They are perhaps the most highly detailed of all the sets of state mathematics standards, greatly facilitating item evaluation. And they have been judged as the best available mathematics standards among all sets of state standards, even better than those from Japan.[4]

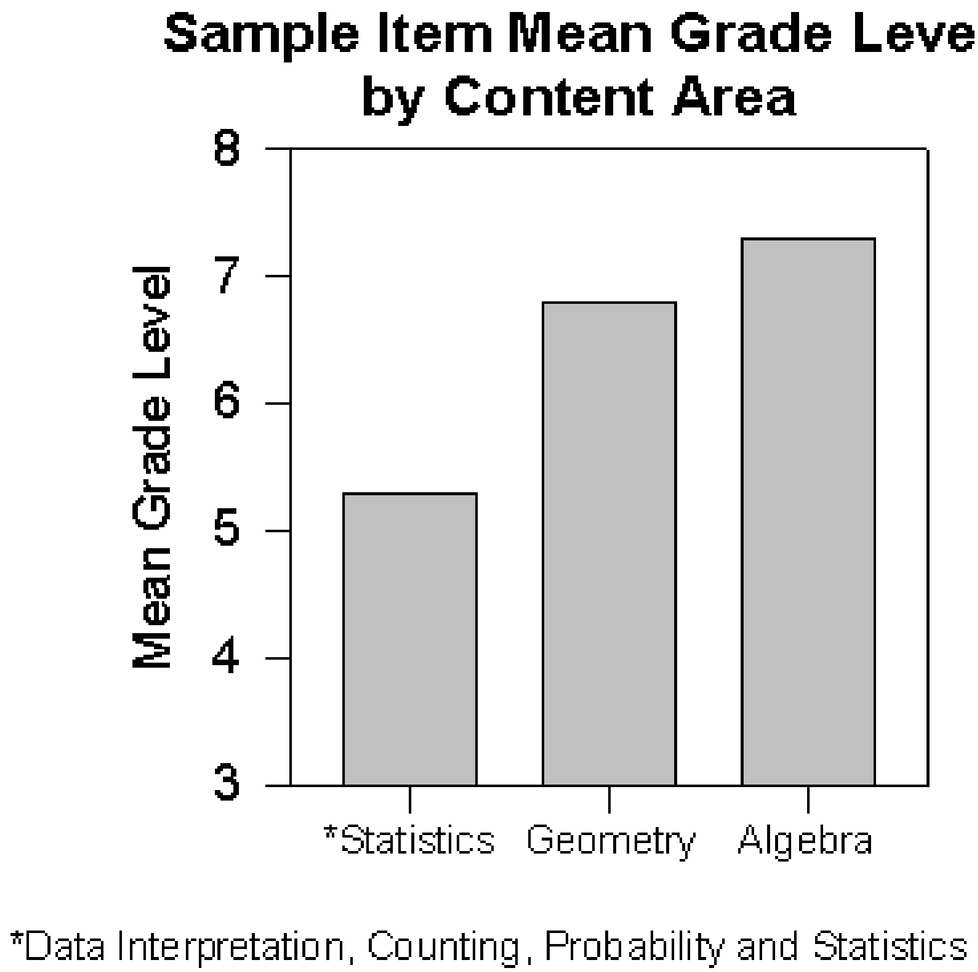
The authors independently judged the grade level of every sample ELM item using California's new mathematics standards as the criteria. The inter-rater reliability was r=.80, which indicates a reasonably high degree of agreement. Where there were differences, the mean of the two ratings was used.

The item ratings ranged from 4th grade to 10th grade, although most (72%) were judged as measuring standards at



the 6th to 8th grade levels. The overall mean grade level rating was 6.92 (SD=1.47). Overall, 68% of items fell below 8th grade level, 24% of items were at the 8th grade level, and 8% of items fell above 8th grade level.

The mean grade level rating for the statistical item group (5.3) was significantly lower than the mean for items with geometry (6.8) or algebra (7.3) content ($p<.05$).



Given the test requirements, a passing grade requires at least some minimal degree of competence in introductory algebra. It should be noted that this is clearly a higher requirement than one often sees for high school graduation, such as in the Texas TAAS examination.[5] On the other hand, passing such an examination is no assurance of the knowledge and skills needed for further courses involving quantitative reasoning. Indeed, success on the ELM might signal readiness to study high school geometry or intermediate algebra, but should not be taken as an indicator of readiness for pre-calculus, calculus, other demanding mathematics courses, or other courses requiring a strong mathematics base.

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1 *Focus on Mathematics: Entry Level Mathematics Placement Examination*. CSU Academic Affairs, Office of the Chancellor, Phillip Emig, CSU Faculty Consultant in Mathematics
http//www.co.calstate.edu/aa/ar/FOM.pdf

2 *Fall 1998 Freshman Remediation Rates Campus and Systemwide*, CSU
http//www.asd.calstate.edu/remrates98sys.htm

3 *Mathematics Content Standards for California Public Schools: Kindergarten Through Grade Twelve*, California State Board of Education
http://www.cde.ca.gov/re/pn/fd/documents/math-stnd.pdf

4 *State Mathematics Standards*, Ralph A. Raimi and Lawrence S. Braden, Fordham Report Volume 2, Number 3, March 1998.
http//www.edexcellence.net/standards/math.html

5 *Statewide Mathematics Assessment in Texas*, P. Clopton, W. Bishop, and D. Klein
http//www.mathematicallycorrect.com/lonestar.htm

Integrated Mathematics in LAUSD

Summary

In California, the *integrated mathematics* option refers specifically to an alternative to the algebra 1, geometry, algebra 2 secondary sequence wherein districts are allowed to provide the same content but in a different sequence over three years. All mathematics programs for K-7 are integrated in that topics from each strand of mathematics are included each year. Because the secondary integrated programs make heavy use of pedagogical approaches often called *reform mathematics*, these terms have unfortunately been used interchangeably. This confuses the issues.

With respect to the secondary *integrated mathematics* programs in use in LAUSD:

> The content of these courses is not equivalent to the content required by the state standards.
>
> > No integrated 1 books were approved by the state under AB2519 - indeed most were not even submitted - as they are unsatisfactory relative to the state learning standards.
> >
> > The integrated programs cannot be legitimately certified as aligned with the state standards.
>
> Students in these programs learn less mathematics than those in traditional programs.
>
> > LAUSD students in integrated mathematics score lower than those in traditional mathematics in grades 8, 9, and 10 on the state standards-based tests according to state records.
> >
> > This deficit is true for economically disadvantaged students as well as others.
> >
> > This deficit is true for LEP students as well as others.
> >
> > This deficit is true for male and female students.
>
> These programs produce students who are less well prepared.
>
> > Integrated mathematics has been promoted through LA-SI (the Los Angeles Systemic Initiative, a federally funded project to implement integrated math programs) in schools around the district. Of the eleven schools associated with LA-SI the longest, all but one have experienced decline in SAT participation over the past two years. The average decline is 12% as reported by the IAU (the "Independent Analysis Unit" of LAUSD) to the Board in a report of May 12, 1999.
> >
> > SAT administration across all LA-SI focal schools is down about 5% while it is up roughly 5% in non-focal schools. The SAT math average in focal schools is 445 while at the non-focal schools have an average of 462.

With respect to *reform mathematics*, the programs approved by the state for K-8 include a range of reform approaches and frequently note their alignment with NCTM. These approaches will still be available even when programs that fail to align with the standards are avoided.

Integrated Mathematics in LAUSD

by Paul Clopton
Member, LAUSD Mathematics Curriculum Committee

Introduction

Historically, secondary mathematics in California has been taught using the courses algebra 1, geometry, and algebra 2. More recently, some schools have switched to courses that mix these topics across three courses. This is called *integrated mathematics* and is an optional sequence in the state Mathematics Framework. However, the integrated programs in use differ in many respects beyond the sequence of topic presentation, and thus *integrated* has taken on other meanings that have to do with pedagogy, presentation style, and other factors.

The new California Mathematics Standards and the Mathematics Framework require a mixture of topics from the strands of mathematics for all students in grades K-7. Districts may use either the traditional or the integrated *sequence* starting in grade 8. The standards stipulate exactly the same objectives for either sequence - that students learn the required mathematics.

To go along with this option, the standards-based portion of the state testing program (STAR) in mathematics has two choices for grades 8 to 10 - algebra 1, geometry, and algebra 2 or integrated 1, integrated 2, and integrated 3. The two sequences contain exactly the same items overall, but they are assigned in a different sequence across the three years.

Also, in grades 8 to 10, only students enrolled in the corresponding traditional or integrated sequence take the standards-based part of the exam. In grade 11, all students take a cumulative form of the standards-based exam covering all of these topics, regardless of what mathematics courses they have taken.

In general, the performance on these standards-based examinations has been poor. This is expected since students have not previously been expected to meet the new standards throughout their academic careers. Achievement in LA has been poor as well. However, certain comparisons are already possible given the baseline test results from the spring 1999 test administration.

Results for LAUSD

The California data file for these test results gives means for the standards-based mathematics tests in grades 8 to 10 only for those students who are "on-track" for meeting the standards, meaning that they are taking the first year in grade 8, or the second year in grade 9, or the third year in grade 10. Here are the average number of correct answers for all of LAUSD

Grade	Traditional	Integrated
8	21.4	19.2

9	23.0	21.6
10	22.5	20.0

On average, the traditional sequence scores are about 10% higher than the integrated sequence scores. We cannot be certain that the curriculum accounts for this difference, since we don't know the characteristics of the students or teachers in each case. However, these results suggest that the integrated programs are less effective than the traditional ones in LAUSD.

Results for Economically Disadvantaged Students

The integrated programs also show weaker results for disadvantaged students. The STAR data file does not contain information on ethnic minorities, but it does summarize scores for economically disadvantaged students vs other students. Both groups achieved lower scores with integrated programs. This is not consistent with the idea that the integrated mathematics programs are better for the disadvantaged students. These results are consistent with the idea that the integrated programs lack equivalent mathematical content.

	Economically Disadvanted		All Others	
Grade	Traditional	Integrated	Traditional	Integrated
8	20.0	18.3	23.8	21.0
9	20.9	20.3	25.1	23.6
10	20.3	19.2	24.0	21.8

Results for LEP Students

From the state data file, it is also possible to inspect the LAUSD results for limited English proficiency students (LEP) compared to other students. Again, both groups achieved lower scores with integrated programs than with traditional programs across the three grade levels.

	LEP Students		All Others	
Grade	Traditional	Integrated	Traditional	Integrated
8	17.0	15.7	22.3	19.9
9	18.5	17.9	23.8	22.3
10	18.8	16.8	23.0	20.4

Results for Male and Female Students

The state data file breaks down scores by student gender. Again, both groups achieved lower scores with integrated programs than with traditional programs across the three grade levels.

	Female Students		Male Students	
Grade	Traditional	Integrated	Traditional	Integrated

8	21.3	18.8	21.5	19.8
9	22.4	21.1	23.8	22.2
10	21.6	19.7	23.4	20.4

Integrated and Traditional High Schools

It is possible to characterize high schools as traditional or integrated based on the tests taken by the students (counts of tests taken are given even when the scores are not reported). In this example, schools were identified as traditional if at least 75% of these augmented tests were in the traditional sequence, and they were called integrated if at least 75% of the tests were in the integrated sequence. These high schools were then compared on the basis of their average scores for the 11th grade where all students take the same standards-based mathematics exam. The results were:

	Traditional	Integrated
Number of Schools	38	28
Average Number Correct	16.1	15.0

Again, we cannot be certain about the actual cause of this difference, but the result favors the traditional approach. What about the "middle" group, those with somewhere between 25% and 75% traditional score reports? There were 15 high schools in this group with an average of 15.1 correct.

The Stanford 9 scores for these same high schools give an indication of achievement on a less rigorous assessment. The results using the NPR for the average student at each school are:

	Economically Disadvantaged		All Others	
Grade	Traditional	Integrated	Traditional	Integrated
9	38.6	35.9	40.1	33.5
10	35.3	33.3	36.8	31.3
11	41.9	37.5	41.9	34.2

The results for the percentage of students above the 50th percentile are:

	Economically Disadvantaged		All Others	
Grade	Traditional	Integrated	Traditional	Integrated
9	30.9	28.3	32.7	24.0
10	29.3	27.6	31.2	24.6
11	36.0	31.5	36.0	25.9

In all cases, economically disadvantaged or not, the means for the traditional program schools are higher than the means for the integrated program schools.

LA-SI Schools

The influx of integrated mathematics programs in LAUSD high schools is related to involvement with LA-SI (the Los Angeles Systemic Initiative, a federally funded project to implement integrated math programs). Schools with the longest involvement are Phase I schools. Of the eleven Phase I schools, all but one have experienced decline in SAT participation over the past two years, some rather substantially; 12% is the average reported by the IAU (the "Independent Analysis Unit" of LAUSD) to the Board in a report of May 12, 1999.

According to the IAU numbers, SAT performance across all of the LA-SI Focal schools is down about 5% in the number of takers and has a math average of 445 while the number of takers is up roughly that same 5% at the non-Focal schools with an average of 462. Neither of these numbers is terribly impressive but they suggest a reason why several high schools have abandoned integrated mathematics and are returning to more traditional programs.

The IAU only looked at the two years 1996-1998 but a longer perspective on these LA-SI Phase I schools is informative. Palisades is not on the state's data base because of its conversion to magnet status but data from the other ten is available on the Internet. That data starts with 1992, the year in which two of them, Roosevelt and Marshall, became pilots for an integrated program called IMP. From 1992 to 1998, the data year of the IAU report, these ten schools dropped an average of 13 points in their SAT math scores while experiencing a 30% overall drop in SAT participation. These numbers compare with the overall statewide math SAT average holding steady at 516 while participation increased by 24%.

Another useful measure of success is the Entry Level Mathematics Exam (ELM) required of students admitted to any CSU campus. The most recent data at that website is for those students admitted sometime during the 1997-8 year. A successful mathematics assessment is an SAT math score of 550 or a passing score on the ELM. For the LA-SI Phase I schools, the collectively failure rate was 78%. This compares with a statewide failure rate of 55%.

Alignment with Standards

The integrated secondary programs in use in LAUSD not only mix up the order of topic presentation, they also reduce the level of mathematics covered. The state has recently approved 5 algebra 1 programs under AB2519, while no integrated 1 programs were approved. In general, the integrated secondary programs were not even submitted because of their lack of alignment with the state standards. The district cannot legitimately certify these integrated programs as being aligned to the state standards.

Reform Methods without Integrated Secondary Programs

Even without these integrated secondary programs, LAUSD students will still have integrated content in grades K-7. Even without these integrated secondary programs, schools will be able to select books with varying degrees of reform mathematics methods at all grade levels. Indeed, the state-approved texts often note their inclusion of these new methods and make reference to the NCTM which is recognized for promoting these methods. Many include leading NCTM members as authors.

LAUSD can comply with state requirements and still encourage classroom teachers to use their own professional judgment in selecting the best methods for meeting the needs of their students. Indeed, this is exactly what is recommended in the state mathematics framework.

Los Angeles Times
Friday, September 17, 1999

L.A.'S MATH PROGRAM JUST DOESN'T ADD UP

Education: We're starting with basics for reading; why not give students the basics of arithmetic, algebra and geometry, too?

By DAVID KLEIN and R. JAMES MILGRAM

The new Los Angeles Unified School District Board of Education deserves praise and encouragement for its efforts to improve student academic achievement. Unfortunately, in the case of mathematics education, the board is getting bad advice from district staff.

Phonics and other basic language skills have received well-deserved national attention in recent years. As a result, "whole language" is disappearing from the curriculum. By contrast, "whole math," the philosophical sibling of whole language, is still entrenched in district schools.

The Los Angeles Systemic Initiative, or LASI, is a multiyear, federally funded district program with the worthy goal of improving mathematics and science education. The problem is that LASI has done more harm than good. The initiative's recommendations have caused many district schools to abandon credible arithmetic, algebra and geometry instruction. LASI has implemented the worst mathematics curricula that we are aware of, and we are aware of many due in part to our service on the California Content Review Panel for K-8 mathematics books. In that capacity, we made recommendations to the state Board of Education for statewide adoption on a huge number of math textbooks submitted by publishers--finding only a small fraction of these worthy of use by California students.

LASI has promoted an experimental K-6 math curriculum, Mathland, which has no textbooks for students. Its manual for teachers tells them not to explain the standard algorithms of arithmetic to children. In other words, children are not taught the traditional procedures for addition, subtraction, multiplication and division. Nowhere in any of these K-6 materials is the usual way to multiply two numbers, like 35 times 76, ever explained.

For high school, LASI recommends so-called integrated math curricula such as Interactive Mathematics Program. Like other integrated math programs, IMP suppresses basic algebra at all grade levels. For example, it delays an important eighth-grade algebra topic, called the quadratic formula, until the 12th grade. This defect alone puts Los Angeles students at a serious disadvantage on the California standardized testing and reporting, or STAR, exam that tests this topic in the eighth grade.

Mathland and IMP are not the only questionable programs implemented by LASI in Los Angeles schools. All of LASI's recommendations are problematic. The heavy emphasis on calculators is particularly damaging. This often results in students needing their calculators for even the most rudimentary figuring. It is our view that calculators should be used sparingly in grades 6-12 and not at all in grades K-5. The base 10 structure of our number system together with the standard arithmetic algorithms carry the seeds of algebra. Depriving children of mastery of arithmetic closes doors to more advanced mathematics courses in ways that district staff members do not seem to understand.

Statistics from the U.S. Department of Education show that success in secondary school algebra is the single greatest predictor of success in college--not just for engineering and science majors, but for majors in all fields.

Particularly troubling to us is the justification for LASI's watered-down mathematics programs as reported in The Times in August. An LASI supporter is quoted as saying, "There's a move to eliminate anything but old-style math. But it's only striking against inner-city schools where kids need a different approach--they need to see, touch and feel what they are learning."

We vigorously disagree. Independent of skin color and wealth, students need the same rigorous foundations, including the all important "old-style math" subjects of arithmetic, algebra and geometry. The legendary Jaime Escalante, depicted in the movie "Stand and Deliver," catapulted his disadvantaged students to national prominence using "old-style math." The high-achieving African American and Latino students at Bennett-Kew Elementary School in Inglewood provide another example. Sacramento Unified School District abandoned the faddish LASI-style curricula for its multiethnic students and increased its first and second grade SAT-9 test scores by more than 16 percentile points this year.

Data from the recent STAR exam show that students taking integrated math courses in California--such as those promoted by LASI--scored lower than their counterparts enrolled in traditional math courses.

All of the mathematicians who served on the Content Review Panel for the State Board agree about what constitutes a good mathematics curriculum. We urge the new Los Angeles school board to set aside the recommendations of LASI, and seek advice from the broader mathematics community instead.

David Klein, a CSUN Mathematics Professor, was appointed by the State Board of Education to evaluate mathematics teacher professional development programs.

R. James Milgram is a Stanford University mathematics professor who regularly advises the state Board of Education on math issues.

Old Math, Good Math

A recent Los Angeles Times lead editorial (January 29, 2000) was devoted to concerns about mathematics education in LA Unified School District. The Times said, in part:

> ***... Math experts agree that students will be able to learn algebra in secondary school only if they master basic math skills like multiplication tables in elementary school.***
>
> ***Many children throughout California, however, are denied such mastery because their school districts adhere to an experimental teaching method called integrated math. A method that disdains the notion of adults hierarchically imparting knowledge to kids, integrated math does not require students to memorize multiplication tables, compute fractions or learn other basic skills essential to algebraic success. It's often rightly derided as "fuzzy math" because of its murky goals ...***
>
> ***... Reform must also take hold in the California State University system, where some schools of education eschew traditional math concepts like memorizing multiplication tables and allow prospective teachers to use calculators on their final exam in basic arithmetic.***
>
> ***The governor's algebra academies are an important step, but the schools need to put a good grounding in math basics back into elementary classrooms as well.***

Mathematically Correct presents

A COMPARISON OF THE LAUSD MATH STANDARDS AND THE CALIFORNIA MATH STANDARDS

Introduction: Below is a comparison of The California Mathematics Academic Content Standards and the Mathematics Student Learning Standards for the Los Angeles Unified School District. The LAUSD standards provide guidelines only for grades 3, 7, 9, and expectations for high school graduates.

The California Mathematics standards were approved by the California Board of Education in February 1998. The LAUSD mathematics standards were recommended by the Los Angeles Systemic Initiative and were unanimously approved (along with Science, History/Social Science, and Language Arts standards) by the Los Angeles Board of Education in the Spring of 1996 for implementation in the 1996/97 academic year.

Shortly after the adoption of the California Math Standards by the California Board of Education, LAUSD Superintendent of Schools, Ruben Zacarias, issued a memorandum stating that

> "the LAUSD Standards include and go beyond the State Board standards."

No adjustment of LAUSD's math standards are necessary, according to Mr. Zacarias, as he explained that,

> "the high expectations for student achievement set forth by the [LAUSD] school board and the Superintendent will be met by implementing the standards-based curriculum recommended by the Los Angeles Systemic Initiative."

Mr. Zacarias further elaborated in his memorandum that textbooks aligned with the new California State Standards would have to be supplemented to "rise" to the level of the LAUSD math standards.

Mathematicians from five universities in the Los Angeles area challenged Zacarias' evaluation of the two sets of standards. They explained to the LAUSD Board of Education that exactly the opposite of Zacarias' assertions is true: the California Mathematics Standards are vastly superior to those of LAUSD. In broad terms, the LAUSD standards are so vague as to be almost meaningless. Use of calculators is required in the third grade, undermining the mastery of essential basic skills (whereas California's math standards will not allow calculators for examinations based on those standards in the elementary school grades). Many important and fundamental topics in mathematics are not even mentioned in the LAUSD standards.

An open letter in support of the California math standards was endorsed by more than 100 California mathematicians, including the chairs of the math departments at Stanford University, Caltech, UC Irvine, UC Riverside, Cal State Los Angeles, the vice president of the American Mathematical Society and a former president of the Mathematical Association of America. In a recent independent evaluation commissioned by the Fordham foundation, Prof. Raimi and Mr. Braden conducted a review of the mathematics standards for 46 states and the District of Columbia, as well as Japan. California's new board-approved mathematics standards received the highest score.

It is commonly agreed that the two most important things that students gain from the study of mathematics are the basic numeric tools needed for survival in modern society and an increased ability to make reasoned decisions. But the study of mathematics does more than this. The precision of thought and training in abstraction that one learns from mathematics are intellectual tools of profound importance.

This being so, it is reasonable to expect precision, clarity, and complete statements in the mathematics standards for LAUSD.

To illustrate the lack of these qualities, consider the LAUSD standard:

> "3. Solve problems based on algebraic relationships and functions; explore the relationship between the symbolic mathematical form of a function (expressed in equalities or inequalities) and a two- or three- dimensional graph of that function."

It would be unreasonable to assume that this actually meant something like "solve 2-step linear equations" or "solve systems of 2 linear equations in two unknowns".

It is difficult to make sense of standards like #3 above. The precision in communication, that mathematics seeks to achieve, requires a comparison of the California State Mathematics Standards with the Los Angeles Unified School District Standards not by what might be meant, but by what is actually stated.

With this in mind we now turn to a number of basic topics in the K-12 mathematics curriculum and compare the two documents directly. For each mathematical category listed below, all relevant LAUSD standards appear in the left column, and all relevant California standards appear in the right column. The most blatant short-coming of the LAUSD math standards requires no table for comparison; TRIGONOMETRY IS ENTIRELY MISSING FROM THE LAUSD MATH STANDARDS.

Number Line

Los Angeles	**California**
	Grade 4, Number Sense 1.5 interpret different meanings for fractions including parts of a whole, parts of a set, indicated division of whole numbers and quantities (and measures) between whole numbers on a number line; and relate to *simple decimals on a number line* 1.8 use concepts of negative numbers (e.g., on a *number line*, in counting, in temperature, "owing") 1.9 identify the relative position of fractions, mixed numbers, and decimals to two decimal places *on the number line* Grade 4, Statistics, Data Analysis and Probability 1.1 formulate survey questions, systematically collect and *represent data on a number line*, and coordinate graphs, tables and charts Grade 5, Number Sense 1.5 identify and represent positive and negative integers, decimals, fractions and mixed numbers on a *number line* Grade 6, Number Sense 1.1 compare and order positive and negative fractions, decimals, and mixed numbers and *place them on a number line* Grade 7 Number Sense 2.5 understand the meaning of the absolute value of a number, interpret it as the distance of the number from zero on a number line and determine the absolute value of real numbers

Development of Use of Negative Numbers	
Los Angeles	**California** Grade 4 Number Sense 1. Students understand place value of whole numbers and decimals to two decimal places, how these relate to simple fractions, and begin to work with *negative* numbers 1.8 use simple concepts of *negative* numbers (e.g., on a number line, in counting, in temperature, "owing") 3. Students solve problems involving addition, subtraction, multiplication and division of whole numbers, including the addition and subtraction of *negative* numbers, and understand the relationships among the operations. Grade 5 Number Sense 1. Students compute with very large and very small numbers, positive and *negative* numbers, decimals and fractions and understand the relationship between decimals, fractions and percents. 1.3 understand and compute squares and cubes of non-*negative* whole numbers; compute examples as repeated multiplication 1.5 identify and represent positive and *negative* integers, decimals, fractions and mixed numbers on a number line 2.1 add, subtract, multiply and divide with decimals and *negative* numbers and verify the reasonableness of the results Grade 6 Number Sense 1.1 compare and order positive and *negative* fractions, decimals, and mixed numbers and place them on a number line 2.3 solve addition, subtraction, multiplication and division problems, including those arising in concrete situations that use positive and *negative* numbers and combinations of these operations Grade 7 Number Sense 1.1 read, write and compare rational numbers in scientific notation (positive and *negative* powers of 10), approximate numbers using scientific notation 2.1 understand *negative* whole number exponents. Multiply and divide expressions involving exponents with a common base 2.2 interpret positive whole number powers as repeated multiplication and *negative* whole numbers as repeated division or multiplication by the multiplicative inverse. simplify and evaluate expressions that include exponents

Fractions Grades K-3

Los Angeles	California
Grade 3 37. Apply the basic operations (addition, subtraction, multiplication, and division) using whole numbers and *simple fractions* (halves, fourths); use rounding to the tens, hundreds, and thousands as an estimation strategy to check the reasonableness of results.	Grade 2 Number Sense 4. Students understand that *fractions* and decimals can refer to parts of a set and parts of a whole. 4.1 recognize, name and compare unit *fractions* up to 1/12 4.2 recognize *fractions* of a whole and parts of a group (e.g., 1/4th of a pie, 2/3'rds of 15 balls) 4.3 know that when all fractional parts are included, such as four-fourths, the result is equal to the whole Grade 3 Number Sense 3. Students understand the relationship between whole numbers, simple *fractions* and decimals. 3.1 compare *fractions* represented by drawings or concrete materials to show equivalency, and to add and subtract simple *fractions* in context (e.g., 1/2 of a pizza is the same amount as 2/4 of another pizza that is the same size; show that 3/8 is more than 1/8) 3.2 add and subtract simple *fractions* (e.g. determine that 1/8 + 3/8 is the same as 1/2) 3.4 know and understand that *fractions* and decimals are two different representations of the same concept (e.g., 50 cents 1/2 of a dollar, 75 cents is 3/4 of a dollar)

Fractions Grades 4-7

Los Angeles	California
Grade 7 25. Add, subtract, multiply, and divide using whole numbers, integers, primes, factors, multiples, *fractions*, decimals, rational numbers, exponents, and scientific notation; estimate and check the reasonableness of results. 33. Use deductive and inductive reasoning to solve mathematical problems; apply proportional reasoning to examine the relationships among *fractions*, decimals, and percents through examples involving rates, ratios, proportions, and scales.	Grade 4 Number Sense 1. Students understand place value of whole numbers and decimals to two decimal places, how these relate to simple *fractions*, and begin to work with negative numbers 1.5 interpret different meanings for *fractions* including parts of a whole, parts of a set, indicated division of whole numbers and quantities (and measures) between whole numbers on a number line; and relate to simple decimals on a number line 1.6 write tenths and hundredths in decimal and fraction notation and know fraction/decimal equivalents for halves and fourths (e.g., 1/2 = 0.5 or.50; 7/4 = 1 3/4 = 1.75) 1.7 write the fraction represented by a drawing of parts of a figure; represent a given fraction using drawings 1.9 identify the relative position of *fractions*, mixed numbers, and decimals to two decimal places on the number line Grade 5 Number Sense 1. Students compute with very large and very small numbers, positive and negative numbers, decimals and *fractions* and understand the relationship between decimals, *fractions* and percents. 1.2 interpret percents as part of a hundred; find decimal and percent equivalents for common *fractions*; explain why they represent the same value; and compute a given percent of a whole number 1.5 identify and represent positive and negative integers, decimals, *fractions* and mixed numbers on a number line 2. Students perform calculations and solve problems involving addition, subtraction and simple multiplication and division of *fractions* and decimals. 2.3 solve simple problems including ones arising in concrete situations involving the addition and subtraction of *fractions* and mixed numbers (like and unlike denominators of 20 or less) and express answers in simplest form 2.4 understand the concept of multiplication and division of *fractions*

	2.5 compute and perform simple multiplication and division of *fractions* and apply these procedures to solving problems Grade 5 Statistics, Data Analysis and Probability 5ST1.3 use *fractions* and percentages to compare data sets of different size Grade 6 Number Sense 1. Students compare and order *fractions*, decimals, and mixed numbers. They solve problems involving *fractions*, ratios, proportions, and percentages. 1.1 compare and order positive and negative *fractions*, decimals, and mixed numbers and place them on a number line 2.1 solve problems involving addition, subtraction, multiplication and division of *fractions* and explain why a particular operation was used for a given situation 2.2 explain the meaning of multiplication and division of *fractions* and perform the calculations (e.g., 5/8 divided by 15/16 = 5/8 x 16/15 = 2/3) 2.4 determine the least common multiple and greatest common divisor of whole numbers. Use them to solve problems with *fractions* (e.g., to find a common denominator in order to add two *fractions* or to find the reduced form for a fraction) Grade 7 Number Sense 1.2 add, subtract, multiply and divide rational numbers, integers, *fractions* and decimals and take rational numbers to whole number powers 1.3 convert *fractions* to decimals and percents and use these representations in estimation, computation and applications 1.5 know that every fraction is either a terminating or repeating decimal and be able to convert terminating decimals into reduced *fractions* 2. Students use exponents, powers, and roots and use exponents in working with *fractions*. 2.2 add and subtract *fractions* using factoring to find common denominators 2.3 multiply, divide, and simplify *fractions* using exponent rules

Percent, Interest, Compound Interest

Los Angeles	**California**
Grade 7 33. Use deductive and inductive reasoning to solve mathematical problems; apply proportional reasoning to examine the relationships among fractions, decimals, and *percents* through examples involving rates, ratios, proportions, and scales.	Grade 5, Number Sense 1. Students compute with very large and very small numbers, positive and negative numbers, decimals and fractions and understand the relationship between decimals, fractions and *percents*. They understand the relative magnitudes of numbers. 1.2 interpret *percents as part of a hundred*; find *decimal and percent equivalents for common fractions; explain why they represent the same value; and compute a given percent of a whole number* Grade 5, Statistics, Data Analysis and Probability 1.3 use *fractions and percentages to compare data sets of different size* Grade 6, Number Sense 1. Students compare and order fractions, decimals, and mixed numbers. They *solve problems involving fractions, ratios, proportions, and percentages*. 1.4 *calculate given percentages of quantities and solve problems involving discounts at sales, interest earned and tips*

	Grade 6, Statistics, Data Analysis and Probability 3.3 represent probabilities as ratios, proportions, and decimals between 0 and 1, *and percents between 0 and 100* and check that probabilities computed are reasonable; know how this is related to the probability of an event not occurring Grade 7, Number Sense 1.3 *convert fractions to decimals and percents* and use these representations in estimation, computation and applications 1.6 calculate *percent of increases and decreases* of a quantity 1.7 solve problems that involve discounts, markups, commissions, profit and *simple compound interest* Algebra I 15. Students apply algebraic techniques to rate problems, work problems, and percent mixture problems.

Lines and Linear Equations Grade K-3

Los Angeles	**California**
Grade 3 40. Use the geometric concepts of space and form to construct, describe, and compare the properties of one-, two-, and three-dimensional figures such as *line* segments, circles, simple polygons, and solids.	Grade 2, Statistics Data Analysis and Probability 2.1 recognize, describe, extend and explain how to get the next term in *linear patterns* (e.g., 4, 8, 12 ...; the number of ears on 1 horse, 2 horses, 3 horses, 4 horses) Grade 3, Algebra and Functions 2.2 extend and recognize a *linear pattern* by its rules (e.g., the number of legs on a given number of horses can be calculated by counting by 4s or by multiplying the number of horses by 4) Grade 3, Statistics, Data Analysis and Probability 1.3 summarize and display the results of probability experiments in a clear and organized way (e.g., use a bar graph or a *line* plot) 1.4 use the results of probability experiments to predict future events (e.g., use a *line plot* to predict the temperature forecast for the next day)

Lines and Linear Equations Grade 4-7

Los Angeles	California
	Grade 4, Measurement and Geometry 2. Students use two-dimensional coordinate grids to represent points and graph *lines* and simple figures. 2.1 draw the points corresponding to *linear relationships* on graph paper (e.g., draw the first ten points for the equation y = 3x and connect them using a straight *line*) 2.2 understand that the length of a *horizontal line segment* equals the difference of the x-coordinates 2.3 understand that the length of a *vertical line segment* equals the difference of the y-coordinates 3. Students demonstrate understanding of plane and solid geometric objects. They use this knowledge to show relationships and solve problems. 3.1 identify *lines that are parallel and perpendicular* Grade 5, Algebra and Functions 1.5 solve problems involving *linear functions* with integer values, write the equation, and graph the resulting ordered pairs of integers on a grid Grade 5, Measurement and Geometry 2.1 measure, identify and draw angles, *perpendicular and parallel lines*, rectangles and triangles, using appropriate tools (e.g., straight edge, ruler, compass, protractor and drawing software) Grade 6, Algebra and Functions 1. Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, *solve simple linear equations* and graph and interpret their results. 1.1 write and *solve one-step linear equations* in one variable Grade 7, Algebra and Functions 3. Students graph and interpret *linear and some non-linear functions*. 3.1 graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems 3.2 plot the values from the volumes of a 3-D shape for various values of its edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and a varying length equilateral triangle base) 3.3 *graph linear functions*, noting that the vertical change (change in y-value) per unit horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph 3.4 plot values of the quantities whose ratio is always the same (cost vs. number of an item, feet vs. inches, circumference vs. diameter of a circle). *Fit a line to the plot* and understand that the slope of the line equals the quantities. 4. Students *solve simple linear equations and inequalities* over the rational numbers. 4.1 solve *two-step linear equations and inequalities in one variable* over the rational numbers, interpret the solution(s) in terms of the context from which they arose and verify the reasonableness of the results 4.2 solve multi-step problems involving rate, average speed, distance and time, or direct variation Grade 7, Measurement and Geometry 3.3 know and understand the Pythagorean Theorem and use it to find the length of the missing side of a right triangle and lengths of other *line segments*, and, in some situations, empirically verify the Pythagorean Theorem by direct measurement 3.6 identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and how two or more objects are related in space (e.g., *skew lines*, the possible ways three planes could intersect)

Lines and Linear Equation Grades 8-12

Los Angeles	California
	Algebra I 4. Students simplify expressions prior to *solving linear equations and inequalities in one variable* such as 3(2x-5) + 4(x-2) = 12. 5. Students solve multi-step problems, including word problems, involving *linear equations and linear inequalities in one variable*, with justification of each step. 6. Students *graph a linear equation*, and compute the x- and y- intercepts (e.g., graph 2x + 6y = 4). They are also able to sketch the *region defined by linear inequality* (e.g., sketch the region defined by 2x + 6y < 4). 7. Students verify that a *point lies on a line given an equation of the line*. Students are able to *derive linear equations using the point-slope formula*. 8. Students understand the concepts of *parallel and perpendicular lines and how their slopes are related*. Students are able to find the *equation of a line perpendicular to a given line that passes through a given point*. 9. Students *solve a system of two linear equations in two variables* algebraically, and are able to interpret the answer graphically. Students are able to use this to *solve a system of two linear inequalities in two variables*, and to sketch the solution sets. Geometry 7. Students prove and use theorems involving the properties of *parallel lines cut by a transversal*, the properties of quadrilaterals, and the properties of circles. 16. Students perform basic constructions with straightedge and compass such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line. 17. Students prove theorems using coordinate geometry, *including the midpoint of a line segment*, distance formula, and *various forms of equations of lines and circles*. Algebra II 2. Students *solve systems of linear equations and inequalities (in two or three variables)* simultaneously, by substitution, graphically, or with matrices. Trigonometry 7. Students know that the *tangent of the angle a line makes with the x-axis is equal to the slope of the line*.

Quadratics

Los Angeles	California
	Algebra 1 14. Students solve a *quadratic* equation by factoring or completing the square. 19. Students know the *quadratic* formula and are familiar with its proof by completing the square. 20. Students use the *quadratic* formula to find the roots of a second degree polynomial and to solve *quadratic* equations. 21. Students graph *quadratic* functions and know that their roots are the x-intercepts. 22. Students use the *quadratic* formula and/or factoring techniques to determine whether the graph of a *quadratic* function will intersect the x-axis in zero, one, or two points. 23. Students apply *quadratic* equations to physical problems such as the motion of an object under the force of gravity. 25.3 Given a specific algebraic statement involving linear, *quadratic* or absolute value expressions, equations or inequalities, students determine if the statement is true sometimes, always, or never. Algebra 2 8. Students solve and graph *quadratic* equations by factoring, completing the square, or using the *quadratic* formula. Students apply these techniques in solving word problems. They also solve *quadratic* equations in the complex number system. 9. Students demonstrate and explain the effect changing a coefficient has on the graph of *quadratic* functions. That is, students can determine how the graph of a parabola changes as a, b, and c vary in the equation y = a(x-b)2 + c. 10. Students graph *quadratic* functions and determine the maxima, minima, and zeros of the function. 16. Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the *quadratic* equation representing it. 17. Given a *quadratic* equation of the form ax2 + by2 + cx + dy + e = 0, students can use the method of completing the square to put the equation into standard form and can recognize whether its graph is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation. Mathematical Analysis 5.1 Students can take a quadratic equation in two variables, put it in standard form by completing the square and using rotations and translations if necessary, determine what type of conic section the equation represents, and determine its geometric components (foci, asymptotes, etc.). 5.2 Students can take a geometric description of a conic section (e.g. the locus of points whose sum of its distances from (1, 0) and (-1, 0) is 6), and derive a quadratic equation representing it.

Exponents, Roots and Logarithms

Los Angeles	**California**
Grade 7 25. Add, subtract, multiply, and divide using whole numbers, integers, primes, factors, multiples, fractions, decimals, rational numbers, *exponents*, and scientific notation; estimate and check the reasonableness of results.	Grade 5 Number Sense 1.4 determine the prime factors of all numbers through 50 and write numbers as the product of their prime factors using *exponents* to show multiples of a factor (e.g., 24 = 2 x 2 x 2 x 3 = 2^3 x 3); Grade 7 Number Sense 2. Students use *exponents, powers, and roots* and use *exponents* in working with fractions. 2.1 understand *negative whole number exponents*. Multiply and divide expressions involving *exponents with a common base* 2.3 multiply, divide, and simplify fractions using *exponent rules* 2.4 use the inverse relationship between *raising to a power* and *root extraction* for perfect square integers; and, for integers which are not square, determine without a calculator, the two integers between which its *square root* lies, and explain why Grade 7 Algebra and Functions 2.1 interpret positive whole number *powers* as repeated multiplication and negative whole numbers as repeated division or multiplication by the multiplicative inverse. simplify and evaluate expressions that include *exponents* 2.2 multiply and divide monomials; extend the process of taking *powers* and *extracting roots* to monomials, when the latter results in a monomial with an integer *exponent* Algebra 1 2. Students understand and use such operations as taking the opposite, reciprocal, *raising to a power*, and *taking a root*. This includes the understanding and use of the *rules of exponents*. Algebra 2 7. Students add, subtract, multiply, divide, reduce and evaluate rational expressions with monomial and polynomial denominators, and simplify complicated fractions including fractions with *negative exponents* in the denominator. 11. Students *prove simple laws of logarithms*. 11.1 Students understand the inverse relationship between *exponents and logarithms*, and use this relationship to solve *problems involving logarithms and exponents*. 11.2 Students judge the validity of an argument based on whether the properties of real numbers, *exponents, and logarithms* have been applied correctly at each step. 13. Students use the *definition of logarithms* and the product formula for logs to *translate between logarithms in any bases*. 14. Students understand and use the *properties of logarithms* to simplify logarithmic numeric expressions and identify their approximate values. 15. Students determine if a specific algebraic statement involving rational expressions, *radical expressions, logarithmic or exponential functions*, is sometimes true, always true, or never true. Note: Additional standards involving exponential distributions and functions include: Probability and Statistics standard 4 Probability and Statistics-Advanced standard 7 Calculus standard 4.4

Triangles Grades K-3

Los Angeles	California
(There is no mention of triangles. See polygons below)	Kindergarten Measurement and Geometry 2.1 identify and describe common geometric objects (e.g., circle, *triangle*, square, rectangle, cube, sphere, cone) Kindergarten Statistics, Data Analysis and Probability 1.2 identify, describe and extend simple patterns involving shape, size, or color such as circle, *triangle*, or red, blue Grade 1 Measurement and Geometry 2.1 identify *triangles*, rectangles, squares and circles, including the faces of three-dimensional objects Grade 2 Measurement and Geometry 2.1 describe and classify plane and solid geometric shapes (e.g., circle, *triangle*, square, rectangle, sphere, pyramid, cube, rectangular prism) according to the number and shape of faces, edges and vertices 2.2 put shapes together and take them apart to form other shapes (e.g., two congruent right *triangles* can form a rectangle) Grade 3 Measurement and Geometry 2.2 identify attributes of *triangles*, (e.g., two equal sides for the isosceles *triangle*, three equal sides for the equilateral *triangle*, right angle for the right *triangle*)

Triangles Grades 4-7

Los Angeles	California
(There is no mention of triangles. See polygons below)	Grade 4 Measurement and Geometry 3.7 know the definition of different *triangles* (equilateral, isosceles, scalene) Grade 5 Measurement and Geometry 1.1 derive and use the formula for the area of right *triangles* and of parallelograms by comparing with the area of rectangles (i.e., two of the same *triangles* make a rectangle with twice the area; a parallelogram is compared to a rectangle with the same area found by cutting and pasting a right *triangle*) 2.1 measure, identify and draw angles, perpendicular and parallel lines, rectangles and *triangles*, using appropriate tools (e.g., straight edge, ruler, compass, protractor and drawing software) 2.2 know that the sum of the angles of any *triangle* is 180 degrees and the sum of the angles of any quadrilateral is 360 degrees and use this information to solve problems Grade 6 Algebra and Functions 3.1 use variables in expressions describing geometric quantities, e.g., P = 2w + 2l, A = 1/2 bh, C = (pi) d which give the perimeter of a rectangle, area of a *triangle*, and circumference of a circle, respectively Grade 6 Measurement and Geometry

	2.2 use the properties of complimentary and supplementary angles and of the angles of a *triangle* to solve problems involving an unknown angle 2.3 draw quadrilaterals and *triangles* given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles *triangle*) Grade 7 Algebra and Functions 3.2 plot the values from the volumes of a 3-d shape for various values of its edge lengths (e.g., cubes with varying edge lengths or a *triangle* prism with a fixed height and a varying length equilateral *triangle* base) Grade 7 Measurement and Geometry 2.1 routinely use formulas for finding the perimeter and area of basic two-dimensional figures and for the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, *triangles*, circles, prisms, cones and circular cylinders 3.3 know and understand the Pythagorean Theorem and use it to find the length of the missing side of a right *triangle* and lengths of other line segments.

Triangles Grades 8-12

Los Angeles	**California**
(There is no mention of triangles. See polygons below)	Geometry 5. Students prove *triangles* are congruent or similar and are able to use the concept of corresponding parts of congruent *triangles*. 6. Students know and are able to use the *triangle* Inequality Theorem. 10. Students compute areas of polygons including rectangles, scalene *triangles*, equilateral *triangles*, rhombi, parallelograms, and trapezoids. 12. Students find and use measures of sides, interior and exterior angles of *triangles* and polygons to classify figures and solve problems. 15. Students use the Pythagorean Theorem to determine distance and find missing lengths of sides of right *triangles*. 18. Students know the definitions of the basic trigonometric functions defined by the angles of a right *triangle*. They also know and are able to use elementary relationships between them, (e.g., tan(x) = sin(x)/cos(x), (sin (x))2 + (cos (x))2 = 1). 19. Students use trigonometric functions to solve for an unknown length of a side of a right *triangle*, given an angle and a length of a side. 20. Students know and are able to use angle and side relationships in problems with special right *triangles* such as 30-60-90 *triangles* and 45-45-90 *triangles*. Trigonometry 12. Students use trigonometry to determine unknown sides or angles in right triangles. 14. Students determine the area of a triangle given one angle and the two adjacent sides. Note: Additional standards involving trigonometry include: Trigonometry standards 3.2 8, 9, 10, 11, and 19 Mathematical Analysis standard 2 Calculus standards 4.4, 17, 18, and 20

Polygons

Los Angeles	California
Grade 3 40. Use the geometric concepts of space and form to construct, describe, and compare the properties of one-, two-, and three-dimensional figures such as line segments, circles, simple *polygon*s, and solids.	Grade 3 Measurement and Geometry 1.3 find the perimeter of a *polygon* with integer sides 2.1 identify and describe and classify *polygon*s (including pentagons, hexagons and octagons) Grade 6 Number Sense 1.3 use proportions to solve problems (e.g., determine the value of N if 4/7 = N/21, find the length of a side of a *polygon* similar to a known *polygon*). Use cross-multiplication as a method for solving such problems, [understanding it as multiplication of both sides of an equation by a multiplicative inverse.] Geometry 10. Students compute areas of *polygon*s including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids. 12. Students find and use measures of sides, interior and exterior angles of triangles and *polygon*s to classify figures and solve problems. 13. Students prove relationships between angles in *polygon*s using properties of complementary, supplementary, vertical, and exterior angles. 21. Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed *polygon*s of circles.

Any Further Topics Relating to Linear Algebra

Los Angeles	California
	LINEAR ALGEBRA The general goal in this discipline is that students learn the techniques of matrix manipulation so as to be able to solve systems of linear equations in any number of variables. Linear Algebra is most often combined with another subject, such as Trigonometry, Mathematical Analysis, or Pre-Calculus. 1. Students solve simultaneous linear equations in any number of variables using Gauss-Jordan elimination. 2. Students interpret linear systems as coefficient matrices and the Gauss-Jordan method as row operations on the coefficient matrix. 3. Students reduce rectangular matrices to row echelon form. 4. Students perform addition on matrices and vectors.

	5. Students perform matrix multiplication, multiply vectors by matrices and by scalars. 6. Students demonstrate understanding that linear systems are either inconsistent (no solutions), have exactly one solution, or have infinitely many solutions. 7. Students demonstrate understanding of the geometric interpretation of vectors and vector addition (via parallelograms) for vectors in the plane and in three dimensional space. 8. Students interpret the solution sets of systems of equations geometrically. For example the solution set of a single linear equation in two variables is interpreted as a line in the plane, and the solution set of a two by two system is interpreted as the intersection of a pair of lines in the plane. 9. Students demonstrate understanding of the notion of the inverse to a square matrix, and apply it to solve systems of linear equations. 10. Students compute the determinants of 2 by 2 and 3 by 3 matrices, and are familiar with their geometric interpretations as area and volume of the parallelepipeds spanned by the images under the matrices of the standard basis vectors in 2-dimensional and 3-dimensional spaces. 11. Students know that a square matrix is invertible if, and only if, its determinant is non-zero. They can compute the inverse to 2 by 2 and 3 by 3 matrices using row reduction methods or Cramer's rule. 12. Students compute the scalar (dot) product of two vectors in n-dimensional space, and know that perpendicular vectors have zero dot product.

Topics Which Either do not Exist or Have Nothing to do with Mathematics

Los Angeles	**California**
11. Analyze how inventions, discoveries, and events influence the development of mathematical theories and how mathematics continues to respond to changing societal, cultural, and technological forces. 23. Analyze the influences that historical events, scientific discoveries, and social changes have had and continue to have on the development of mathematical theories. 35. Compare and describe the number systems and mathematical concepts, for example, the Pythagorean Theorem, developed in different civilizations, historical periods, and cultures. 47. Compare the use of various number systems (for example, Hindu-Arabic, Roman, tally, etc.) from different historical periods.	

Remarks: The two standards 33 and 40 in the LAUSD mathematics standards together seem to bear the brunt of the content requirements there. Note that they each appear twice in the lists above, and in three of the four areas where they appear, they are the ONLY STANDARD from the LAUSD standards which mentions the topic.

It is perhaps also worth noting that standard 40 appears in the third grade standards and standard 33 in the seventh grade standards. In particular, this must minimize the level at which 40 applies. So we may reasonably infer that the expected background for students in LAUSD in lines and linear equations as well as their knowledge of such figures as polygons is expected to be negligible. Of course, while it could be argued that polygons are not central to applications of mathematics in the world and in the workplace, the same CANNOT be said for lines and linear equations. These are BASIC. Moreover, standard 33 only refers to *line segments* in geometric figures. So it can fairly be said that there is absolutely no discussion of linear equations, systems of linear equations, or related topics in these entire standards. It is hard to find the words to describe this situation.

Similarly, having a discussion of the essential topics of percent, interest, and compound interest confined to a single standard would imply, at best, minimal competence with these concepts. But these concepts are likely to influence every one of the major financial transactions for each of these students throughout their lives.

Some Remarks from Professor Ralph Raimi

(Ralph Raimi, together with Larry Braden, authored
the recent report State Mathematics Standards published by
the Fordham Foundation, March, 1998, Vol.2, #3)

It is hard to find parallel statements by which to compare the LA Standards and the CA Standards point-for-point. In general, the LA Standards are too vague to admit comparison with *anything*. And sometimes they are downright foolish, as in my first example.

LA Grade 7:

> "(35) Compare and describe the number systems and mathematical concepts, for example, the Pythagorean Theorem, developed in different civilizations, historical periods, and cultures."

This has *no* correspondent in the CA Standards, and I will not waste your time describing its fatuousness. These kids are 12 years old! They don't know how to place Napoleon and Julius Caesar in correct chronological order, and many of their teachers don't, either. Read it and weep. Do you suppose they intend to have the kids start with Neugebauer's *The Exact Sciences in Antiquity*? (The Dover edition is only $7.95, cheaper than algebra titles.)

Here is the nearest thing to a pair for comparison, for Grade 7:

Los Angeles	**California**
28. Identify, describe, compare, and classify geometric figures; apply geometric properties and relationships to solve problems; and use geometric concepts as a means to describe the physical world. *[At the Grade 7 level this is about all there is on geometry, except for the historical-philosophical item (35) Compare and describe the number systems and mathematical concepts, for example, the Pythagorean Theorem, developed in different civilizations, historical periods, and cultures.]*	2. Students compute the perimeter, area and volume of common geometric objects and use these to find measures of less common objects; they know how perimeter, area, and volume are affected under changes of scale. 2.1 routinely use formulas for finding the perimeter and areas of basic two-dimensional figures and for the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and circular cylinders 2.2 estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking them up into more basic geometric objects 2.3 compute the length of the perimeter, the surface area of the faces, and the volume of a 3-D object built from rectangular solids. They understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor 2.4 relate the changes in measurement under change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 12 square inches, 1 cubic inch = 2.54 cubic centimeters) 3. Students know the Pythagorean Theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures. 3.1 identify and construct basic elements of geometric figures, (e.g., altitudes, midpoints, diagonals, angle bisectors and perpendicular bisectors; and central angles, radii, diameters and chords of circles) using compass and straight-edge 3.2 understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections 3.3 know and understand the Pythagorean Theorem and use it to find the length of the missing side of a right triangle and lengths of other line segments, and, in some situations, empirically verify the Pythagorean Theorem by direct measurement 3.4 demonstrate an understanding of when two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures 3.5 construct two-dimensional patterns for three-dimensional models such as cylinders, prisms and cones 3.6 identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and how two or more objects are related in space (e.g., skew lines, the possible ways three planes could intersect)

It is possible that LA believes its Standard (28) quoted above includes all the things detailed in the new CA Standards, but I don't believe it, and belief is not a good test of a contract anyway.

Other particularly vague items in the LA Standards, that should be held up to scorn, and which could, with effort, be found to correspond with some rather extensive and particular listings in the CA Standards, are:

Graduate Level, items 8,9,10

> 8. Investigate the relationship between mathematical models and real-life problems by using hands-on materials and/or current technology such as calculators and computer modeling.

> 9. Make and test conjectures (inductive and deductive), construct simple arguments, validate solutions, and apply conclusions to various real-world situations.
>
> 10. Make connections among related mathematical concepts and apply these concepts to other content areas and the world of work.

9th Grade Level, items 21,22

> 21. Apply inductive and deductive reasoning and problem-solving strategies such as analysis of patterns, properties, relations of number systems, to validate solutions and apply conclusions to various real-world situations.
>
> 22. Make connections among mathematical concepts and apply these concepts to other content areas and to real-life situations.

Notice: Item 16

> 16. Use inductive and deductive reasoning and concepts of coordinate and transformational geometry to analyze geometric relationships, validate formal and informal proofs, and solve problems in geometric relationships such as congruency and similarity.
>
> > This item fails to specify whether Euclid's axiom system, and deductive proof, are intended anywhere. The phrase "deductive and inductive reasoning" is borrowed from NCTM and means nothing.

7th grade level, item 29

> 29. Apply a variety of discrete structures (series, sequences, matrices, and tree diagrams) to find possible combinations and arrangements in a problem situation.

3rd grade level, item 46

> 46. Make connections among mathematical concepts and relate them to concepts in other content areas and in daily life.

Lack of Substance in LAUSD Standards

LAUSD staff members have maintained that the LAUSD mathematics standards are aligned with the California mathematics standards, and LAUSD Superintendent of School Ruben Zacarias claimed in his "Informative" that the LAUSD standards are even superior. This claim is based on wildly inflated interpretations of the LAUSD standards. For example, it might be claimed that standards #3 and #15 above subsume all California math standards related to linear equations, quadratics, simultaneous equations, perhaps even linear algebra, and all possible functions and all possible relations.

> 3. Solve problems based on algebraic relationships and functions; explore the relationship between the symbolic mathematical form of a function (expressed in equalities or inequalities) and a two- or three-dimensional graph of that function.
>
> 15. Identify patterns, functions, and other algebraic relationships including inequalities; use tables, graphs, and equations to model functional relationships in real-life situations; apply knowledge of functions to analyze and interpret problems, predict solutions, and create algebraic algorithms to solve problems."

Unfortunately, it is very likely that arguments of this type will continue to be made by LAUSD staff. The contention that these standards go far beyond the state standards is based on **vagueness**. However, vagueness is not a virtue in standards of learning for mathematics. A document seeking to detail the MATHEMATICS that students need to know at various points in their K-12 educations must describe that material precisely.

This simply does not happen in the LAUSD standards.

Fundamentally, the writing here is so sloppy and imprecise that it is impossible to give the standards any reasonable meaning within the context of mathematics.

The vacuousness of the LAUSD math standards facilitates poor achievement of LAUSD students in mathematics. Since these standards can mean whatever a particular reader wants them to mean, they are not standards at all. They serve only to protect poor achievement in mathematics -- the status quo for LAUSD.

Partial Transcript: LAUSD Board Meeting, July 14, 1998

Diana Dixon-Davis: My name is Diana Dixon-Davis. I have three sons. Two are still in LAUSD. I'm elected parent representative of school based management, school site council at Chatsworth High School and Lawrence Middle School, and I'm also demographer, epidemiologist and have a Masters Degree in those fields.

I'm here to talk about math options at the secondary level. In April, we did call and we ended up speaking before the curriculum committee earlier this month [actually, May 14], and I did want to thank David Tokofsky and John Liechty, and Bob Collins for hearing our plea and helping us with some of our concerns.

However, there are still larger issues left, and this is concerning integrated math, traditional math, and the wholesale adoption of one over the other.

I was talking to a teacher the other day and they said, "You know, every time LAUSD has a problem, instead of trying to figure out what's causing the problem and teaching the program more effectively and seeing why some kids succeed and others fail, the solution is throw it all out. Start something new." Unfortunately, that "start something new" right now is LASI's program on integrated math. Integrated math [i.e., LASI] is less than one twentieth of one percent of all the LAUSD money per year. They have a $15 million grant spread over 3 [really 5] years, and yet this tail, let alone, actually a hair on the tail of a dog, is wagging the entire district. And they have decided that they will have everybody adopt integrated math.

In your own publication [holds up document], it says, as of now 337 schools have enrolled in the LASI project. Another 200 schools are expected to join next year, with the final 50 joining the year after. The hair of the dog is literally wagging the dog.

We are worried about this because there is no data, no research, and yet we are going for 100% adoption of an untried program. This is much like what happened with Whole Language. We had a program adopted without real data and research behind it. We kept wondering what's going on? Why are we doing this?

We have major conflicts of interest. Sarah Munshin, the author of the book that's being promulgated by LASI, is also the major consultant to our cluster--to LASI. I was told you don't do this. Unfortunately she is still a major consultant to LASI and to our cluster. Secondly, LASI is paying for 40% of our new textbooks. That's an incentive in any high school because we're always strapped for cash for buying books. And when they say if you adopt this program, we'll pay for 40% of your books, all of a sudden people start adopting it.

Some of the major problems with the integrated math have been iterated before: it's a three year program, you can't move in and out, most of the best schools do not use integrated math. In fact, our best schools, our gifted magnets, do not use integrated math. And the 100 best schools in the country, in a recent Newsweek article, they found of the schools that were reached, 75% use only traditional math, 5 schools offered both, and one school was the only one that used integrated math whole cloth.

[Note: Newsweek, March 30, 1998 listed the "Top 100 High Schools" in the nation according to its own criteria in the article "Class Struggle," by Jay Mathews. A group of parents and teachers in Juneau, Alaska attempted to contact these schools to find out which math programs they use. Only 30 schools responded. Of these, 24 said they offered only traditional math programs, 5 schools offered both traditional and integrated math, and one school said that it offered only integrated math, but with modifications to suit different students' needs and abilities--David Klein]

I had a lot more things to say. I think this is an issue that needs more than 3 minutes here at the board. I think we really need to have a wholesale discussion of the problems with integrated math, the need for accommodating students' needs as they move around the districts, and also if they move into other school districts in other states where integrated math is not yet accepted. We need real data, real research.

Just one parting comment, I'm sorry. LASI's own data--I got a copy of their report under the table--shows that students taking integrated math are doing less well than students who are taking traditional math, number one. Number two, minority students are doing less well. Latino and Asian students are doing less well than they were before with traditional math. And the only students who are benefiting are African American students.

And I think we need to really seriously look at a program that is only benefiting 10% of our students and leaving 90% behind. Thank you.

Barbara Beaudreax (LAUSD Vice President, and acting chair for this session): I have another speaker on the same subject, Dr. David Klein?

David Klein: I have some handouts for the Board. How do you handle that?

Ruben Zacarias (Superintendent of LAUSD): (gesturing) She'll get them from you.

David Klein: My name is David Klein. I'm a professor of mathematics at Cal State Northridge. I'm a strong supporter of public education. I want to see the best for LAUSD.

My purpose here is to discuss the LAUSD math standards and to urge you to replace them by the California math standards.

To be useful, standards need to be specific and give grade by grade benchmarks. They have to be sufficiently clear so that parents, students, teachers, and administrators know exactly what students are expected to know and be able to do. And they have to be measurable.

Unfortunately, the LAUSD math standards fail in each of these respects. They are repetitive. They are vague, and they're not measurable. You can't tell whether the students are meeting the standards or not. And they fall far below the level of the California math standards and are certainly not in conformity with them.

The California math standards, by contrast, are clear, specific, testable. They're internationally competitive. They give specific benchmarks for each year. They specify what a student has to know and be able to do. But they place no restrictions on the teaching method. They contain a balance of concepts, basic skills, and high level problem solving skills.

As an aid to understanding the comparison, I've given you a table--that's this particular handout [holds up document].

There are a number of categories for each of the standards. You see that in most of the categories, the LAUSD standards are blank. They simply don't address the issues. For example, there's no trigonometry specified whatsoever in the LAUSD standards in all of the grades K through 12. Is that the will of the Board that trigonometry is now no longer part of the education system in K-12?

Triangles. The word "triangle" simply does not appear, at all, in the standards. Calculators are encouraged even at the early grades, first, second, and third grade, by the LAUSD standards.

Whether or not LAUSD adopts the California math standards, the STAR test next year will be based, in part, on them. And I think our district will be much better off if we prepare in advance for that. If you adopt the California math standards, you will not be making a mistake. They've received national acclaim. The Fordham Foundation [holds up document], of which I've given you a portion of the report, commissioned a study of 46 states, the District of Columbia, and Japan. California's math standards ranked number one out of all of those. I've also given you a letter that I wrote which is signed by 100 California mathematicians which expresses support for the California math standards. The CSU Chancellor has also strongly supported them.

Valerie Fields (LAUSD Board Member): I don't know who's the new chairman of the ICSA [sp?]. George? But I think this certainly would be appropriate for the curriculum and instruction committee to go into some depth.

I wanted to assure Ms Davis that I had a meeting just last week with John Liechty and Bob Collins on this subject because I too was concerned that we were going to go down the path of Whole Language--only this time, Whole math--and I want to make sure--two things--one is that we always provide the traditional...

David Tokofsky (LAUSD Board member): Options.

Valerie Fields: ...math in every school so that any child who has a special ability in math or a special interest in math can take calculus or trig or whatever he or she wants to take. And those children, who are perhaps less adept, can continue with LASI. They assured me that is the case and would continue to be the case. I also feel we should not adopt, districtwide, any program, until after it's been piloted and there has been a reasonable evaluation about how kids are doing.

If they're doing better, terrific. If they're not, then we shouldn't adopt it. If we could make that recommendation to Mr. Kiriyama, that this be a topic for investigation by the curriculum and instruction committee...

Barbara Boudreaux (Vice President of the LAUSD Board): Mr. Superintendent?

Valerie Fields: ...both this topic and the topic...

Barbara Boudreaux: Thank you, we'll make...

Valerie Fields: Pardon?

Barbara Boudreaux: We'll make that recommendation.

Valerie Fields: Yes. And the topic of our standards and how they stack up with California's standards. Certainly we want to have the strongest standards possible.

Barbara Boudreaux: Mr. Superintendent, we have someone in the audience who's from the LASI office, division of instruction, Ms MacIver [Note: the spelling of this name is a guess]. She might have something to share to these comments.

Ms MacIver: Good evening Madam Vice President of the Board, members of the Board, and Superintendent of the district. I'm representing the division of instruction. As you know, Carmen Schroeder and many of the other administrators of the division of instruction are in Palm Springs doing leadership. Carol Takemotto waited as long as she could and asked me to respond. This is not our first encounter with Ms Diana Dixon-Davis or with Dr. Klein. And what they would like for me to do is to request that they put their statements in writing and present them to us with their sources, citing the sources of their data, and to give the specifics--because I do know that something that Ms Davis Dixon said about the LASI report was not quite directly quoted--so that we can respond in writing to the Board. And therefore take a look. And looking at the Stanford-9, since we only have the 96-97 data, we have not yet had a chance to analyze the 97-98, it would be unfair to make a motion, or to make a decision at this point that what we are trying to do is not helping the students. So, we would like to have their information in writing and we would like the opportunity to come back to the board in writing. That would include the division of instruction and LASI

Barbara Boudreaux: Mr. Superintendent?

Ruben Zacarias: I thank you for your comments. I totally agree with Ms Fields that this is a proper and important subject to be fully discussed at a board subcommittee meeting. ICSA [sp?], it's called, the curriculum committee? I would also say to you that as an educator, I believe in offering as many options as possible to our students. It shouldn't necessarily be an "either or." You can have both because different students learn in different ways with different approaches, so I would hope, Madam Chair, that the board takes this important subject up for full discussion.

Barbara Boudreaux: Mr. Kiriyama, while you were out, we were thinking of having this as a part of the agenda for your committee meeting, LASI

Valerie Fields: and the standards

Barbara Boudreaux: and the standards. Jeff has a comment.

Jeff Horton (LAUSD Board member): I think we should have it on every year. I mean, we have had it in every year. We had an extensive discussion this year under David's [Tokofsky] leadership. We had several discussions of it in the instruction committee when I was chair--I mean, we've had a lot of discussion of this and seen a lot of data--and I mean--which doesn't really support what we heard tonight. But we have to do it again, I agree. I mean, we have to be constantly evaluating. I just heard from both the speakers some real misunderstandings as to what it's even about, and I think the board--maybe what we need is a special committee of the whole, because I think the whole board needs to know. It is the districts effort to improve math, science, and technology instruction for all students, especially those groups that have not benefited from the traditional approach.

Ms MacIver: Could I make one final...

Barbara Boudreaux: Excuse me. Mr. Tokofsky you wanted...

David Tokofsky: The division did produce a document I believe--correct me if I'm wrong--that compares the district standards and the state standards, and how our standards match with the state's, right?

Ms MacIver: And my understanding is the assessment that ours went beyond the state standards.

David Tokofsky: But there is a document that could be provided to the public that would-- because one thing is of concern that we mentioned before is that there's a whole bunch of grants that the district will apply for and if we don't match the state standards, we're ineligible, and that's why, I think, we had to do that. And if it's going to be subjected to the rigors of professors of math--that would be great--who've worked on the state standards and compared. But if you could have Ms Schroeder provide that--I think she gave it to me last week and it might be available for all Board members and the public.

Valerie Fields: And Dr. Klein

David Tokofsky: Yeah, the public

Barbara Boudreaux: Thank you

Ms MacIver: May I make one last comment. I think what we want to be concerned with is also equity and making sure we don't decide that students at one part of the district will be given traditional education and students at another part will be given something different. And in having meetings with persons form the UC system, the integrated mathematics and science is accepted at an equal level for the a-f requirements the traditional algebra, geometry, algebra II.

David Tokofsky: So we can make sure all children get triangles.

Barbara Boudreaux: Right. Let me say that I agree with all of the comments form the Superintendent and Ms Fields and Jeff and all the Board members that we need this as a topic for our agenda in the future and also making sure that we have correct data so that no misinformation goes out.

David Tokofsky: There's an evaluation that they provided too, from an outside evaluator.

[new topic]

The Freedom to Agree

by David Klein

The L. A. School District Board of Education shows its appreciation for open-minded debate. These folks decide how math is taught to your kids.

Several years ago, the National Science Foundation awarded the Los Angeles Unified School District $15 million to replace secondary school courses in algebra and courses in geometry with "integrated math." The grant, called the Los Angeles Systemic Initiative or LA-SI, is also intended to promote elementary school curricula, like "MathLand," which de-emphasize arithmetic and push "group discovery" learning with calculators. LA-SI has similar goals for science education.

Integrated math is unpopular with parents. It superficially combines statistics, geometry, and algebra in each of the middle and high school grades. Students are deprived of the concentrated exposure to algebra, geometry, and trigonometry required to pursue scientific careers, or even to become scientifically literate citizens. It is a dumbed-down version of high school mathematics intended to equalize mathematics achievement among all students. Critics, like myself, have termed it "fuzzy math" (detailed descriptions are available at the website of a parents group called Mathematically Correct, which is critical of integrated math.

The motivation for fuzzy math is a deep-seated fear among many liberals that minority students are just too dumb to learn real math. So, to equalize student outcomes, we'd better get rid of unpleasant topics like algebra, or at least dilute them. This is the reigning perversion of egalitarianism in liberal education circles. Liberals have revealed their racism and ceded the high ground in education to the conservatives without so much as a struggle. Education leaders regard parental requests for algebra courses for their children as interference. LAUSD, with federal backing, knows what's best and everybody else has the freedom to agree. No other freedom is allowed.

But parents of school children failed to exercise their LAUSD-given freedom to agree during a meeting at a public school in the San Fernando Valley. On June 15, a group of 50 to 100 parents, teachers, and LA-SI leaders met at Nobel Middle School to discuss the implementation of integrated math. As a mathematics professor from the nearby Northridge campus of California State University, I was asked by two parents to attend. At my request, Ali Zakeri, another mathematics professor from Northridge, also attended that evening.

Prior to the meeting, a parent and local school activist, Diana Dixon-Davis, and I urged the LAUSD Board of Education to adopt the California Mathematics Standards and to make available Algebra I courses to students who request them. A sympathetic assistant superintendent, John Liechty, appropriated approximately $9,000 to each of three middle schools so that they could each offer a single Algebra I course, in addition to their LA-SI-sanctioned integrated math series.

But the principal of Nobel wanted no part of Algebra I. She would not even acknowledge that she had money approved for an Algebra I class. The June 15 meeting was an indoctrination session for integrated math.

The first hour was spent on the predictable dog and pony show, complete with overhead projectors, to demonstrate the futility of algebra and the glories of integrated math. Driven by the $15 million, fuzzy math teachers from other schools, LA-SI leaders, and even a district cluster leader lectured the audience that Algebra I is no good and integrated math is great. It was explained that within five years, all secondary math courses in LAUSD would become integrated math courses. Parents knew that the top performing high schools in LAUSD don't use integrated math, but comments from the audience were suppressed.

At one point during the presentation, the outgoing head of the math department at Chatsworth High, one of the leaders of the pro-integrated math movement, approached me from behind my seat and demanded to know where I got the copy of the integrated math textbook under discussion. It was sitting on my lap, and she demanded that I give it to her immediately. I had never before met her and only learned her identity a few days later. I refused to give her the book, and Diana Dixon-Davis explained that she had borrowed it from one of the schools. She signed out for it and my possession of it was above board. Evidently the math teacher did not want me even to see this math text, since I am a known non-believer of the true religion. She left in a huff.

Then three-by-five cards were passed around. These were for written questions. Verbal questions were not to be allowed by edict of the principal. Diana Dixon-Davis bravely interrupted the speakers on several occasions and read aloud statistics contradicting the claims of the high priests of fuzz. These interruptions were not appreciated by the truth monitors.

As the little cards were being collected, I stood up and objected to the process. I pointed out that it was designed to stifle any and all legitimate criticisms of integrated math and filter all questions through LA-SI leaders. I said that Prof. Zakeri and I came, not to ask questions on three-by-five cards, but rather to criticize the integrated math books used by this particular school.

The principal said that we would not be allowed to speak. But when several parents demanded that we be heard, she relented. Ali and I walked to the front of the auditorium. When we reached the podium, the principal insisted on knowing what we were going to say before she would hand me the microphone. I told her that I was going to criticize integrated math and speak of the virtues of a traditional course in algebra. She refused to give me the microphone, and almost on cue, one of the true-believers insisted that it would be unfair to allow us to speak.

That was that. We returned to our seats. Several parents objected once again. The principal said that we could come to her office and express our concerns privately the next day. When a few parents said they wanted to hear us, she said they too could come to her office.

I stood up and said that Ali and I would definitely not return the following day. The principal then said we could speak at the end of the three-by-five question-and-answer-session. About this time a policeman walked into the room and stood in the back.

The praise of integrated math droned on and the last three-by-five card was finally read. At that point a parent said that he wanted to hear what the "opposition" had to say. The principal said that it would not be fair for her to allow Ali and me to talk, since we were not previously scheduled.

Objections to the censorship were stronger this time. One of the parents demanded a vote. There was a spontaneous show of hands overwhelmingly in favor of letting us speak. I introduced myself, explained the importance of algebra and the weaknesses of integrated math. I explained how a concentrated course in algebra is better for those who intend to go into math, science, or engineering. I talked a little about the excellent new California math standards. I introduced Ali Zakeri as one of 20 recent recipients in the U.S. of the Mathematical Association of America distinguished teaching award. He continued on the same theme, indicating the superficiality of integrated math compared to a good algebra course.

Then we had a question and answer session. One of the fuzzy math advocates demanded an example of how integrated math was more superficial than the traditional sequence. I talked about the treatment of the quadratic formula and explained in detail how it was splattered over three years in the integrated sequence and still not done properly. LA-SI leaders moaned audibly in derision. All the while the policeman was standing in the back of the room staring at me and Ali. An education professor from CSUN, with a child attending the school, stood up and explained that education professors at CSUN disagree with the "subject matter professors" and that from the point of view of research on how children learn, integrated math is better. These are the people who train our teachers.

Eventually the fuzzies took over the microphone again and resumed efforts to get the crowd to worship the goals of the $15 million NSF grant. I wish now in retrospect I had said something about the power of money to force programs in school districts. $15 million buys many true believers. Unfortunately, I didn't think of it at the time.

Ali and I left at this point. The situation was chaotic with parents not knowing what to believe. But as we left, the high priests were once again baptizing the parents in the true religion. I don't know what the outcome will be. Diana Dixon-Davis remained and fought it out. She was tersely instructed by the cluster leader not to announce that money had been allocated for any Algebra I course, unless she had proof.

She obtained that proof the following day in the form of a memorandum of understanding from Asst. Superintendent John Liechty. I have learned that some parents are now adamantly demanding a real algebra course instead of the NSF-funded stupification program. But I'm inclined to believe that the LA-SI leaders will ultimately have their way. Algebra and geometry as individual subjects will be abolished through government power. The sole right of parents is to agree.

CPR

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David Klein is a mathematics professor at California State University, Northridge, and a member of the Green Party. Despite considering himself firmly on the left, he finds much to criticize in liberal education policy.

LAUSD's Refusal to Adopt the California Mathematics Standards

by David Klein
Professor of Mathematics
California State University, Northridge

BACKGROUND

Shortly after the adoption of the California Math Standards by the California Board of Education, LAUSD Superintendent of Schools, Ruben Zacarias, issued an Informative stating that *the LAUSD Standards include and go beyond the State Board standards*.

No adjustment of LAUSD's math standards is necessary to accomodate the California math standards, according to the Informative. It explains that:

> *... the high expectations for student achievement set forth by the [LAUSD] school board and the Superintendent will be met by implementing the standards-based curriculum recommended by the Los Angeles Systemic Initiative*.

The Los Angeles Systemic Initiative (LASI) is a project to impose an integrated math curriculum in LAUSD. Among other weak curricula, LASI promotes the highly discredited elementary school curriculum, MathLand.

The LAUSD Informative further claims that textbooks aligned with the new California State Standards would have to be supplemented to rise to the level of the LAUSD math standards.

In response to this statement, on March 16, 1998 a number of mathematicians and others familiar with the California Mathematics Standards released a document entitled A Comparison of the LAUSD Math Standards and the California Math Standards.

This study was based on the idea that such an analysis *requires comparison of the California Mathematics Standards with the Los Angeles Unified School District Standards not by what might be meant, but by what is actually stated*.

On this basis it was concluded that the LAUSD Standards are woefully lacking in critical mathematical content areas and lacking in precision, clarity and completeness.

In the conclusion, the comparison of the math standards notes:

The vacuousness of the LAUSD math standards facilitates poor achievement of LAUSD students in mathematics. Since these standards can mean whatever a particular reader wants them to mean, they are not standards at all. They serve only to protect poor achievement in mathematics -- the status quo for LAUSD.

The comparison document also forewarned of *wildly inflated interpretations of the LAUSD standards. For example, it might be claimed that standards #3 and #15 ... subsume all California math standards related to linear equations, quadratics, simultaneous equations, perhaps even linear algebra, and all possible functions and all possible relations*.

LAUSD STAFF RESPOND

At the behest of LASI, Ruben Zacharias, LAUSD Superintendent, released a second Informative to the LAUSD Board of Education on June 4, 1998. It state, *There has recently been a great deal of confusion about the LAUSD mathematics standards, their rigor, and how they align with the California state standards.* The LAUSD/LASI Informative continues, *While at first glance, the LAUSD standards and the state standards appear to be fairly different, this is largely due to formatting choices*.

The LAUSD/LASI Informative undermines its own purpose by reinforcing many of the public criticisms of the LAUSD Standards. For example:

- Aside from a brief statement that students should know basic numerical skills, the Informatives acknowledges that students really are allowed to use calculators from the earliest ages -- third grade and below. In fact, the LAUSD math standards themselves explicity require the use of technology *such as calculators or computers* in its third grade benchmark. The LAUSD math standards are in conflicht with California state policy which does not allow calculator usage on the STAR exams in the elementary grades.

- The LAUSD/LASI Informative states that *LAUSD has written broad statements at only the selected benchmark grades of third, seventh, ninth and twelfth*. In other words, these grades are treated minimally, while other grades are missing altogether.

- The second Informative admits that the LAUSD Standards lack not only Trigonometry, they lack second year Algebra.

Thus, the LAUSD Standards have serious defects - defects that do not disappear even with this second attempt to deny them.

THE LAUSD COMPARISON ANALYZED

The document, A Comparison of the LAUSD Math Standards and the California Math Standards, dispels any notion that LA standards are even remotely comparable to the California Mathematics Standards. Even though it was intended to rebutt criticisms of the LAUSD math standards, a careful reading of the June 4 LAUSD/LASI Informative itself provides evidence that:

- The LAUSD standards present some required topics too late

- The LAUSD standards omit some required topics

- The LAUSD standards are unclear and subject to a wide range of interpretation

- The LAUSD standards omit grade-by-grade benchmarks and contradict the California standards

LAUSD standards present some required topics too late

The LAUSD/LASI Informative includes detailed tables which juxtapose LAUSD Math Standards with California Math Standards. In the first table, LAUSD mathematics graduation requirements are equated with California algebra I standards and geometry standards. These California algebra I and geometry standards are designed to target 8th and 9th grade students, but LASI identifies these as graduation (i.e., 12th grade) standards. This is a clear (though perhaps unintended) admission by LASI and LAUSD staff that the LAUSD standards expect less of students than the California Math Standards. Here is a list of California algebra and geometry standards which, according to the LASI/LASUD Informative, LAUSD requires only AFTER the 9th grade:

Algebra I

1. Students identify and use the arithmetic properties of subsets of integers, rational, irrational and real numbers. This includes closure properties for the four basic arithmetic operations where applicable.
1.1 Students use properties of numbers to demonstrate that assertions are true or false.
3. Students solve equations and inequalities involving absolute values.
4. Students simplify expressions prior to solving linear equations and inequalities in one variable such as $3(2x-5) + 4(x-2) = 12$.
19. Students know the quadratic formula and are familiar with its proof by completing the square.
23. Students apply quadratic equations to physical problems such as the motion of an object under the force of gravity.
25. Students use properties of the number system to judge the validity of results, to justify each step of a procedure and to prove or disprove statements.
25.1 Students use properties of numbers to construct simple valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.
25.2 Students judge the validity of an argument based on whether the properties of the real number system and order of operations have been applied correctly at each step.
25.3 Given a specific algebraic statement involving linear, quadratic or absolute value expressions, equations or inequalities, students determine if the statement is true sometimes, always, or never.

Geometry

7. Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.
11. Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.
13. Students prove relationships between angles in polygons using properties of complementary, supplementary, vertical, and exterior angles.
14. Students prove the Pythagorean Theorem.
17. Students prove theorems using coordinate geometry, including the midpoint of a line segment, distance formula, and various forms of equations of lines and circles.

21. Students graph quadratic functions and know that their roots are the x-intercepts.

The LAUSD standards omit some required topics

The LAUSD/LASI Informative acknowledges that topics of Algebra II and above are not covered in the LAUSD standards. But this admission does not go far enough. Nowhere in any of the tables of the LAUSD/LASI Informative are the following California algebra I and geometry standards assigned counterparts in the LAUSD math standards. These topics are entirely omitted by the LAUSD Math Standards:

Algebra I

9. Students solve a system of two linear equations in two variables algebraically, and are able to interpret the answer graphically. Students are able to use this to solve a system of two linear inequalities in two variables, and to sketch the solution sets.
10. Students add, subtract, multiply and divide monomials and polynomials. Students solve multistep problems, including word problems, using these techniques.
11. Students apply basic factoring techniques to second and simple third degree polynomials. These techniques include finding a common factor to all of the terms in a polynomial and recognizing the difference of two squares, and recognizing perfect squares of binomials.
14. Students solve a quadratic equation by factoring or completing the square.

Geometry

1. Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.
2. Students write geometric proofs, including proofs by contradiction.
3. Students construct and judge the validity of a logical argument. This includes giving counter examples to disprove a statement.
18. Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them, (e.g., $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$).
19. Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.
20. Students know and are able to use angle and side relationships in problems with special right triangles such as 30-60-90 triangles and 45-45-90 triangles.

LAUSD standards are unclear and subject to a wide range of interpretation

The LAUSD/LASI Informative arbitrarily juxtaposes state standards with LAUSD standards in an effort to demonstrate consistency. However, it is evident that the Informative takes great liberties in interpreting the meaning of the LAUSD standards. For example, the LAUSD standard number 28 states:

> *Identify, describe, compare, and classify geometric figures; apply geometric properties and relationships to solve problems; and use geometric concepts as a means to describe the physical world.*

This could mean:

2. Students describe and compare the attributes of plane and solid geometric figures and use their understanding to show relationships and solve problems.
2.1 identify and describe and classify polygons (including pentagons, hexagons and octagons)
2.2 identify attributes of triangles, (e.g., two equal sides for the isosceles triangle, three equal sides for the equilateral triangle, right angle for the right triangle)
2.3 identify attributes of quadrilaterals (e.g., parallel sides for the parallelogram, right angles for the rectangle, equal sides and right angles for the square)
2.4 identify right angles in geometric figures or in appropriate objects and determine whether other angles are greater or less than a right angle
2.5 identify, describe and classify common three-dimensional geometric objects (cube, rectangular solid, sphere, prism, pyramid, cone, cylinder)
2.6 identify the common solid objects that are the component parts needed to make a more complex solid object

However, LASI AND LAUSD STAFF claim that it means:

2. Students compute the perimeter, area and volume of common geometric objects and use these to find measures of less common objects; they know how perimeter, area, and volume are affected under changes of scale.
2.1 routinely use formulas for finding the perimeter and area of basic two-dimensional figures and for the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, cones and circular cylinders
2.2 estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking them up into more basic geometric objects
2.3 compute the length of the perimeter, the surface areas of the faces, and the volume of a 3-d object built from rectangular solids. They understand that when the length of all dimensions are doubled or tripled, the unit measures are increased by the same factor
2.4 relate the changes in measurement under change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 12^2 square inches, 1 cubic inch = 2.6^3 cubic centimeters)
3. Students know the Pythagorean Theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures.
3.1 identify and construct basic elements of geometric figures, (e.g., altitudes, midpoints, diagonals, angle bisectors and perpendicular bisectors; and central angles, radii, diameters and chords of circles) using compass and straight-edge
3.2 understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections
3.3 know and understand the Pythagorean Theorem and use it to find the length of the missing side of a right triangle and lengths of other line segments, and, in some situations, empirically verify the Pythagorean Theorem by direct measurement
3.4 demonstrate an understanding of when two geometrical figures are congruent

> and what congruence means about the relationships between the sides and angles of the two figures
> **3.5** construct two-dimensional patterns for three-dimensional models such as cylinders, prisms and cones
> **3.6** identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and how two or more objects are related in space (e.g., skew lines, the possible ways three planes could intersect)

The LAUSD standard quoted above is a **seventh grade** standard. The first list is from the state's **third grade** standards, while the second is from the state's **seventh grade** standards. The LAUSD standard is not particularly better aligned with the state **seventh grade** standards than with the state **third grade** standards. LASI has demonstrated in the Informative that the LAUSD standards are not adequately specific.

LAUSD standards omit grade-by-grade benchmarks and contradict the California standards

The LAUSD/LASI Informative admits that, *LAUSD has written broad statements at only the selected benchmark grades of third, seventh, ninth, and twelfth.* This obvious difference leaves the students of Los Angeles without a road map for their progress - **most of the grades are omitted**. In addition, the California state mandated STAR exams will not allow calculator usage in the first six grades. This contradicts the LAUSD third grade benchmark which prematurely requires children to use calculators in the early grades, Kindergarten, first, second, and third grades.

The LAUSD/LASI Informative even contradicts itself.

On page 2, the LAUSD/LASI Informative indicates, in a chart, that the LAUSD math standards include topics which the California math standards do not. According to the chart, discrete mathematics is included in the LAUSD standards, but not in the California State standards. But the LAUSD/LAUSD Informative then lists topics in discrete mathematics from the California math standards on the very next page, contradicting its own assertion. The treatment of discrete mathematics in the California math standards is superior to the treatment in the LAUSD standards.

The only remaining standards included in the LAUSD document, but missing in the California math standards, correspond to those noted in A Comparison of the LAUSD Math Standards and the California Math Standards under the heading, "Topics Which Either do not Exist or Have Nothing to do with Mathematics."

CONCLUSION

LASI's attempt to present the LAUSD math standards in a positive light cannot mask the deficiencies. The district's own comparison actually confirms criticisms found in A Comparison of the LAUSD Math Standards and the California Math Standards.

Why would LASI go to such lengths to defend its obviously inferior math standards? Perhaps it is merely pride of authorship. Perhaps LASI's hollow claims are dictated by LAUSD'S need to meet or surpass the state standards in order to secure future grant funding. Perhaps the district is trying to justify itself knowing that there will eventually be a review by the state. In any case, however, the district would be well-advised simply to adopt the state standards and be done with it. Afterall, ... *you can't fool all of the people all of the time.* (Abraham Lincoln, 1864)

LAUSD's current resistance to the state standards, on the advice of LASI, is likely to cause delays in revising LAUSD curriculum to meet the new state standards. This will undoubtedly contribute to an even worse showing in 1999 on the STAR exams relative to other school districts in California than occurred this year. Other districts have already either aligned their math standards with the state standards or adopted the state standards outright.

LAUSD cannot succeed merely by claiming to have high expectations. In the strongest possible terms, the Los Angeles Unified School District must make it official and clear to all involved - students, parents, teachers, principals, administrators and the public - that district students are expected to meet the California Mathematics Standards. The continuing resistance to adopting the California mathematics standards serves only to save face for LASI and district staff, but it does so at the cost of a proper mathematics education for the children of Los Angeles.

LAUSD Curriculum Committee Hearing
David Tokofsky, Chair
May 14, 1998

Diana Dixon-Davis (parent activist): Thank you very much for letting all of us address you now. We're here to talk about the integrated math program and the traditional math program. I have three sons. One has already graduated from LAUSD and another at Lawrence Middle school and a third at Chatsworth High School. With me today are Rona Greenstadt, who is a parent and a teacher--she teaches at LAUSD and she's also a parent of a student at Chatsworth High School and she has been heading our Chatsworth assessment and evaluation committee, where we've looked into the issue of traditional math and integrated math at the high school.

And also Dr. David Klein is with me. He is from Cal State Northridge. He is a professor of mathematics there and he was one of the evaluators of the newly adopted standards and he's going to be talking about the standards and their relationship to the curriculum.

Why are we here? Well, for this whole past year we had a surprise. In September of 1997, all of a sudden we walked into the high school and the middles school and in the 8th grade they were only offering integrated math, no algebra I. In the high school similarly all the students who had not started a math program were put into integrated math. It was 100% adoption with no options for traditional or integrated math. And we asked, why such a sudden change? And we were told, well, that the math cadre, the cluster administrator, and the principal had decided that they were going to eliminate all traditional math and put in its placed integrated math. And I asked what is the data? Why are you doing this? What is the purpose? And they said, well, we think integrated math will do better for the kids, will bring more kids in.

We've been working with the LASI and other people on this issue and we feel this is the best way. Well, many of us have some concerns about this and we tried to work within the system for the past year. We are both LEARN and school based management. I sit on both school based councils and we thought all such major decisions would be brought to all the parents in the community and to all the teachers. It was not. It was done by a few people at the top. We don't question the motives of the reformers. We question the procedures. We feel the math cadre has a conflict of interest. The major consultant to the math cadre is one of the co-authors of the book that was adopted. The McDougall integrated math textbook. We feel that, plus the LA-SI providing 60% of the purchase price of the those textbooks also swayed, perhaps, people who could have taken a broader look at this issue.

We are asking that--right now we are finding that integrated math does not cover all the needs of the students. In fact even with the Golden State [exam] s, the student has to take two years of integrated math to get enough to cover algebra I, so there are some deficiencies in this program. When we bring this up, people are very upset.

What do we want out of this? Why are we here?

What we're asking for is that the school board send a clear message that says that you'll meet the needs of all students. You'll both have a traditional math sequence as well as the integrated math sequence. And why do e need both?

First, there are students with different learning styles. I have a student who is learning disabled, who has problems with languages. A language based math series will take away the one thing he's good at, which is mathematics and looking at things theoretically, rather than language based. So, for him, integrated math will be difficult. Secondly, we're not accommodating the fact that we have students moving in and out of our schools all the time. Only half of our students at Chatsworth actually matriculate from Lawrence Middle School and Nobel Middle School. So those 50% of students who come from other schools where algebra is offered, they have to either repeat, lose part of their credit for algebra I, or jump into a class where they've missed the geometry. It's a three year sequence. You're requested to take all three. And you can't just jump in the middle of it unless you take all three. You don't get all the material you need at the end. And another reason is when we do mathematics we're not just doing it for the fun of it. We're getting prepared for advanced math and science. Those students who want to take the AP calculus, physics, the advanced mathematics can and should get a more rigorous dense mathematical program that is offered in traditional math. Integrated math works for another type of student who is less mathematically inclined, who is less able to quickly accumulate the mathematical knowledge. Also, we want good studies done. We want to have--if an adoption is done, 100% adoption--before that is done, let's pilot the program. Let's do real statistical studies with valid criteria to see what works and what doesn't and for whom, and not just throw out the baby with the bath water. We had the experience with Whole Language a couple of years ago and we really don't want to repeat hat again.

Anyway, that's why we're here and thank you for listening to me. And we have two other people.

David Tokofsky (LAUSD board member and chair of the Curriculum Committee): I have David Klein on the list and then Rona Greenstadt. You have approximately five minutes. If you could keep it closer to three, and then if there are any questions you could respond back.

David Klein: I actually have two subjects to address.

David Tokofsky: O.K.

David Klein: One is very simple. I came to urge you to adopt the California math standards in place of the LAUSD math standards. The CA math standards are vastly better. It's no comparison. In a few days, there will be a comparison going up on a website for you to look at, which will be contributed to by several world leading mathematicians who are concerned about the issue, since LAUSD is such a large school district.

I don't know if you're aware of it, but the LAUSD math standards entirely omit trigonometry. Is it the intention of LAUSD that high school graduates should have zero exposure to trigonometry? There are numerous defects within the LAUSD standards. Calculators are encouraged in the 3d grade, which undermines mastery of arithmetic which is a foundation--particularly fractions--for algebra in the middle schools and later. Algebra is under emphasized. The standards are vague and repetitive. Frankly, it's an embarrassment. There are many mathematicians who feel the same way.

By contrast, more than 100 California mathematicians endorsed a letter I wrote--an open letter to the new chancellor of the CSU system--in support of the CA math standards. Those are excellent world-class standards which LAUSD would do very well to adopt. I have copies of an op-ed piece I wrote for the L.A. Times a week and a half ago on this subject, which I'd like to leave for the board.

David Tokofsky: O.K.

David Klein: The second topic I came to address is the issue of integrated math versus the more traditional way of breaking math up into algebra I, geometry, and algebra II. Diana Dixon-Davis asked me to look at some of the books which are used for integrated math, and I did so as a professional mathematician. And I can tell you I was horrified. The integrated approaches that I have seen in the books that were given to me are incoherent, disorganized, and they omit crucial topics. They present mathematical topics in a way which doesn't make any sense. Let me read to you a quote from a former chairman of the Sanford math department who continues as a professor. Ralph Cohen wrote this past August in a letter to the Academic Standards Commission:

"[Some have the view] that the Japanese and others have done better than the Americans (and particularly Californians) in achievement levels because they treat subjects in an 'integrated' fashion. This is nonsense. Just look at their textbooks! The difference is that they treat subject carefully and thoroughly, unlike any American integrated text I have seen (and I have seen far too many)."

David Tokofsky: Thank you very much. Rona Greenstadt, welcome.

[Rona Greenstadt spoke in support of choice between integrated math and "stand alone" math (meaning alg I, Geometry, alg II). She asked who makes the decision to use one or the other].

David Tokofsky: Mr. Collins, would you like to briefly respond or give any insight or direction? And we now have also back Ms Joan Evens from standards, and the LASI math and science folks, Carol.

Bob Collins: Sure, let me introduce it and then maybe Carol [Takemotto] can help. The integrated math and science programs came about as part of the national reform movement in those subject areas and sponsored by the National Science Foundation. One of the reasons that that program became so important was what we saw throughout this country as a failure of traditional mathematics for large numbers of students. In fact, we know in public education today--although alot of individuals will talk about how successful traditional mathematics is--we know 40, 50 percent of students fail algebra in the United States on a regular basis, and not just in LAUSD.

David Tokofsky: That's cause it's in Arabic.

Bob Collins: (laughs) Probably. And there probably are alot of reasons for that. But the reform movement in mathematics addresses two critical issues:

One, a change in pedagogy along with a change in curriculum

And two, it addresses the issue of all students being successful in higher levels of math.

We know that that in many cases is an idea that alot of people reject: that only certain people can be successful in higher levels of math. But the reality is, in America today, if we're not able to have youngsters successful in algebra and geometry, and algebra II a b in alot of school districts in this country, they're not even going to graduate, and so there's a need to do that.

David Tokofsky: Bob, I think the people who spoke left, so we can be real brief.

Diana Dixon-Davis: We're here!

Bob Collins: Carol, do you want to add on?

David Tokofsky: I think the key questions to answer are, who makes the decision at cluster and school site level? Is there supposed to be a choice--the options of both full paths at both schools? What is, from your point of view, the message and policy of the board with respect to those things? Also, the issue on the standards was, is there a consistency or conflict between the CA standards and the district standards? And specifically, I guess, that would even raise my question. Yesterday, Gov. Wilson's budget comments were specifically designed to take away funding from school districts that do not follow the CA state math standards. So that becomes now not just a curricular issue, but a financial issue. Then there was an issue of the books and an allegation that there is a financial conflict of interest.

[Carl Takemotto from LASI addressed these and other questions and denied any conflict of interest]

Joan Evans (director of standards, curriculum, and assessment): As Mr. Collins has already identified, the purpose of standards based education was to clearly improve student achievement. With that in mind, the first thing this district did following the adoption of their standards that met all of the ten criteria established by the National Education Goals Panel as certified by the Council for Basic Education, was we aligned--when the state adopted their standards--we aligned each of the district standards with the state standards. It only made sense to us that that alignment would be a critical step in the process because following the state adoption, we understand, of the state standards, the curriculum frameworks are going to be revised to reflect those state standards, as well as the adoption in the state's assessment test to measure those standards. And that's why as the norm referenced test, the Stanford-9 is being adjusted to reflect then those state standards. Obviously it would behoove us, and that's why we did in fact insure that our curriculum that would be based on the state's curriculum framework would address those state standards.

We also then took the next step which is, beyond the norm-referenced test we have in this district a performance based assessment, and we have made sure that that performance based assessment will also, in part, address those state standards. Additionally, our standards meet the national standards, because, for us, our students need to do more than just pass a test.

David Tokofsky: So, in a sense what you're saying is the accusation of being repetitive and vague standards--what you're saying is that's a perception, but what we have, in fact, came out in early point in our standards development in the district, and as things change at state in terms of the test, and now they're more clear and precise standards that we will have to revisit our standards and make them more clear and precise?

Joan Evans: No. Let me clarify that.

David Tokofsky: And then if you could transition to the fact that the Governor has tagged lots of money...

Joan Evans: That's why.

David Tokofsky: ...to state math standards and....

Joan Evans: That's why. We're totally connected to that. OK. For economic reasons, but for educational reasons as well. This district never intended to do what the state had always told us that they were going to do, which was to identify specific grade level standards for students and performance standards.

This district according to Goals 2000 Education America, was required to identify benchmark standards. One at the elementary level, one at the middle grade level, one at the senior high which would in fact clearly provide the benchmarks for what this district's direction would be in terms of standards.

We always knew because we worked in close collaboration with the State Department of Education that they were going to establish the performance standards at each grade level. Our standards, because we served on an ongoing basis on each of the state conferences, hearings, and discussions of their standards and the implications in terms of implementation of them, we know where they differentiate and we know, in fact, that our standards, like I have said, have met already the national criteria. That is not our concern.

Our concern is with the implementation because we know that our standards push our kids to the level of application. So, in fact, they will assure that in the event that we ever are involved in a national test, our students will meet those national standards and/or currently as they are involved in advanced placement.

David Tokofsky: I think we took a position at the federal level against the national test in math, didn't we, as a district?

Joan Evans: Given the current administration and, of course, from a historic point of view, if that changes, as you can well appreciate Mr. Tokofsky, that may be another point of consideration. But our students currently do take a national test which is the advanced placement test and that's why it was most important for us to insure that our students meet national standards.

David Tokofsky: If Dr. Klein would like to respond for about two minutes, otherwise we'll take the responses by letter and consider it as Board members.

David Klein: You won't be able to find a mathematician or scientist alive who would agree with that.

David Tokofsky: With what?

David Klein: With the equivalence of the LAUSD standards with the CA standards. They are day and night. They are vastly different. It's not a matter of perception whether or not there is trigonometry there, for example. That's not just my opinion; you can read it. It's going to go up on the web, many, thousands of people are going to see it in the next few days. It's clear and it's obvious that the LAUSD standards are very very weak.

You know, just speaking of test scores, I come from the CSU system. Statewide this year, 54% of our entering freshmen had to take remedial math, mostly at the 7th grade level. In the L.A. area, it's much higher. Cal State Northridge it's 67% of the entering freshmen--from this school district--have to take very remedial math. I'm talking about adding fractions and so on. The percentages have been increasing each year since 1989 and that correlates very well with the implementation of this reform math stuff.

Bob Collins: Dr. Klein, can I ask you a question? On the CSU math test itself, what is the highest level of mathematics tested on that test?

David Klein: It's, uh, elementary algebra, a little bit of geometry uh, ...

Bob Collins: My review of it when we took a look at it is there's some trigonometry, some calculus on the test.

David Klein: Calculus? I don't think so.

Bob Collins: I'm just concerned because the CSU standards, or not the CSU standards, [rather] the entrance standards are the minimum standards and I really want to take a look at the test itself.

David Klein: I think you should. It's called the ELM, Entry Level Math Exam.

Bob Collins: Correct. uh really test material that our students aren't required to learn as a condition of entering CSU, but that's another issue we need to look at with CSU.

David Klein: Well, they should be required to learn. We're talking about adding fractions, doing very elementary things...

David Tokofsky: I think this debate could be solved by having a sample of the test given to board members, and [if] somebody in the margins wants to take a look at the level, we'll take a look at it. But I appreciate your presentation. Anything else you'd like to submit, come back in another meeting. We will continue to look at it. We have a concern, obviously, both on a financial level, and a curricular level on what you've mentioned.

David Klein: Just a final note. I'm really tired of failing two thirds of my calculus students, and I wish they'd learn some algebra and arithmetic when they come out of LAUSD.

David Tokofsky: OK, OK.

John Liechty (director of Middle Schools): I just feel that's a cold shot. If you're failing 2/3 of your kids, we ought to be looking at a whole instructional program. I didn't know we went into business to fail kids. And this pointing the blame kind of thing, really--I gotta tell you, as the father of four children in this system that went through college--I take issue with. Because what I'm seeing right now is this pointing. This should be about our children and it should be about us coming together collaboratively, and not about the adult agendas.

Obviously the work load that we have in the public sector--going on to the university--is very critical, but the dignity and worth of our children should never be determined by whether they can enter or pass a math test at the college level that somebody else created for them.

What we've got in this industry, folks--I need to say to say it because I'm the father of four--is we've got a system that's not inclusive; it's exclusive. And we're trying to build from K-12 a different culture that we can include more youngsters in. And I thought we need to work together in doing that, and not this, "I'm failing two thirds of my kids."

I'd feel very unsuccessful as a teacher, and did, if I had to fail 2/3 of my youngsters because we weren't talking together about building programs that we could believe.

[Applause from staff in the room]

David Tokofsky: OK, thank you very much, John.

Valerie Fields (LAUSD Board member): But I do want to have for my own comfort level, the transiency problem and whether or not the board should have a policy that we must offer a choice at every school, so that if a kid moves from Lawrence to Paul Revere, they can have the same offerings.

[John Liechty agreed and said this would be a major focus this year]

David Tokofsky: Thank you very much, John. And again, we do have both an instructional mission which John passionately presented, and we also have the financial concern which is to say that much, including one of the challenge grants that will be put before us--or one of the grants--is directly related to whether or not we are in accord with the state math standards, and if we are not in accord, we will learn the hard way. Just as the audit on Assembly Bill 3482 on phonics and Whole Language will check whether or not we purchased the correct materials as instructed by law, we will also be held accountable if the state math standards are such, and we are ignoring them--that we would lose any resources, past, present, and future, for being out of compliance with the state math standards. So, having concluded with that, and the hour is moving on...

[new topic]

San Diego City Schools
Mathematics
Content and Performance Standards

Board Review Copy, January 1998

Rigorous academic standards are essential for educational improvement. Coupled with appropriate instructional materials, instruction, and assessment, standards have the power to transform schooling and learning. The task of the Mathematics Grade-level Standards Committee was to draft rigorous, world-class mathematics content and performance standards for all San Diego City Schools students, grades K–12. Please keep the following in mind:
• The standards are an attempt to define excellence, not minimum competencies.
• The standards balance facts, skills, procedural knowledge, conceptual understanding, problem solving, application, reasoning, and mathematical communication.
• The standards define a core of essential knowledge and skills, but do not limit or dictate instructional strategies for ensuring students achieve them. The *how-to* is left to the professionals in the classrooms.
• The standards describe the content for each grade level or course. It is important to note that at this point, there has been no attempt to indicate which standards or content topics are most important for each grade level.
• The standards are designed to ensure that as students progress through the grades, they are effectively prepared for and successful in an increasingly rigorous mathematics curriculum.

In creating the standards, the committee carefully analyzed and drew from the best components of a variety of successful local, state, national, and international standards models. Model standards reviewed include the National Council of Teachers of Mathematics (NCTM) Standards, the California Academic Standards Commission Mathematics Content Standards, the Challenge Standards for Student Success, the Mathematically Correct Standards, the Education Round Table Standards for High School Graduates, New Standards, and the San Diego City Schools Consultation Draft standards. In addition, the committee reviewed state standards from Colorado, Maryland, Virginia, and Washington and district standards from Boston, Massachusetts; Charlotte-Mecklenburg, North Carolina; Chicago, Illinois; Jefferson County, Colorado; Los Angeles, California; Minneapolis, Minnesota; Milwaukee, Wisconsin; Philadelphia and Pittsburgh, Pennsylvania; and Rochester, New York. Local school standards from San Diego and international models including Japan also were carefully reviewed.

Adoption of increasingly rigorous math standards has important implications and inherent challenges:
• Current students, at the time the new standards are implemented, will not have had the benefit of beginning and progressing through a math program based upon these rigorous standards. As a result, it is anticipated that some will not be adequately prepared to meet the standards in the very first years of implementation and will need additional time and support to be successful.
• Instructional materials and evaluation tools must be changed or adopted to reflect and align with the new standards. Standards can only chart the course. Appropriate instructional materials supporting the achievement of the standards must be readily available and teacher- and student-friendly, if all students are to achieve these rigorous standards.
• The adoption of increasingly rigorous mathematics standards will result in need for intensified training in mathematics content for both new and experienced elementary teachers.
• Information and training for parents and community members will be essential.

Organization and Abbreviations
Content and performance standards for kindergarten through grade 7 have been organized into five strands: 1) number sense and operations; 2) function and algebra; 3) measurement and geometry; 4) data analysis, statistics, and probability; and 5) problem solving, mathematical reasoning, and communication. Above grade 7, standards are organized into four courses: algebra, geometry, intermediate algebra, and advanced algebra and trigonometry. The following abbreviations have been used in listing and enumerating the standards:
NO: Number Sense and Operations
FA: Functions and Algebra

MG: Measurement and Geometry
DA: Data Analysis, Statistics, and Probability
PS: Problem Solving, Mathematical Reasoning, and Communication
A: Algebra
G: Geometry
IA: Intermediate Algebra
AT: Advanced Algebra and Trigonometry

A list of key vocabulary is provided for each grade level, K–7. It is important for teachers and students to use appropriate mathematics terms. (Note: The words are not intended as a "spelling list" for students.)

Kindergarten

Number Sense and Operations

Content Standard K–NO1
The student understands and relates a sense of numbers and quantity in useful ways.
Performance Standards
The student will:
K–NO1.1 Count in various ways including counting objects up to 12, counting by ones up to 31 and backwards from 10, skip-counting by fives and tens to 50 and by twos up to 10 (2 to 10 and 1 to 9).
K–NO1.2 Identify written numbers from 0 to 31.
K–NO1.3 Select the correct numeral to indicate a quantity from 0 to 9, tracing over and writing the numeral.
K–NO1.4 Select a reasonable order of magnitude from two given quantities for a familiar situation (e.g., 5 or 50 jellybeans in a jar).
K–NO1.5 Identify ordinal positions from first to fifth, using concrete objects.
K–NO1.6 Compare two sets of 10 or fewer concrete items and identify one as containing more, less, or the same as the other set.
K–NO1.7 Divide a set of two, four, six, or eight concrete objects into equal halves.
K–NO1.8 Identify a penny, nickel, dime, quarter, and one-dollar bill.
K–NO1.9 Identify the cent sign and write amounts to nine cents using the cent sign.

Content Standard K–NO2
The student understands and describes simple addition and subtraction situations.
Performance Standards
The student will:
K–NO2.1 Identify one more and one less for numbers from one to nine.
K–NO2.2 Add and subtract whole numbers using up to 10 concrete items.

Functions and Algebra

Content Standard K–FA1
The student asks and answers questions involving quantities given a picture or situation.
Performance Standards
The student will:
K–FA1.1 When prompted by a picture or situation, identify whether things would be added or subtracted.
K–FA1.2 Identify and interpret + and – symbols.

Content Standard K–FA2
The student identifies, describes, and extends simple patterns.
Performance Standards
The student will:
K–FA2.1 Sort objects by attribute (e.g., color, shape, size) and identify the attribute.
K–FA2.2 Find the element of a set that does not belong and explain why it does not belong.
K–FA2.3 Identify and describe patterns of symbols, shapes, and objects (e.g., patterns on a calendar; counting by fives and tens; circle-circle-square, circle-circle-square, ….)
K–FA2.4 Extend and create simple patterns of symbols, shapes, and objects.

Measurement and Geometry

Content Standard K–MG1
The student selects appropriate tools and language to compare quantities and measure length, weight, capacity, time, and temperature.
Performance Standards
The student will:
K–MG1.1 Identify the instruments used to measure length (ruler), weight (scale), time (clock, calendar), and temperature (thermometer).
K–MG1.2 Make direct comparisons or use nonstandard units to measure length/height (shorter, longer, taller), weight (lighter, heavier), temperature (colder, hotter).
K–MG1.3 Compare the volumes of two given like-shaped containers by using concrete materials (e.g., jellybeans, sand, water, rice).

Content Standard K–MG2
The student demonstrates an understanding of time, using major units on clocks and calendars.
Performance Standards
The student will:
K–MG2.1 Tell time to the nearest hour, using analog and digital clocks, and demonstrate under-standing of morning, afternoon, and night.
K–MG2.2 Describe and tell time using calendars (e.g., days of the week, months of the year, seasons).

Content Standard K–MG3
The student identifies common geometric objects in the environment and describes their features.
Performance Standard
The student will:
K–MG3.1 Identify, describe, and draw or construct two-dimensional geometric objects (circle, triangle, square, rectangle) in the environment (e.g., clock faces, doorways) and describe their positions (e.g., next to, top, bottom).

Data Analysis, Statistics, and Probability

Content Standard K–DA1
The student collects and records information about the environment.
Performance Standards
The student will:
K–DA1.1 Notice and talk about quantities in the environment (e.g., How many students are wearing red? Brown? Blue?).
K–DA1.2 Collect information and record the results, using objects, pictures, or picture graphs.

Problem Solving, Mathematical Reasoning and Communication

Content Standard K–PS1
The student counts, adds, and subtracts as appropriate in problem solving.
Performance Standards
The student will:
K–PS1.1 Identify situations in which the answers to questions such as How many? How many more? How many altogether? can be obtained by counting, addition, or subtraction; identify the units needed for the answer (e.g., a number of apples); model the problem with concrete objects; and state the solution.

K–PS1.2 Restate in his or her own words problems involving counting, addition, or subtraction of small quantities.
K–PS1.3 Make up problems that can be solved by counting, addition, and subtraction.

Content Standard K–PS2
The student interprets and uses basic logical statements in informal language.
Performance Standard
The student will:
K–PS2.1 Identify true and false statements involving the use of *either, or, both*, and *and*, and the quantitative terms *all, some,* and *none.*

Content Standard K–PS3
The student communicates knowledge of basic skills, conceptual understanding, and problem solving, and demonstrates understanding of the mathematical communications of others.
Performance Standards
The student will:
K–PS3.1 Use appropriate mathematical vocabulary (see kindergarten Key Vocabulary).
K–PS3.2 Show ideas with a variety of concrete materials (connecting cubes, pattern blocks, buttons, beads, color tiles, etc.) and by pasting paper representations of materials.
K–PS3.3 Explain strategies used in solving problems and share ideas orally when probed by the teacher or when dictating to an adult for recording.
K–PS3.4 Understand and follow oral directions for appropriate mathematics activities.

Key Vocabulary

Number Sense and Operations

–, +, addition, answer, cent, dime, dollar, equal, estimation, fewer, halves, less, more, nickel, numeral, ordinal, penny, quarter, same, set, subtraction

Functions and Algebra

–, +, added, attribute, calendar, circle, color, element, objects, patterns, set, shape, size, square, subtracted, symbols

Measurement and Geometry

>, <, =, afternoon, analog, bottom, calendar, circle, clock, clock face, colder, compare, day, difference, digital, doorway, geometry, heavier, height, hotter, hour, larger, length, lighter, month, morning, next to, night, place value (ones, tens), rectangle, ruler, scale, seasons, shorter, smaller, square, sum, taller, temperature, thermometer, time, top, triangle, volume, week, weight, year

Data Analysis, Statistics, and Probability

environment, graph, quantity

Problem Solving, Mathematical Reasoning, and Communication

addition, all, and, both, counting, either, false, none, or, solution, some, subtraction, true

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Grade 1

Number Sense and Operations

Content Standard 1–NO1
The student identifies and represents the number and order of objects.
Performance Standards
The student will:
1–NO1.1 Count forward by twos, fives, and tens to 100 and count backwards from 100 by ones.
1–NO1.2 Read and write numerals from 0 through 100.

1–NO1.3 Read and write numbers from 0 to 20 as words.
1–NO1.4 Identify the ordinal positions *first* through *twenty-first* using concrete objects and pictures.
1–NO1.5 Locate, sequence, and represent whole numbers on a number line.
1–NO1.6 Compare two sets of up to 12 objects and tell which has more and how many more.
1–NO1.7 Use the symbols >, <, and = to compare two sets or pictures of sets of up to 12 objects and two numbers from 0 to 100.
1–NO1.8 Identify the number of pennies equivalent to a nickel, a dime, and a quarter and compute sums of money up to $1.00.
1–NO1.9 Show different combinations of coins that equal the same amount of money up to $1.00 and use the dollar sign.

Content Standard 1–NO2
The student demonstrates an understanding of base-ten place value for two-digit numbers.
Performance Standards
The student will:
1–NO2.1 Identify the place value of each digit in numbers up to 100.
1–NO2.2 Use words, models (e.g., groups of objects), and expanded form (e.g., 24 means 20 + 4 or two bundles of 10 rods and 4 additional rods) to represent two-digit numbers.
1–NO2.3 Mentally identify *one more than, one less than, ten more, ten less,* with numbers and sums up to 100.
1–NO2.4 Select a reasonable order of magnitude from three given quantities for a familiar situation (e.g., 5, 50, and 500 jellybeans in a jar).

Content Standard 1–NO3
The student demonstrates the meaning of addition *(putting together, increasing)* and subtraction *(taking away, comparing, adding on)* and uses those operations to solve problems.
Performance Standards
The student will:
1–NO3.1 Recall basic addition facts (sums 0–18) and the corresponding subtraction facts.
1–NO3.2 Add three single-digit numbers.
1–NO3.3 Add and subtract two-digit numbers both horizontally and vertically, without regrouping.
1–NO3.4 Solve story and picture problems involving one-step solutions, using basic addition and subtraction facts.

Content Standard 1–NO4
The student demonstrates a beginning understanding of the concept of division.
Performance Standards
The student will:
1–NO4.1 Identify *one-half, one-third,* and *one-fourth,* using concrete materials or pictures.
1–NO4.2 Divide groups of objects into equal sets.

Functions and Algebra

Content Standard 1–FA1
The student creates and solves problems using words, symbols, drawings, and objects.
Performance Standards
The student will:
1–FA1.1 Solve simple addition (sums 0–18) and corresponding subtraction equations with a blank, box, or other symbol in any position, such as 2 + 5 =___ , 7 – ___ = 5, ___ – 2 = 5.
1–FA1.2 Write and solve number sentences that express relationships involving addition and subtraction, including the correct interpretation and use of the +, –, and = symbols.

Content Standard 1–FA2
The student sorts objects and completes and describes patterns involving numbers, shape, size, rhythm, and color.
Performance Standards
The student will:
1–FA2.1 Sort concrete objects according to two attributes (such as color and shape).
1–FA2.2 Recognize, describe, and extend a wide variety of patterns, including size, color, shape, and quantity, and including increasing, decreasing, and repeating patterns, using concrete materials and patterns.
1–FA2.3 Identify the common property of the elements of a set, select matching additions to the set, and identify the item that does not belong in a set.

Measurement and Geometry

Content Standard 1–MG1
The student compares objects according to length, weight, and capacity using direct comparison, non-standard and standard units.
Performance Standards
The student will:
1–MG1.1 Estimate and then count the number of cubes in a rectangular box.
1–MG1.2 Compare weights of objects using a balance scale.
1–MG1.3 Measure and draw line segments in inches and centimeters.
1–MG1.4 Estimate and measure length in inches and weight in pounds.
1–MG1.5 Measure perimeter to the nearest inch and centimeter.
1–MG1.6 Order similar objects (e.g., smallest to largest, longest to shortest, lightest to heaviest).
1–MG1.7 Estimate and measure volume in cups and identify a cup, pint, quart, and gallon.

Content Standard 1–MG2
The student demonstrates an understanding of time and temperature and tells time to the half-hour.
Performance Standards
The student will:
1–MG2.1 Name the days of the week and the months of the year, both in order and out of sequence.
1–MG2.2 Tell time to the half-hour, using analog and digital clocks.
1–MG2.3 Orient events in time using *today, yesterday,* and *tomorrow*; *morning* and *afternoon*; *this morning* and *yesterday morning,* etc.
1–MG2.4 Associate temperature in degrees Fahrenheit with weather (e.g., 85° is really hot, 50° is colder).

Content Standard 1–MG3
The student identifies common geometric objects in the environment and describes their features.
Performance Standards
The student will:
1–MG3.1 Demonstrate orientation and relative position, such as *closed/open, on/under/over, in front of/in back of, between, in the middle of, next to, beside, inside/outside, around, far from/near to, above/below, to the right of/to the left of,* and *here/there.*
1–MG3.2 Identify and describe three-dimensional objects (cube, sphere, cone, cylinder), including identifying the faces as two-dimensional shapes (triangles, rectangles, squares, or circles).
1–MG3.3 Draw and describe triangles, squares, rectangles, and circles according to number of sides and corners.
1–MG3.4 Identify geometric shapes that have symmetry.
1–MG3.5 Identify congruent shapes and designs.

Data Analysis, Statistics, and Probability

Content Standard 1–DA1
The student collects, reads, interprets, and compares data from simple graphs.
Performance Standards
The student will:
1–DA1.1 Gather, record, read, interpret, and compare data in bar graphs, tally charts, and picture graphs.
1–DA1.2 Compare data using the concepts of *largest, smallest, most often,* and *least often.*

Problem Solving, Mathematical Reasoning, and Communication

Content Standard 1–PS1
The student shows critical evaluation of available information in problem situations involving counting, addition, and subtraction.
Performance Standards
The student will:
1–PS1.1 Identify quantities and units given as facts in a problem situation, determine which facts are relevant or irrelevant to the solution, and explain why some facts are irrelevant by showing that they are not required for the solution.
1–PS1.2 Identify whether or not a problem situation contains sufficient information to arrive at an answer and be able to explain what is lacking when there is insufficient information.

Content Standard 1–PS2
The student uses problem-solving strategies involving addition, subtraction, and basic measurements.
Performance Standards
The student will:
1–PS2.1 Formulate and carry out a plan to solve a problem by acting out the problem, using manipulatives or models, or drawing a picture.
1–PS2.2 Create a problem and state it orally when given a picture, a model, or a real-life situation and identify problem solutions as correct or incorrect.

Content Standard 1–PS3
The student communicates knowledge of basic skills, conceptual understanding, and problem solving and demonstrates understanding of mathematical communications of others.
Performance Standards
The student will:
1–PS3.1 Use appropriate mathematical vocabulary (see grade 1 Key Vocabulary).
1–PS3.2 Show ideas by drawing, using words and numbers, and by building with a variety of concrete materials, such as connecting cubes, pattern blocks, buttons, beads, and color tiles, and by pasting paper representations of materials.
1–PS3.3 Explain strategies used in solving problems and share ideas orally when probed by the teacher, and in writing and drawing.
1–PS3.4 Follow oral directions for appropriate mathematics activities.

Key Vocabulary

Number Sense and Operations

>, <, =, answer, backward, bundle, comparing, difference, digit, dime, dollar sign, equal, equivalent, expanded form, forward, fourth, half, horizontally, less than, locating, more than, nickel, ordinal, pennies, place value (ones, tens, hundreds), quarter, rod, sequencing, set, sum, symbol, third, vertically

Functions and Algebra

+, –, =, addition, attribute, color, decreasing pattern, element, equation, function, increasing pattern, number sentence, pattern, property, quantity, repeating pattern, set, shape, size, subtraction

Measurement and Geometry

above, afternoon, analog, around, balance, behind, below, beside, between, centimeter, circle, closed, comparing, cone, congruent, corner, cube, cup, customary, cylinder, day, design, digital, estimating, face, Fahrenheit, far from, gallon, half-hour, heaviest, here, inch, in back, in front, in the middle of, inside, largest, lightest, line segment, longest, metric unit, month, morning, near, next to, on, open, outside, over, perimeter, pint, position, pound, quart, rectangular, rectangle, scale, shape, shortest, side, smallest, sphere, square, symmetry, there, three-dimensional, to the left of, to the right of, tomorrow, triangle, two-dimensional, under, volume, week, weight, year, yesterday

Data Analysis, Statistics, and Probability

bar graph, data, gathering, largest, least often, most often, picture graph, smallest, tally chart

Problem Solving, Mathematical Reasoning, and Communication

insufficient, irrelevant, quantity, relevant, solution, sufficient, unit

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Grade 2

Number Sense and Operations

Content Standard 2–NO1
The student extends an understanding of number and place value to exact and approximate whole numbers to 1,000.

Performance Standards
The student will:
2–NO1.1 Count forward and backwards by ones and count forward by twos, fives, and tens, starting at an arbitrary number.
2–NO1.2 Count by threes and fours to 48 and hundreds and fifties to 1,000.
2–NO1.3 Read and write numbers from 0 to 1,000.
2–NO1.4 Read and write numbers from 0 to 100 as words.
2–NO1.5 Write two- and three-digit numbers in expanded form (e.g., 567 = 500 + 60 + 7).
2–NO1.6 Use symbols and words (>, <, or =; *greater than*, *less than*, or *equal to*) to compare two whole numbers between 0 and 1,000.
2–NO1.7 Round numbers from 0 to 1,000 to the nearest 10 or 100.
2–NO1.8 Identify odd and even numbers from 0 to 100.
2–NO1.9 Identify the place value for each digit up to the hundreds.
2–NO1.10 Identify *one more*, *one less*, *ten more*, *ten less*, *one hundred more*, and *one hundred less* than a given number (solution in the range 0 to 1,000).

Content Standard 2–NO2
The student computes with whole numbers using addition, subtraction, multiplication, and division and uses those operations to solve problems.
Performance Standards
The student will:
2–NO2.1 Recall basic addition facts (sums 0–20) and the corresponding subtraction facts.
2–NO2.2 Add and subtract numbers on paper (sums to 1,000) with and without regrouping.
2–NO2.3 Add three two-digit numbers on paper with and without regrouping.
2–NO2.4 Use paper and pencil, mental computation, or concrete materials to estimate sums to 1,000 and the corresponding differences.
2–NO2.5 Solve word problems involving sums to 1,000 and corresponding differences.
2–NO2.6 Model products and quotients (e.g., with manipulatives) using repeated addition, arrays, counting by multiples, finding area, repeated subtraction, equal sharing, and forming equal groups.
2–NO2.7 Identify the part of a set and/or region that represents *one-half*, *one-third*, *one-fourth*, *one-eighth*, and *one-tenth* and write the corresponding fraction.
2–NO2.8 Recognize the multiplication sign, know what the terms *factor* and *product* mean in multiplication, and demonstrate that multiplication represents repeated addition.
2–NO2.9 Multiply single-digit numbers by 0, 1, 2, and 10.

Content Standard 2–NO3
The student counts and compares amounts of money.
Performance Standards
The student will:
2–NO3.1 Use a collection of coins and one-dollar bills to count, compare, and make change.
2–NO3.2 Read and write amounts of money using dollar and cent signs and the decimal point.
2–NO3.3 Show different combinations of coins and bills that equal the same amount of money (e.g., 15 quarters = $3.75).

Functions and Algebra

Content Standard 2–FA1
The student creates and solves problems involving addition and subtraction by modeling, representing, and interpreting number relationships.
Performance Standards
The student will:
2–FA1.1 Identify and use the inverse relationship between addition and subtraction to solve problems such as 4 + ___ = 7 and ___ + 3 = 7 and 7 – ___ = 3.
2–FA1.2 Use and describe the commutative (order) and associative (grouping) property of addition.
2–FA1.3 Use simple properties of addition and subtraction to devise algorithms or check results.
2–FA1.4 Identify and use the multiplication properties of zero and one.
2–FA1.5 Identify problem situations that match or do not match a given number sentence.

Content Standard 2–FA2
The student demonstrates an understanding of patterns and how they grow and describes them in general ways.
Performance Standards
The student will:
2–FA2.1 Recognize, describe, extend, and explain how to get the next term in patterns that grow in a linear way (e.g., 4, 8, 12, …; the number of ears on 1, 2, 3, 4, … horses).

2–FA2.2 Create and solve problems involving simple number patterns.
2–FA2.3 Identify and correct errors in patterns.

Content Standard 2–FA3
The student demonstrates an understanding of how two or more quantities are related to one another and how change in one causes change in another.
Performance Standards
The student will:
2–FA3.1 Demonstrate that a number is related to other numbers by being their sum, difference, or product (e.g., find all pairs of whole numbers whose sum is 14).
2–FA3.2 Explain how a change in input affects the output in a simple relationship (e.g., How far you can go depends on how much gasoline is in your car).

Measurement and Geometry

Content Standard 2–MG1
The student estimates, measures, and demonstrates understanding of the concepts of *length, area, volume,* and *capacity.*
Performance Standards
The student will:
2–MG1.1 Select appropriate formal or informal units to measure length, area, and volume and predict whether the measure will be greater or smaller when a different unit is used.
2–MG1.2 Determine the perimeter of a region by measuring or by adding given measures.
2–MG1.3 Estimate and determine the area and volume of figures by covering them with squares or counting the cubes.
2–MG1.4 Estimate and measure volumes in cups, pints, quarts, gallons, and liters and compare these volumes using the concepts of *more, less,* and *equivalent.*
2–MG1.5 Estimate and measure weight in pounds and kilograms.

Content Standard 2–MG2
The student demonstrates an understanding of the concept of time and temperature and the relationship among different units of time and tells time to the nearest quarter hour or five minutes.
Performance Standards
The student will:
2–MG2.1 Measure and tell time to the nearest quarter-hour or five minutes, using analog and digital clocks; compare time related to events (e.g., before/after, shorter/longer); and write time to the quarter-hour.
2–MG2.2 Solve simple problems of elapsed time to the hour.
2–MG2.3 Use *a.m./p.m.* and *noon/midnight.*
2–MG2.4 Order events by time sequence, identify equivalent periods of time (weeks in a year, days in a month, minutes in an hour), determine past and future days of the week, and identify specific dates on a calendar.
2–MG2.5 Write the date using words and numbers or only numbers.
2–MG2.6 Measure and record temperature in degrees Fahrenheit and Celsius to the nearest two degrees, using the degree sign.

Content Standard 2–MG3
The student identifies and describes the elements that compose common figures in the plane and common objects in space.
Performance Standards
The student will:
2–MG3.1 Compare and contrast plane and solid geometric shapes (circle/sphere, square/cube, triangle/pyramid, and rectangle/rectangular solid).
2–MG3.2 Identify, describe, and classify solid geometric shapes according to the number and shape of faces, edges, bases, and vertices.
2–MG3.3 Put shapes together to form larger or other shapes (e.g., two congruent right triangles form a rectangle) and find basic shapes within more complex figures (e.g., a hexagon is made up of six triangles).
2–MG3.4 Recognize and create symmetrical shapes and designs and identify and draw lines of reflection.
2–MG3.5 Identify lines as *horizontal, vertical, perpendicular,* and *parallel.*
2–MG3.6 Draw or construct congruent shapes and designs.

Data Analysis, Statistics, and Probability

Content Standard 2–DA1

The student collects, records, organizes, and displays data, noting patterns and making simple predictions.
Performance Standards
The student will:
2–DA1.1 Collect, record, and organize data to answer a question and display data from simple experiments using dot plots and bar graphs (including those with categorical or simple whole-number scales).
2–DA1.2 Make simple interpretations from data given in tables, picture graphs, or bar graphs.
2–DA1.3 List the possible outcomes of a simple event (e.g., tossing a coin, pulling colored marbles out of a bag) and predict whether outcomes are certain, likely, unlikely, or impossible.

Problem Solving, Mathematical Reasoning and Communication

Content Standard 2–PS1
The student uses appropriate problem-solving strategies.
Performance Standards
The student will:
2–PS1.1 Use addition to verify subtraction and vice versa.
2–PS1.2 Estimate to identify solutions that are reasonable or unreasonable in magnitude.
2–PS1.3 Solve addition and subtraction problems using data from simple charts, picture graphs, and number sentences (e.g., Find the answer for $4 + x =$ ___ when $x = 2$).
2–PS1.4 Recognize phrases in problem situations that suggest whether addition, subtraction, multiplication, or division is required and explain how the operations would be used to find the solutions.

Content Standard 2–PS2
The student identifies unions and intersections of sets.
Performance Standards
The student will:
2–PS2.1 Identify elements that belong in two sets (e.g., blue objects and triangles) and identify all elements that belong to the combination of two sets (e.g., the group of objects that is either red or square).
2–PS2.2 Use Venn diagrams to represent unions and intersections.

Content Standard 2–PS3
The student makes extensions of a problem and connections from one problem to another.
Performance Standards
The student will:
2–PS3.1 Identify relevant and irrelevant information in the statements of problem situations.
2–PS3.2 When directed to do so, show the solution with another material or in another way.
2–PS3.3 Compare the problem to other problems already solved and tell how it is related.

Content Standard 2–PS4
The student communicates knowledge of basic skills, conceptual understanding, and problem solving and demonstrates understanding of mathematical communications of others.
Performance Standards
The student will:
2–PS4.1 Use appropriate mathematical terms, vocabulary, and language, based on prior conceptual work (see grade 2 Key Vocabulary).
2–PS4.2 Show ideas in a variety of ways, including drawings, words, numbers, symbols, simple graphs and tables, and building with a variety of concrete materials.
2–PS4.3 Explain strategies used in solving problems and share ideas orally when probed by the teacher and in writing and drawing.
2–PS4.4 Present ideas appropriately for a particular audience or purpose.
2–PS4.5 Understand oral and simple written directions for appropriate mathematics activities.

Key Vocabulary

Number Sense and Operations

1,000, >, <, =, addition, area, array, cent sign, combination, computation, decimal, difference, division, dollar, equal to, even, expanded form, factor, hundred, less than, greater than, multiplication, odd, one-eighth, one-fourth, one-half, one-tenth, one-third, point, product, quotient, range, region, regrouping, repeated addition, rounding, subtraction, sum, symbol, ten, word

Functions and Algebra

algorithms, associative property, commutative property, difference, input, inverse relationship, linear, multiplication properties, number pattern, one, output, product, properties of addition, properties of subtraction, simple relationship, sum, zero

Measurement and Geometry

after, a.m., analog, area, bases, before, Celsius, circle, congruent, corner, cube, cup, date, day, degree sign, design, digital, edges, elapsed time, equivalent, faces, Fahrenheit, future, gallon, geometric, geometry, greater, horizontal, hour, kilogram, length, less, line of reflection, liter, longer, measurement, midnight, minute, month, more, noon, parallel, past, perimeter, perpendicular, pint, plane, p.m., pound, predicting, pyramid, quart, quarter hour, rectangle, rectangular solid, region, shape, shorter, smaller, solid, sphere, square, symmetry, temperature, time, triangle, week, vertical, vertices, volume, weight, year

Data Analysis, Statistics, and Probability

bar graphs, category, certain, data, dot plots, experiments, impossible, likely, picture graph, table, unlikely

Problem Solving, Mathematical Reasoning, and Communication

addition, division, elements, estimating, intersection, irrelevant, multiplication, number sentences, operations, relevant, sets, solutions, subtraction, union, Venn diagram

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Grade 3

Number Sense and Operations

Content Standard 3–NO1
The student extends an understanding of number and place value to exact and approximate whole numbers to 999,999 and demonstrates an understanding of negative numbers, using a number line.
Performance Standards
The student will:
3–NO1.1 Read and write numbers from 0 to 999,999 with digits and words.
3–NO1.2 Identify numbers as even or odd.
3–NO1.3 Write numbers in expanded form to 999,999 and identify the place value for each digit up to the hundred-thousands.
3–NO1.4 Compare two whole numbers between 0 to 999,999 using symbols (>, <, or =) and words (*greater than, less than,* or *equal to*).
3–NO1.5 Round a whole number to the nearest 10, 100, or 1,000.
3–NO1.6 Use and interpret negative numbers using a number line.
3–NO1.7 Use a number line to solve simple subtraction problems with differences in the range of -10 to -1.

Content Standard 3–NO2
The student estimates, calculates, and demonstrates understanding of sums, differences, products, and quotients and the mutual relationships between them.
Performance Standards
The student will:
3–NO2.1 Complete addition problems (up to 100,000) of any two whole numbers and the corresponding subtraction problems.
3–NO2.2 Mentally estimate a sum to 999 and the corresponding difference.
3–NO2.3 Use mental computation strategies to simplify addition and subtraction problems (e.g., 102 + 84 = 100 + 86).
3–NO2.4 Recall multiplication up to 10 times 10 and corresponding division facts.
3–NO2.5 Multiply by 10, 100, and 1,000 mentally.
3–NO2.6 Estimate and find the product of two whole numbers, in which one factor is nine or less and the other is a multi-digit number up to four digits.
3–NO2.7 Identify perfect squares (and square roots) to 100 and recognize the exponent 2 as indicating a number squared.
3–NO2.8 Divide dividends up to four digits by one-digit divisors, with and without remainders.

3–NO2.9 Check division by multiplying and adding any remainder.
3–NO2.10 Solve two-step word problems.

Content Standard 3–NO3
The student demonstrates an understanding of decimals and simple fractions.
Performance Standards
The student will:
3–NO3.1 Identify fractions (to tenths) represented by drawings or concrete materials and represent a given fraction using both concrete materials and symbols.
3–NO3.2 Identify and write mixed numbers.
3–NO3.3 Demonstrate equivalent fractions (for example, 1/2 = 3/6), using concrete materials.
3–NO3.4 Use the signs >, <, and = to compare fractions with like denominators.
3–NO3.5 Compare the numerical values of two fractions having like denominators, using concrete materials.
3–NO3.6 Add and subtract with proper fractions having like denominators of 10 or less.
3–NO3.7 Read and write decimals to the hundredths.
3–NO3.8 Add and subtract decimals to hundredths.
3–NO3.9 Order money amounts written as decimals; count, compare, and make change for amounts up to $20.00, using a collection of coins and bills.
3–NO3.10 Model, estimate, and solve problems involving addition, subtraction, multiplication, and division of money amounts in decimal notation (for multiplication and division, use whole number multipliers and divisors).

Functions and Algebra

Content Standard 3–FA1
The student selects appropriate symbols, operations, and properties to represent, describe, simplify, and solve simple number relationships.
Performance Standards
The student will:
3–FA1.1 Use variables as place holders for numbers in sentences involving addition, subtraction, multiplication, or division ($m + 2 = 6$; $2 \cdot K = 12$; $7 + 8 = N$).
3–FA1.2 Express simple unit conversions in symbolic form (e.g., # inches = # feet • 12).
3–FA1.3 Identify and describe the commutative and associative properties of multiplication and the special properties of 0 and 1.
3–FA1.4 Write one or more number sentences to represent a word problem and solve and interpret the arithmetic answer(s) in the context of the problem.
3–FA1.5 Identify and use the inverse relationships between multiplication and division (e.g., 3 x 7 = 21, so 21 ÷ 3 = ____) to compute and check results.

Content Standard 3–FA2
The student represents and interprets a quantitative relationship in a formula, table, or graph.
Performance Standards
The student will:
3–FA2.1 Use a function rule to solve simple problems (e.g., the function rule is "adding 7" so 1 ->•8, 3 ->•10, 12 ->•19) and graph the resulting ordered pairs of whole numbers on a grid.
3–FA2.2 Recognize, describe, and extend geometric, simple numeric, and linear patterns.
3–FA2.3 Use simple two-dimensional coordinate systems to find locations on a map (e.g., A4) and represent points and simple figures in a coordinate grid.

Measurement and Geometry

Content Standard 3–MG1
The student makes reasonable estimates and accurately measures distance, area, volume, and weight.
Performance Standards
The student will:
3–MG1.1 Know that 1 foot = 12 inches, 1 yard = 36 inches = 3 feet, 1 meter = 100 centimeters, one meter is a little more than one yard, etc.
3–MG1.2 Measure and draw line segments in inches (to 1/4 inch) and in centimeters.
3–MG1.3 Estimate and measure length in inches, feet, yards, centimeters, and meters and know the abbreviations.
3–MG1.4 Know that 1 quart = 2 pints, 1 gallon = 4 quarts, etc.

3–MG1.5 Estimate and measure liquid volume in cups, pints, quarts, gallons, and liters.
3–MG1.6 Estimate and measure weight in pounds/ounces and grams/kilograms and know the abbreviations.

Content Standard 3–MG2
The student tells time to the nearest minute, solves problems of elapsed time, and measures temperature at, above, or below zero.
Performance Standards
The student will:
3–MG2.1 Read digital and analog clock faces and tell time to the minute in both minutes before and minutes after the hour.
3–MG2.2 Know that 60 seconds equals one minute.
3–MG2.3 Solve problems of elapsed time (e.g., The game started at 1:15 and ended at 3:45. How long was the game? It is 1:15. What time will it be in 22 minutes?).
3–MG2.4 Measure and record temperature (Celsius and Fahrenheit) to the nearest degree at, above, or below zero.

Content Standard 3–MG3
The student analyzes plane and solid geometric objects and their attributes.
Performance Standards
The student will:
3–MG3.1 Use names for lines and line segments (e.g., line AB; segment CD).
3–MG3.2 Identify, name, describe, and classify polygons, including pentagons, hexagons, and octagons.
3–MG3.3 Name and describe (e.g., symmetry, number/shape of faces, straight/curved edges) common three-dimensional geometric objects (cube, rectangular solid, sphere, prism, pyramid, cone, cylinder).
3–MG3.4 Identify attributes of triangles and quadrilaterals; draw and describe the elements that distinguish those figures (e.g., equal sides, perpendicular sides, right angles, parallel sides, vertices).
3–MG3.5 Compute areas of rectangles in square inches and square centimeters.
3–MG3.6 Demonstrate understanding that a given area can be enclosed by several different perimeter lengths and that a given perimeter can encase figures of different areas (e.g., Given 12 square tiles, make all the rectangles you can that use all the tiles and find the perimeter of each rectangle).
3–MG3.7 Find the perimeter and the area of a polygon and give examples when one would want to find perimeter and area.

Data Analysis, Statistics and Probability

Content Standard 3–DA1
The student collects, records, organizes, graphs, and interprets the results of surveys and experiments and communicates the findings clearly.
Performance Standards
The student will:
3–DA1.1 Ask and answer questions relevant to the data presented in charts, tables, and graphs.
3–DA1.2 Predict probable outcomes from a collection of organized data.

Content Standard 3–DA2
The student determines the number of possible outcomes of an event and makes simple predictions.
Performance Standards
The student will:
3–DA2.1 Identify possible outcomes (e.g., with a spinner or die) and predict how likely they are to occur.
3–DA2.2 Find possible combinations and arrangements involving a limited number of items (e.g., How many ways can you line up a blue block, a red block, and a green block?).

Problem Solving, Mathematical Reasoning, and Communication

Content Standard 3–PS1
The student uses multiple solution strategies in simple multiplication and division problems.
Performance Standards
The student will:
3–PS1.1 Use concrete objects or drawings to model a multiplication problem and show that it can be solved by counting, skip counting, repeated addition, and multiplication (and similar demonstrations for simple division problems).
3–PS1.2 Select appropriate information and operations to express and solve word problems.

Content Standard 3–PS2

The student identifies and makes adjustments for units, as needed in solving problems.
Performance Standards
The student will:
3–PS2.1 Identify units for each fact in a problem situation, the units needed in the solution, and make simple unit conversions within a system of measurement (e.g., inches and feet, hours and minutes), as needed to correctly solve problems.
3–PS2.2 Identify unit conversion errors in incorrect problem solutions and correct them.
3–PS2.3 Determine a total cost or amount as a function of the number of units and the unit cost or value.

Content Standard 3–PS3
The student defends reasoning when finding solutions to problems generated from grade 3 key math content.
Performance Standards
The student will:
3–PS3.1 Use patterns to verify that all combinations have been found, when required to do so as part of the problem solution.
3–PS3.2 Make up and use a variety of strategies and approaches to solving problems and understand approaches that others use.
3–PS3.3 Use one strategy to verify a solution obtained by another strategy.

Content Standard 3–PS4
The student communicates knowledge of basic skills, conceptual understanding, and problem solving and demonstrates understanding of mathematical communications of others.
Performance Standards
The student will:
3–PS4.1 Use appropriate mathematical terms, vocabulary, and language, based on prior conceptual work (see grade 3 Key Vocabulary).
3–PS4.2 Show ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and building with a variety of concrete materials.
3–PS4.3 Explain strategies used in solving problems, show ideas, and support solutions with evidence, in both oral and written form.
3–PS4.4 Present ideas appropriately for a particular audience or purpose.
3–PS4.5 Understand oral and written directions for appropriate mathematics activities.

Key Vocabulary

Number Sense and Operations

decimal point, denominator, dividend, division, divisor, equivalent fraction, exponent (x 2), even, fraction, hundreds, hundred thousands, hundredths, making change, mental, mixed number, multiplication, negative, numerator, odd, ones, perfect square, place value, proper fraction, quotient, remainder, simplify, square exponents, square root, tenths, tens, ten thousands, thousands, whole number

Functions and Algebra

arithmetic, associative property, commutative property, division, function rule, geometric, grid, interpret, inverse relationships, linear pattern, multiplication, ordered pair, simple numeric, special properties of 0 and 1, subtraction, symbolic form, unit conversion, word problem, variables

Measurement and Geometry

above zero, analog, area, at zero, attribute, below zero, Celsius, centimeter (cm), cone, coordinate grid, coordinate system, cube, curved edge, cylinder, digital, elapsed time, element, equal side, Fahrenheit, feet (ft.), foot (ft.), gallon (gal.), gram (g), hexagon, inch (in.), kilogram (kg), line, line segment, liter (l), map, meter (m), minute, number, octagon, ounce (oz.), parallel side, pentagon, perpendicular side, pint (pt.), point, polygon, pound (lb.), prism, pyramid, quadrilateral, quart (qt.), rectangle, rectangular solid, right angle, second, shape of faces, sphere, square centimeter, square inch, straight edge, symmetry, three-dimensional, triangle, vertex, vertices, yard (yd.)

Data Analysis, Statistics, and Probability

arrangements, possible combinations, predicting, probable outcome, variable

Problem Solving, Mathematical Reasoning and Communication

chart, combination, counting, diagram, division, error, feet, function, graph, hour, inch, measurement, minute, multiplication, number, picture, repeated addition, skip counting, solution, strategy, symbol, table, unit conversion, value, verify, word, word problem

Grade 4

Number Sense and Operations

Content Standard 4–NO1
The student extends understanding of number and place value to include decimals to thousandths.
Performance Standards
The student will:
4–NO1.1 Read, write, order, compare (in symbols and words), and round numbers from .001 to over 1,000,000.
4–NO1.2 Write numbers in expanded form from .001 to more than 1,000,000 and identify the place value for each digit (e.g., 21.49 = 20 + 1 + .4 + .09).

Content Standard 4–NO2
The student demonstrates an understanding of addition, subtraction, multiplication, and division and uses these operations to estimate or find solutions to problems.
Performance Standards
The student will:
4–NO2.1 Estimate and compute the sum or difference of whole numbers and positive decimals to thousandths.
4–NO2.2 Recall multiplication up to 12 • 12.
4–NO2.3 Estimate and multiply by two-digit and three-digit numbers.
4–NO2.4 Create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 4 digits or less (two-step including multiplication and division).
4–NO2.5 Multiply mentally by 10, 100, 1,000, and 10,000.
4–NO2.6 Identify multiples of a given number and common multiples of two given numbers.
4–NO2.7 Identify factors of a given number and common factors of two given numbers.
4–NO2.8 Write a cube number as repeated multiplication.
4–NO2.9 Estimate and divide dividends up to four digits by one- digit divisors.

Content Standard 4–NO3
The student identifies, represents, interprets, and uses fractions and decimals.
Performance Standards
The student will:
4–NO3.1 Identify and write equivalent fractions.
4–NO3.2 Put fractions in lowest terms.
4–NO3.3 Change improper fractions to mixed numbers and vice versa.
4–NO3.4 Compare the numerical values of mixed numbers and fractions having like and unlike denominators of 12 or less and find their approximate location on a number line.
4–NO3.5 Add and subtract with fractions having like and unlike denominators of 12 or less.
4–NO3.6 Multiply and divide a fraction or a decimal by a whole number.
4–NO3.7 Read and write decimals as fractions (for example, 0.39 = 39/100) and vice versa.
4–NO3.8 Relate fractions to decimals, using concrete objects.
4–NO3.9 Determine and express simple ratios.

Content Standard 4–NO4
The student demonstrates an understanding of negative integers and their relationship to positive integers.
Performance Standards
The student will:
4–NO4.1 Interpret negative integers (to 20) as temperatures below zero, distances below sea level, distances to the left of zero on an integer number line, or solutions to problems in which a larger number is subtracted from a smaller number.
4–NO4.2 Model the concept that a whole number and its opposite add to be zero.

Functions and Algebra

Content Standard 4–FA1
The student uses and interprets variables, mathematical symbols, and properties to write and simplify expressions and sentences.
Performance Standards
The student will:
4–FA1.1 Model, describe, and use the distributive property.
4–FA1.2 Write and simplify numerical expressions involving one or more of the following operations: +, –, •, and ÷.
4–FA1.3 Develop, interpret, and use formulas (e.g., A = bh) to answer questions about quantities and their relationships.
4–FA1.4 Recognize the addition of a negative number as the subtraction of a positive number.
4–FA1.5 Use mental computation strategies for multiplication, for example, breaking a problem into partial products: 3 x 27 = (3 x 20) + (3 x 7) = 60 + 21 = 81.

Content Standard 4–FA2
The student uses information given verbally, in tables, and on graphs to create and interpret other representations of the same information.
Performance Standards
The student will:
4–FA2.1 Describe and explain the effect that the variation of one quantity has on the variation of a second quantity.
4–FA2.2 Graph and name ordered pairs on a four-quadrant coordinate grid.

Measurement and Geometry

Content Standard 4–MG1
The student measures, estimates, and demonstrates an understanding of the relationships between distance, capacity, and weight measures.
Performance Standards
The student will:
4–MG1.1 Estimate and measure length in parts of an inch (1/2, 1/4, and 1/8), inches, feet, yards, millimeters, centimeters, and meters.
4–MG1.2 Estimate and measure areas, recognizing and using appropriate units in both the metric and customary system, e.g., square centimeters (cm2), square meters (m2), square kilometers (km2), square inches (in2), square yards (yd2), square miles (mi2).
4–MG1.3 Estimate and measure liquid capacity in teaspoons, tablespoons, cups, pints, quarts, gallons, milliliters, and liters and know the abbreviations.
4–MG1.4 Estimate and measure weights in pounds/ounces and in grams/kilograms.
4–MG1.5 Know the following equivalences among U. S. customary units of measurements and solve problems involving changing units of measurements: 1 ft. = 12 in., 1 yd. = 3 ft. = 36 in., 1 mi. = 5,280 ft, = 1,760 yd., 1 lb. = 16 oz., 1 ton = 2,000 lb., 1 cup = 8 fl. oz., 1 pt. = 2 c., 1 qt. = 2 pt., 1 gal. = 4 qt.
4–MG1.6 Know the following equivalences among metric units of measurement and solve problems involving changing units of measurement: 1 cm = 10 mm, 1 m = 1,000 mm = 100 cm, 1 km = 1,000 m, 1 cg = 10 mg, 1 g = 1,000 mg = 100 cg, 1 kg = 1,000 g, 1 cl =10 ml, 1 liter = 1,000 ml = 100 cl.
4–MG1.7 Estimate the conversion between ounces and grams, pounds and kilograms, inches and centimeters, yards and meters, miles and kilometers, and quarts and liters.

Content Standard 4–MG2
The student measures and draws line segments and measures, draws, identifies, and classifies angles.
Performance Standards
The student will:
4–MG2.1 Estimate, draw, measure, and classify right, acute, or obtuse angles and associate an angle with an amount of turning (1/4 turn with 90 degrees, 1/2 turn with 180 degrees).
4–MG2.2 Identify and draw points, line segments, rays, and lines that appear to be parallel, perpendicular, or intersecting (using appropriate words and symbols).

Content Standard 4–MG3
The student draws, analyzes, and classifies plane and solid geometric objects and determines and differentiates between their attributes.
Performance Standards
The student will:
4–MG3.1 Classify and draw quadrilaterals as squares, rectangles, rhombi, parallelograms, trapezoids, or triangles (scalene, isosceles, equilateral and right, acute, obtuse).
4–MG3.2 Determine and use formulas to find the perimeter of polygons and the areas of squares, rectangles, and triangles.

4–MG3.3 Demonstrate by example the radius, diameter, circumference, and area of a circle.
4–MG3.4 Determine whether the sides of a plane figure and the edges or faces of a solid object are congruent, parallel, or perpendicular. (e.g., the ceiling is parallel to the floor; the wall is perpendicular to the floor.)

Data Analysis, Statistics and Probability

Content Standard 4–DA1
The student collects, records, graphs, and interprets data and communicates the findings clearly.
Performance Standards
The student will:
4–DA1.1 Pose questions that can only be answered by collecting data and select ways of systematically collecting data to answer them.
4–DA1.2 Read, interpret, and display data in line plots, graphs, tables, or charts .
4–DA1.3 Identify verbal, numerical, and graphical representations that convey the same information.
4–DA1.4 Read and interpret scales on axes, maps, and drawings.

Content Standard 4–DA2
The student determines the possible outcomes of an event and the probability of an event occurring.
Performance Standards
The student will:
4–DA2.1 Determine (using concrete materials or lists) all possible outcomes of an event involving four or fewer items (e.g., the number of ways Jose, Mary, and Flo can finish first, second, and third in a race) and explain a process for ascertaining that there have been no omissions or duplications.
4–DA2.2 Determine and explain how likely an event is (e.g., If I have a bag with four red marbles and five blue marbles, I have five chances out of nine of choosing a blue marble.) and interpret this as the probability that the event will occur.

Problem Solving, Mathematical Reasoning, and Communication

Content Standard 4–PS1
The student carries out, explains, and justifies his or her work in an organized approach to problem solving using grade 4 mathematics skills.
Performance Standards
The student will:
4–PS1.1 Formulate or reformulate a statement of a problem, identify facts and collect information, consider various solution strategies, attempt problem solutions, and verify the reasonableness of solutions and/or verify the accuracy of the solution using another approach, explaining each step of the process.
4–PS1.2 Draw diagrams to represent problem situations and label the facts and the unknowns on the diagrams.
4–PS1.3 Identify the missing information that is needed to find a solution to a given problem.

Content Standard 4–PS2
The student defends reasoning when finding solutions to problems generated from grade 4 key math content.
Performance Standards
The student will:
4–PS2.1 Use patterns to verify that all combinations have been found, when required to do so as part of the problem solution.
4–PS2.2 Articulate reasoning for each step when a problem requires multiple steps to solve, including showing multiple steps in symbolic form and explaining each symbol or expression.
4–PS2.3 Explain a pattern that can be used in similar situations.
4–PS2.4 Explain how a problem is similar to other problems already solved.
4–PS2.5 Explain how the mathematics used in a problem is like other concepts in mathematics.
4–PS2.6 Explain how a problem solution can be applied to other school subjects in real-world situations.
4–PS2.7 Make a solution into a general rule that applies to other circumstances.

Content Standard 4–PS3
The student communicates knowledge of basic skills, conceptual understanding, and problem solving and demonstrates understanding of mathematical communications of others.

Performance Standards
The student will:

4–PS3.1 Use appropriate mathematical terms, vocabulary, and language, based on prior conceptual work (see grade 4 Key Vocabulary).
4–PS3.2 Show ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.
4–PS3.3 Explain clearly and logically solutions to problems and support solutions with both oral and written evidence.
4–PS3.4 State his or her purpose and address the audience when communicating.
4–PS3.5 Comprehend mathematics from reading assignments and other sources.

Key Vocabulary

Number Sense and Operations

1,000,000, approximate, common denominator, cube number, cube (x3) = x • x • x decimal, denominator, dividend, divisor, equivalent fractions, expanded form, exponent (x3), factors, improper , raction, like, lowest term, mixed number, multiple, negative integer, positive integer, ratio, repeated multiplication, rounding, simple ratio, thousandths, unlike

Functions and Algebra

+, –, x, ÷, additive, distributive property, formula, four-quadrant coordinate grid, graph, indicators of division (÷, /, long-division box), indicators of multiplication (x, dot, parenthesis, juxtaposition, *), inverse, mental computation, negative number, numerical expressions, ordered pair, partial product, positive number, quantity, relationship, variation, x-axis, y-axis, x-coordinate, y-coordinate

Measurement and Geometry

three-dimensional, acute angle, area, centigram (cg), centimeter (cm), circumference, congruent, conversion, cup (c.), customary, diameter, equilateral, equivalence, formula, fluid ounce (fl. oz.), feet (ft.), foot (ft.), gallon (gal.), gram (g), inch (in.), interpreting, intersecting, isosceles, kilogram (kg), kilometer (km), liquid capacity, liter (l), line (and symbolic notation), line segment (and symbolic notation), meter (m), metric, mile (mi.), milligram (mg), milliliter (ml), millimeter (mm), obtuse angle, orientation, ounce (oz.), parallel (p), parallelogram, perimeter, perpendicular (P), pint (pt.), plane, polygon, pound (lb.), quadrilateral, quart (qt.), radii, radius, ray (and symbolic notation), rectangle, reflection, rhombi, rhombus, right angle, rotation, scale, scalene, similar, slides, solid, square, square centimeter, square inch, square kilometer, square meter, square mile, square yard, tablespoon (tbsp.), teaspoon (tsp.), ton (T.), trapezoid, triangle, yard (yd.)

Data Analysis, Statistics, and Probability

chart, collect, data, display, duplication, experiment, formulate, graph, graphing, interpret, likelihood of an event, line plot, mean, median, mode, omission, possible outcome, probability, range, recording, survey, systematically, table

Problem Solving, Mathematical Reasoning, and Communication

chart, combination, diagram, formulate, graph, model, number, picture, solution, symbol, table, word

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Grade 5

Number Sense and Operations

Content Standard 5–NO1
The student extends understanding of number and place value including very large and very small numbers.
Performance Standards
The student will:
5–NO1.1 Estimate, determine, and interpret the meaning of very large numbers (e.g., How many dollars is a million pennies?).
5–NO1.2 Read, write, and interpret whole number powers of 10 (e.g. 104 = 10,000).
5–NO1.3 Round decimals (and decimal quotients) to the nearest tenth; to the nearest hundredth; to the nearest thousandth.
5–NO1.4 Move the decimal point when dividing by 10, 100, or 1,000.

Content Standard 5–NO2
The student extends understanding of addition, subtraction, multiplication, and division with whole numbers and uses these operations to solve problems.
Performance Standards
The student will:
5–NO2.1 Multiply and divide whole numbers.
5–NO2.2 Determine the greatest common factor and the least common multiple of two or three whole numbers.
5–NO2.3 Identify prime and composite numbers up to 100.
5–NO2.4 Solve division problems with remainders by rounding a decimal quotient.

Content Standard 5–NO3
The student demonstrates an understanding of fractions and decimals and computes with them.
Performance Standards
The student will:
5–NO3.1 Determine the least common denominator (LCD) of fractions with unlike denominators.
5–NO3.2 Compare fractions with like and unlike denominators, using the signs >, <, and =, and *greater than, less than,* and *equal to*.
5–NO3.3 Add and subtract with fractions and mixed numerals (with like and unlike denominators), with and without regrouping, and express answers in simplest form.
5–NO3.4 Multiply mixed numbers and fractions.
5–NO3.5 Write fractions as decimals (e.g., 1/4 = 0.25; 17/25 = 0.68; 1/3 = 0.3333...or 0.33, rounded to the nearest hundredth).
5–NO3.6 Write terminating decimals as fractions.
5–NO3.7 Add and subtract decimals through hundred-thousandths.
5–NO3.8 Estimate and find the product of two numbers expressed as decimals through thousandths.
5–NO3.9 Given a dividend expressed as a decimal, estimate and find the quotient through ten-thousandths and a whole number.
5–NO3.10 Arrange fractions, mixed numbers, and decimals in order, represent them on a number line, and explain the process used.
5–NO3.11 Find the given percent of a number.
5–NO3.12 Express equivalences between fractions, decimals, and percentages, and know the percentage equivalent for halves, quarters, fifths, and tenths.

Content Standard 5–NO4
The student models, explains, and computes with positive and negative integers.
Performance Standards
The student will:
5–NO4.1 Illustrate and determine the value of a sum of a positive and a negative integer or two negative integers.
5–NO4.2 Identify or represent integers on a number line.
5–NO4.3 Read, write, and order positive and negative decimals to the nearest hundred thousandth.
5–NO4.4 Compare the value of two negative or positive decimals through hundred-thousandths using the symbols >, <, or =, and *greater than, less than,* and *equal to*.

Functions and Algebra

Content Standard 5–FA1
The student describes and uses properties and variables to express a verbal, numeric, geometric, or graphical relationship; models; and solves algebraic equations in one variable.
Performance Standards
The student will:
5–FA1.1 Identify the reciprocal of a given fraction; show that the product of a given number and its reciprocal = 1.
5–FA1.2 Use a letter to represent an unknown number and write and simplify a simple algebraic expression in one variable for a given situation.
5–FA1.3 Evaluate simple algebraic expressions by substitution.
5–FA1.4 Solve one-step linear equations in one variable involving whole number coefficients and rational solutions.
5–FA1.5 Solve simple problems involving rate of speed, unit cost, or unit weight.
5–FA1.6 Know the names and use of the commutative and associative properties for addition, and the commutative, associative, and distributive properties for multiplication, and illustrate understanding by usage and by identifying examples and counterexamples

Content Standard 5–FA2
The student investigates, describes, and extends numerical and geometric patterns.
Performance Standards

The student will:
5–FA2.1 Identify and describe triangular, square, and cubic numbers, and investigate and extend patterns involving them, using drawings and concrete materials as needed.
5–FA2.2 Create, describe, and extend patterns formed by powers and arithmetic sequences.

Measurement and Geometry

Content Standard 5–MG1
The student measures, compares, and interprets lengths, areas, volumes, weights, and time using appropriate units and tools.
Performance Standards
The student will:
5–MG1.1 Convert to common units of measurement in problems involving addition and subtraction of different units.
5–MG1.2 Differentiate between perimeter, area, and volume and determine which of these is appropriate to use in a given problem.
5–MG1.3 Determine the duration of a time interval (e.g., 11:00 a.m. to 3:30 p.m. the next day).

Content Standard 5–MG2
The student uses cubic units to measure volume and identify and differentiate between one-, two-, and three-dimensional parts of a plane or solid object.
Performance Standards
The student will:
5–MG2.1 Identify the diameter, radius, chord, central angle, arc, circumference, and area of a circle.
5–MG2.2 Model and compute the volume of geometric solids (rectangular solids and prisms), select and use appropriate units in both metric and customary systems (cubic centimeters [cm3], cubic meters [m3], cubic inches [in3], cubic yards [yd3].
5–MG2.3 Identify and determine a method to calculate the surface of a cube and a rectangular solid.
5–MG2.4 Recognize and describe bilateral and rotational symmetry in two- and three-dimensional figures.

Content Standard 5–MG3
The student identifies and draws one-, two-, and three-dimensional geometric objects and uses their properties.
Performance Standards
The student will:
5–MG3.1 Know that regular polygons have sides of equal lengths and angles of equal measure.
5–MG3.2 Identify and draw diagonals of polygons.
5–MG3.3 Estimate, measure, identify, and construct perpendicular and parallel lines, angles, midpoints, and bisectors (perpendicular and angle), using appropriate tools (e.g., straight edge, ruler, compass, protractor, and drawing software).
5–MG3.4 Associate 3/4 turn with 270• and a full turn with 360•.

Data Analysis, Statistics and Probability

Content Standard 5–DA1
The student displays, analyzes, develops questions, interprets, and makes predictions based on sets of data.
Performance Standards
The student will:
5–DA1.1 Formulate questions and answer them by carrying out a survey or experiment, accurately recording data, and clearly communicating the results.
5–DA1.2 Create and interpret circle graphs and display information in alternative graphical formats (e.g., show information in a bar graph as a circle graph).
5–DA1.3 Draw conclusions, note trends, and make a recommendation based on data analysis.
5–DA1.4 Organize and display single-variable data in appropriately designed and labeled tables, charts, and graphs (e.g., use a bar graph, histogram or frequency table for grouped data).
5–DA1.5 Explain what measure of central tendency (mean, median, mode) is most representative for a given set of data.

Content Standard 5–DA2
The student selects and uses a method for determining the likelihood of a simple event.
Performance Standard
The student will:
5–DA2.1 Use experiments with concrete materials (e.g., dice, spinners, cards) to determine a relative frequency for an event and compare that with

the expected probability derived from a tree diagram, sample space, or area comparison.

Problem Solving, Mathematical Reasoning, and Communication

Content Standard 5–PS1
The student translates among verbal, graphic, and symbolic representations of problems involving grade 5 mathematics and their solutions.
Performance Standard
The student will:
5–PS1.1 Translate among problem statements, diagrams, and symbolic expressions for situations involving perimeter, area, and volume and those that involve one- and two-step arithmetic solutions, and explain or justify these translations.

Content Standard 5–PS2
The student recognizes and explains simple logical errors.
Performance Standards
The student will:
5–PS2.1 Recognize that general rules may not apply in all cases (e.g., any number divided by itself equals 1, which does not apply for 0), and that special cases do not necessarily lead to a general rule, (e.g., 32 degrees F = 0 degrees C, does not mean that C = F – 32) and search for counterexamples in these situations.
5–PS2.2 Recognize that stating a premise as false does not guarantee that the conclusion also is false (e. g., All squares have four corners. This object is not a square. Therefore, this object does not have four corners.) and explain the logical error.
5–PS2.3 Recognize the misapplication of a formula or rule in solving a problem (e.g., 50 miles at 10 miles per gallon = 500 miles travel), and explain the error.

Content Standard 5–PS3
The student defends reasoning when finding solutions to problems generated from grade 5 key math content.
Performance Standards
The student will:
5–PS3.1 Defend the choice of using addition, subtraction, multiplication, and/or division of whole numbers and/or fractions and decimals when solving a non-routine problem.
5–PS3.2 Use and invent a variety of approaches and understand and evaluate those of others.
5–PS3.3 Involve problem-solving strategies, such as illustrating with sense-making sketches to clarify situations or organizing information in a table.
5–PS3.4 Explain, where helpful, how to break a problem into simpler parts.
5–PS3.5 Solve for unknown or undecided quantities using algebraic thinking, graphing, sound reasoning, and other strategies.
5–PS3.6 Make sensible, reasonable estimates.
5–PS3.7 Make justified, logical statements.
5–PS3.8 Verify and interpret results with respect to the original problem situation.
5–PS3.9 Generalize solutions and strategies to new problem situations.

Content Standard 5–PS4
The student communicates knowledge of basic skills, conceptual understanding, and problem solving and demonstrates understanding of mathematical communications of others.
Performance Standards
The student will:
5–PS4.1 Use mathematical language and representations with appropriate accuracy, including numerical tables and equations, simple algebraic equations and formulas, charts, graphs, and diagrams (see grade 5 Key Vocabulary).
5–PS4.2 Organize work, explain facets of solution orally and in writing, label drawings, and use other techniques to make meaning clear to the audience.
5–PS4.3 Use mathematical language to make complex situations easier to understand.
5–PS4.4 Exhibit developing reasoning abilities by justifying statements and defending work.
5–PS4.5 Show understanding of concepts by explaining ideas not only to teachers but to fellow students or younger children.
5–PS4.6 Comprehend mathematics from reading assignments and from other sources.

Key Vocabulary

Number Sense and Operations

>, <, =, composite number, decimal, decimal point, decimal quotient, dividend, equal to, factor, greater than, greatest common factor, hundredth, least common denominator (LCD), least common multiple, less than, negative, negative

integer, non-terminating decimal, number, percent (%), place value, positive, powers of 10, prime, product, ratio, reciprocal, rounding, tenth, ten thousandths, terminating decimal, thousandth, unlike denominator

Functions and Algebra

algebraic expression, arithmetic sequence, coefficients, cubic number, fraction, linear equation, multiplicative inverse, power, product, rate, rational solution, reciprocal, square, substitution

Measurement and Geometry

3/4 turn = 270•, full turn = 360•, angle, arc, area of a circle, bilateral, central angle, chord, circumference of a circle, compass, cube, cubic centimeter (cm3), cubic inches (in3), cubic meter (m3) , cubic yard (yd3), customary system, diameter, drawing software, duration, geometric solid, metric, midpoints, parallel line, perpendicular, pi, prism, protractor, radius, rectangular solid, rotational symmetry, ruler, straight edge, surface, time interval

Data Analysis, Statistics, and Probability

area comparison, bar graph, drawing conclusion, expected probability, frequency table, histogram, mean, median, measure of central tendency, mode, note trend, relative frequency, range, sample space, tree diagram

Problem Solving, Mathematical Reasoning, and Communication

area, diagram, false, justified, logical, perimeter, premise, problem statement, reasoning, symbolic, volume, translate

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Grade 6

Number Sense and Operations

Content Standard 6–NO1
The student reads, writes, computes, and estimates whole numbers, fractions, decimals and percentages, and ratios and proportions.
Performance Standards
The student will:
6–NO1.1 Read, write, and compute with whole numbers in scientific notation (e.g., 325,600 = 3.256 • 105).
6–NO1.2 Find common multiples and factors, including the least common multiple and the greatest common factor, using prime factorization.
6–NO1.3 Determine whether a number is a prime number or a composite number and explain the concepts of prime and composite numbers.
6–NO1.4 Write fractions as equivalent terminating or repeating decimals and explain the process used.
6–NO1.5 Interpret and model percentage terms of parts of 100, determine the percentage equivalent to decimals (expressed as tenths or hundredths) and simple fractions (e.g., 1/2 is 50%, 3/5 is 60%, 1/20 = 5%).
6–NO1.6 Identify the reciprocal of a given fraction and know that the product of a given number and its reciprocal is 1 (multiplicative inverse).
6–NO1.7 Illustrate and solve division of fractions (e.g., 5/8 ÷ 3/4 = 5/8 • 4/3 or 20/24 or 5/6).
6–NO1.8 Describe and compare two quantities using ratios and appropriate notations (*a*/*b*, *a* to *b*, *a*:*b*) and give ratios in lowest terms.
6–NO1.9 Illustrate and solve proportions for a missing value (e.g., determine the value of *N* if 4/7 = *N*/21).

Content Standard 6–NO2
The student identifies, represents, adds, subtracts, multiplies, and divides positive and negative rational numbers.
Performance Standards
The student will:
6–NO2.1 Identify, compare, and order rational numbers and represent them on a number line.
6–NO2.2 Illustrate and solve addition, subtraction, multiplication, and division problems using positive and negative numbers, including fractions and decimals.

Functions and Algebra

Content Standard 6–FA1
The student writes verbal expressions/sentences as algebraic expressions/ equations, graphs them, and interprets the results in all three representations.
Performance Standards
The student will:
6–FA1.1 Write and solve one- and two-step linear equations and inequalities in one variable, using strategies involving inverse operations, integers, fractions, and decimals.
6–FA1.2 Apply order of operations and the commutative, associative, and distributive properties to solve problems involving combinations of the four operations.

Content Standard 6–FA2
The student analyzes tables, graphs, and rules to determine functional relationships.
Performance Standards
The student will:
6–FA2.1 Translate between verbal, numeric, graphical, and symbolic representations of linear functional relationships.
6–FA2.2 Demonstrate understanding that rate is a measure of one quantity per unit value of another quantity.
6–FA2.3 Solve multi-step problems involving rates, average speed, distance, and time.

Content Standard 6–FA3
The student investigates and describes geometric and exponential patterns.
Performance Standards
The student will:
6–FA3.1 Apply a functional perspective to questions in geometry (e.g., the number of sides of a regular polygon and the sum of its interior angles).
6–FA3.2 Identify, represent, extend, and create number patterns involving multiples, squares, and cubes.

Measurement and Geometry

Content Standard 6–MG1
The student uses ratios and proportional reasoning to convert within measurement systems.
Performance Standards
The student will:
6–MG1.1 Determine perimeter, area, volume, and circumference of a given geometric figure.
6–MG1.2 Create and solve proportions to convert between units of time (e.g., 60 min./1 hr. = 700 min./*x* hr.).
6–MG1.3 Know estimates of pi and use them to determine the circumference and the area of a circle.
6–MG1.4 Model and compute the volume of a cylinder, selecting and modeling a volume of approximately unit 1 cc/cm, 1 in, 1 ft, 1 m, 1 yd, and select an appropriate unit with which to represent a given volume.
6–MG1.5 Determine whether geometric figures (quadrilaterals and triangles) are similar and write proportions to express the relationships between corresponding parts of similar figures.

Content Standard 6–MG2
The student identifies and describes the properties of and relationships among two- and three-dimensional figures.
Performance Standards
The student will:
6–MG2.1 Identify, describe, and classify angles as vertical, adjacent, complementary, and/or supplementary.
6–MG2.2 Define, describe, and draw polygons, given information about them.
6–MG2.3 Identify and describe properties of simple geometric figures and parallel and perpendicular lines.
6–MG2.4 Identify congruent angles and sides and axes of symmetry and show how congruent plane figures can be made to correspond through reflection, rotation, and translation.
6–MG2.5 Visualize, represent, and interpret two-dimensional views of three-dimensional objects made from rectangular solids.

Data Analysis, Statistics and Probability

Content Standard 6–DA1
The student reads and analyzes data taken from tables or graphs and uses the concept of *average* to answer relevant questions.

Performance Standards
The student will:
6–DA1.1 Identify ordered pairs of data from a graph (bar, circle, line, or broken-line) and interpret the meaning of the data in terms of the situation depicted by the graph.
6–DA1.2 Describe the possible effects of missing or incorrect information.
6–DA1.3 Find and model the mean, median, mode, and range of a set of data.
6–DA1.4 Determine a missing quantity if the mean of a set of quantities is known.

Content Standard 6–DA2
The student determines theoretical and experimental probabilities and uses these to make predictions about events.
Performance Standards
The student will:
6–DA2.1 Carry out and interpret a simulation (by hand or using computer software) to predict the likelihood of an event.
6–DA2.2 Compute the probability of an event using counting principles, sample spaces, and geometric arguments and compare it with results from a simulation.
6–DA2.3 Demonstrate understanding that the sum of the probabilities of all possible events is 1.0, that the probability of any event lies between 0.0 and 1.0 inclusively, and that the probability of an event not happening is 1 minus the probability of the event's happening.
6–DA2.4 Organize and display data in appropriately designed and labeled tables, charts, and graphs.

Problem Solving, Mathematical Reasoning, and Communication

Content Standard 6–PS1
The student incorporates the grade 6 mathematics of proportions, rates, inequalities, and exponents into problem-solving strategies.
Performance Standards
The student will:
6–PS1.1 Identify rates and proportions in problem situations, carry units through all steps of the solution, and describe each step in terms of the units involved, using appropriate mathematical language.
6–PS1.2 Identify and correct errors when rates or proportions are inverted in incorrect problem solutions.
6–PS1.3 Explain and justify each step of a solution to an inequality problem.
6–PS1.4 Describe situations involving patterns of increasing exponents, explain how they relate to repeated multiplication, and illustrate the mathematics of the example symbolically.
6–PS1.5 Extract pertinent information from situations and figures and identify what additional information is needed.
6–PS1.6 Invoke problem-solving strategies, such as illustrating with sketches to help make sense of complex situations or organize information in a table.
6–PS1.7 Solve problems for unknown or undecided quantities, using algebra, graphing, sound reasoning, and other strategies.

Content Standard 6–PS2
The student recognizes and explains simple logic.
Performance Standards
The student will:
6–PS2.1 Use estimation and reasonableness of solutions as a guide and identify and correct errors in the application of scientific notation to problem solutions.
6–PS2.2 Formulate conjectures and argue, short of formal proof, why they must be or seem to be true.
6–PS2.3 Explain, where helpful, how to break a problem into simpler parts.
6–PS2.4 Verify and interpret results of a problem.
6–PS2.5 Generalize solutions and strategies to new problems.

Content Standard 6–PS3
The student communicates knowledge of basic skills, conceptual understanding, and problem solving, and demonstrates understanding of mathematical communications of others.
Performance Standards
The student will:
6–PS3.1 Use mathematical language and representations with appropriate accuracy, including numerical tables and equations, simple algebraic equations, formulas, charts, graphs, and diagrams (see grade 6 Key Vocabulary).
6–PS3.2 Organize work, explain facets of a solution orally and in writing, label drawings, and use other techniques to make meaning clear to the audience.
6–PS3.3 Use mathematical language to make complex situations easier to understand.
6–PS3.4 Exhibit developing reasoning abilities by justifying statements and defending work.

6–PS3.5 Demonstrate understanding of concepts by explaining ideas not only to teachers but to fellow students or younger children.
6–PS3.6 Demonstrate comprehension of mathematics from reading assignments and other sources.

Key Vocabulary

Number Sense and Operations

appropriate notation, common multiples, factor, greatest common factor, least common multiple, negative rational number, number line, positive rational number, prime factorization, proportion, ratio, rational number, reciprocal, repeating decimal, scientific notation, terminating decimal

Functions and Algebra

algebraic expression, associative property, average speed, commutative property, decimal, distance, distributive property, equations, exponent, exponential pattern, fraction, functional relationship, generalize, geometric growth, graphical, inequality, integer, interior angle, inverse operation, linear equation, numeric, quantity, rate, regular polygon, relationship, rule, scientific notation, square root, symbolic, time, variable, verbal

Measurement and Geometry

adjacent, area, axes of symmetry, circumference, classifying angles, cylinder, complementary, congruent, convert, parallel, perimeter, perpendicular, plane figure, polygon, property, proportion, quadrilateral, reflection, regular solid, rotation, similar figures, supplementary, three-dimensional, translation, triangle, two-dimensional, vertical, volume

Data Analysis, Statistics, and Probability

bar, broken line, chart, circle, counting principles, graph, line, mean, median, mode, modeling, ordered pair, predict, sample space, set of data, simulation, theoretical probability

Problem Solving, Mathematical Reasoning, and Communication

accuracy, chart, complex situation, conjecture, diagram, equation, estimation, formula, formulate, generalize, graph, increasing exponents, inequality, interpret, inverted, numerical table, justify, proportion, rate, reasonableness, scientific notation, symbolically, verify

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Grade 7

Number Sense and Operations

Content Standard 7–NO1
The student applies, explains attributes, and computes with real numbers expressed in a variety of forms.
Performance Standards
The student will:
7–NO1.1 Read, write, and compute with rational numbers in scientific notation (positive and negative powers of 10), approximate numbers using scientific notation, and explain the process.
7–NO1.2 Use the laws of exponents to solve problems.
7–NO1.3 Model and express rational numbers as fractions, terminating or repeating decimals, or percentages and describe the equivalence relationship among these representations.
7–NO1.4 Demonstrate understanding of the square root symbol, determine the two integers between which a particular square root lies, and explain how he or she knows.
7–NO1.5 Interpret the absolute value of a real number as its distance from zero on a number line and determine the absolute value of real numbers.

Content Standard 7–NO2
The student uses correct order of operations and number properties to add, subtract, multiply, and divide positive and negative rational numbers,

extract roots, and determine whole number powers of positive rational numbers.
Performance Standards
The student will:
7–NO2.1 Add, subtract, multiply, and divide rational numbers, integers, fractions, and decimals, and raise rational numbers to whole number powers.
7–NO2.2 Explain and use the inverse relationship between exponentiation/root extraction.
7–NO2.3 Explain and apply properties of real numbers (associative, commutative, distributive, identity, inverse) to simplify numerical expressions.

Content Standard 7–NO3
The student reasons proportionately and uses ratios and rates to solve problems.
Performance Standards
The student will:
7–NO3.1 Use proportions to solve problems involving a change of scale (drawing, models, maps) or a comparison of two quantities.
7–NO3.2 Identify and interpret situations involving direct variation and represent these situations on a coordinate graph.
7–NO3.3 Solve consumer application problems involving discount, markup, commission, profit, and simple compound interest.

Functions and Algebra

Content Standard 7–FA1
The student identifies, describes, represents, extends, and creates linear and non-linear number patterns.
Performance Standards
The student will:
7–FA1.1 Identify Pythagorean triples and describe patterns found in Pascal's triangle.
7–FA1.2 Identify, describe, and generalize patterns involving geometric growth, square roots, or exponents.

Content Standard 7–FA2
The student relates the equation, coordinate graph, and set of ordered pairs of a linear function.
Performance Standards
The student will:
7–FA2.1 Graph a linear function in two variables on the coordinate plane, given a set of ordered pairs.
7–FA2.2 Graph a linear function in two variables on the coordinate plane given its equation, slope, and *y*-intercept, or given both its *x*- and its *y*-intercepts.

Content Standard 7–FA3
The student expresses quantitative relationships using algebraic terminology, expressions, equations, and inequalities.
Performance Standards
The student will:
7–FA3.1 Use variable and appropriate operations to write an expression, equation, inequality, or system of equations or inequalities representing a verbal description.
7–FA3.2 Apply the property of real number operations to evaluate algebraic expressions for given replacement values of variables.
7–FA3.3 Discuss the different uses of variables in expressions (e.g., $2w + 2L$), equations (e.g., $y = x – 4$), formulas (e.g., $c = 9d$), and properties (e.g., $a + b = b + a$).

Content Standard 7–FA4
The student interprets and evaluates expressions involving integer powers and roots of monomials.
Performance Standards
The student will:
7–FA4.1 Interpret whole number powers as repeated multiplication, negative integer powers as reciprocals, and evaluate monomials that have them.
7–FA4.2 Simplify square roots of perfect square monomials.

Content Standard 7–FA5
The student writes verbal expressions/sentences as algebraic expressions/equations, graphs them and interprets the results in all three representations.
Performance Standard
The student will:
7–FA5.1 Write and solve two-step linear equations and inequalities in one variable, using strategies involving inverse operations, integers, fractions, and decimals.

Measurement and Geometry

Content Standard 7–MG1
The student chooses appropriate units of measure and uses proportional reasoning to convert within and between measurement and monetary systems.
Performance Standards
The student will:
7–MG1.1 Select, use, and explain a method for comparing weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, 4.5 meters is about 1190 inches).
7–MG1.2 Convert between monetary systems (e.g., US $ and francs).
7–MG1.3 Use rates (e.g., speed, density) and other derived units to solve problems.

Content Standard 7–MG2
The student knows and uses formulas for perimeter, circumference, area, and volume of common geometric objects and uses these to derive methods for finding or approximating measures of less common objects.
Performance Standards
The student will:
7–MG2.1 Estimate and find the area of polygons by subdividing them into rectangles and triangles.
7–MG2.2 Reason proportionately to find measures and the ratios in situations involving similar figures.

Content Standard 7–MG3
The student describes and explains relationships among one-, two-, and three-dimensional objects.
Performance Standards
The student will:
7–MG3.1 Identify and construct line segments, altitudes, medians, angle bisectors, and perpendicular bisectors.
7–MG3.2 Classify quadrilaterals using their properties (deductive reasoning).
7–MG3.3 Use the Pythagorean Theorem to find or approximate the length of the missing side of a right triangle or the diagonal of a square or rectangle.
7–MG3.4 Determine the number of diagonals and the measures of central, interior, and exterior angles of regular polygons.
7–MG3.5 Identify and sketch central and inscribed angles, arc, radii, diameters, and chords of circles.

Data Analysis, Statistics and Probability

Content Standard 7–DA1
The student demonstrates an understanding of displaying, analyzing, and interpreting data he or she has generated or taken from resources.
Performance Standards
The student will:
7–DA1.1 Construct and interpret frequency distributions, line plots, stem-and-leaf plots, box-and-whisker plots, and scattergrams.
7–DA1.2 Determine measures and appropriate uses of central tendencies (mean, median, and mode), frequency, and distribution (range, inter-quartile range) for a set of data.
7–DA1.3 Define, describe, and use algebraic terminology correctly *(equation, inequality, variable, expression, term, constant, coefficient).*

Content Standard 7–DA2
The student demonstrates an understanding of using probability to answer questions about the likelihood of an event.
Performance Standards
The student will:
7–DA2.1 Construct a sample space to determine theoretical and experimental probabilities and represent it in the form of a list, chart, picture, or tree diagram.
7–DA2.2 Determine the probability of a given simple event and express that probability as a ratio decimal or percentage.
7–DA2.3 Identify and describe the number of possible arrangements of several objects, using a tree diagram or the Fundamental Counting Principle.

Problem Solving, Mathematical Reasoning, and Communication

Content Standard 7–PS1
The student uses mathematical techniques and language accumulated by grade 7 in explaining and justifying each step in problem solutions.
Performance Standards
The student will:
7–PS1.1 Appropriately frame the symbolic representation of a problem (e.g., with introductory statements such as "Let *x* equal the quantity of fuel in gallons.").
7–PS1.2 Refer to properties by name or symbolic form (e.g., $ab = ba$) in justifying each step in manipulating an equation or expression.
7–PS1.3 Explain the sequence of steps in solving a problem (e.g., first convert all time values to minutes so that they will be in the same scale; then multiply durations times repetitions to get totals for each event; then add these products to get a grand total; and finally convert to hours and minutes so the solution is easier to comprehend) and state the reasons for selecting a particular sequence.
7–PS1.4 Check solutions (e.g., by working backwards or substituting solutions back into original formulations) and explain how the process verifies the solution.
7–PS1.5 Extract pertinent information from situations and figures and identify what additional information is needed.
7–PS1.6 Invoke problem-solving strategies, such as illustrating with sense-making sketches to clarify situations or organizing information in a table.
7–PS1.7 Solve for unknown or undecided quantities, using algebra, graphing, sound reasoning, and other strategies.

Content Standard 7–PS2
The student recognizes and explains simple logical errors.
Performance Standards
The student will:
7–PS2.1 Integrate concepts and techniques from different areas of mathematics.
7–PS2.2 Make sensible and reasonable estimates.
7–PS2.3 Make justified and logical statements.
7–PS2.4 Generalize solutions and strategies to new problems.
7–PS2.5 Formulate conjectures and argue, short of formal proof, why they must be or seem to be true.
7–PS2.6 Use and invent a variety of approaches and understand and evaluate those of others.

Content Standard 7–PS3
The student communicates knowledge of basic skills, conceptual understanding, and problem solving and demonstrates understanding of the mathematical communications of others.
Performance Standards
The student will:
7–PS3.1 Use mathematical language and representations with appropriate accuracy, including numerical tables and equations, simple algebraic equations, formulas, charts, graphs, and diagrams (see grade 7 Key Vocabulary).
7–PS3.2 Organize work, explain facets of a solution orally and in writing, label drawings, and use other techniques to make meaning clear to the audience.
7–PS3.3 Use mathematical language to make complex situations easier to understand.
7–PS3.4 Exhibit reasoning abilities by justifying statements and defending work.
7–PS3.5 Demonstrate understanding of concepts by explaining ideas not only to teachers but to fellow students or younger children.
7–PS3.6 Demonstrate comprehension of mathematics from reading assignments and from other sources.

Key Vocabulary

Number Sense and Operations

absolute value, associative property, attribute, commission, commutative property, compound interest, consumer application problem, coordinate graph, decimal, direct variation, discount, distributive property, equivalence relationship, exponentiation, exponent, extract roots, identity property, integer, inverse, markup, negative power, number property, numerical expression, order of operation, percent, power, profit, proportionately, ratio, rational number, real number, repeating decimal, scale, scientific notation, square root, symbol, terminating decimal

Functions and Algebra

algebraic expression, algebraic terminology, coefficient, constant, coordinate graph, coordinate plane, cube, decimal, equation, expression, formula, fraction, inequality, integer, inverse operation, linear equation, linear function, monomial, multiple, negative integer power, non-linear, number pattern, ordered pair, Pascal’s triangle, perfect square monomials, property, property of real number operations, Pythagorean triples, quantitative relationships, reciprocals, set of ordered pairs, square, square root, term, system of equations, variable, verbal description, whole number, whole number powers

Measurement and Geometry

altitude, angle bisector, arc, area, central angle, chords of circles, circumference, classify quadrilaterals, diagonal, diameter, exterior angle, inscribed angle, interior angle, line segment, median, one-dimensional object, perimeter, perpendicular bisector, polygon, radii, three-dimensional object, two-dimensional object, volume

Data Analysis, Statistics, and Probability

box-and-whisker plot, central tendencies, distribution, experimental probability, frequency, frequency distribution, Fundamental Counting Principle, inter-quartile range, line plot, mean, median, mode, range, scattergram, stem-and-leaf plot, theoretical

Problem Solving, Mathematical Reasoning, and Communication

chart, complex situation, diagram, equation, expression, formula, generalize, graph, integrate, justify, logical, manipulate, manipulating, numerical table, sequence, solution, symbolic representation, verify

Algebra

According to the notion fostered by more than a century of teaching, algebra is largely a matter of simplifying expressions made up of numbers and letters, solving equations, and learning the rules for manipulating symbols. Although important, these skills by no means encompass the whole of algebra. In fact, courses that emphasize the isolated practice of these skills, outside a context that provides meaning, can deprive students of an opportunity to master some of the most powerful intellectual tools our civilization has developed— algebraic reasoning in its many forms and the use of algebraic representations such as functions, graphs and spreadsheets. Without an understanding of algebra's concepts and symbols, neither higher mathematics nor quantitative science is possible.

To assure that students have an opportunity to master algebra, the following content and performance standards have been developed:

Content Standard A.1: Problem Solving

The student demonstrates problem-solving ability by using algebraic concepts and skills to solve problems for which specific and detailed steps toward solution are not provided. In the process, the student demonstrates mastery of all three elements of finding a solution: formulation, implementation, and conclusion.

Performance Standards

A.1.1 The student represents and solves a variety of textbook and real-life problems involving linear, quadratic, exponential and absolute value expressions algebraically.

A.1.2 The student translates among verbal, graphical, tabular, and symbolic representations of a problem.

Content Standard A.2: Symbol and Symbol Manipulation

The student simplifies expressions and solves problems in symbolic form.

Performance Standards

A.2.1 The student solves linear equations and inequalities in one unknown and applies these skills to solve practical problems.

A.2.2 The student demonstrates an understanding of the different conceptual uses of symbols (e.g., literal numbers, variables) and communicates their meaning.

A.2.3 The student uses algebraic techniques, including linear combinations and substitution, to solve quantitative problems involving systems of two linear equations in two variables and understands the relationship between the graphic and symbolic forms.

A.2.4 The student applies the laws of exponents to perform operations on expressions with integral exponents and uses scientific notation when

appropriate.
A.2.5 The student adds, subtracts, and multiplies polynomials and divides polynomials with monomial and binomial divisors. The student also simplifies algebraic expressions by combining like terms and by addition, subtraction, multiplication, and division of polynomial components of these expressions.
A.2.6 The student factors various factorable polynomials in one or two variables, including trinomials, quadratics, differences of two squares, expressions requiring regrouping or repeated factorizations, and expressions with lead coefficients greater than one and extracts monomial and binomial factors from expressions of the third or fourth degree.
A.2.7 The student uses graphing, factoring, or the quadratic formula to solve problems involving quadratic equations.
A.2.8 The student simplifies square root radical expressions, including those with constants and variables, and expresses given real numbers using radical sign.

Content Standard A.3: Communication and Reasoning

The student demonstrates the ability to reason mathematically and uses appropriate language and symbols to communicate this conceptual understanding of algebraic ideas.
Performance Standard
A.3.1 Using oral and written language, the student justifies steps used in simplifying expressions and solving equations and inequalities. Justifications will include the use of concrete objects, pictorial representation, and the properties of the real number system.

Content Standard A.4: Functions and Relations

The student demonstrates an understanding of the concept of *function.* The student demonstrates this understanding by using it to solve problems and by representing it in multiple ways (graphically, symbolically, tabularly, and verbally as well as with numbers and diagrams). The student also demonstrates facility in using skills that are associated with the concept.
Performance Standards
A.4.1 The students analyzes a given set of data for the existence of a pattern, represents the pattern algebraically and graphically, if possible, and determines whether it is a function.
A.4.2 The student determines the domain and range of a relation, given a set of ordered pairs, and identifies the relations that are functions.
A.4.3 The student recognizes relations that are linear; finds slopes and understands their significance; and determines the three representations of a linear function (graphic, symbolic, and tabular), given specific information about the function.
A.4.4 The student selects, justifies, and applies an appropriate technique to graph a linear function. Techniques include slope intercept, x and y intercepts, and two points.
A.4.5 The student determines the slope of a line when given the symbolic form, the graph, or two points. The student understands the slope as a rate of change between two quantities and recognizes when two lines are perpendicular or parallel.
A.4.6 The student writes the equation of a line when given the graph, two points, or the slope and a point. The student determines the length of a segment with given end points.
A.4.7 The student, given the symbolic or graphic representation, finds the value of x given y, or y given x and finds the zeros algebraically.
A.4.8 The student demonstrates an understanding of families of functions, their characteristics, and the relationships between the various representations.
A.4.9 The student recognize quadratic relationships.
A.4.10 The student analyzes functional relations between quantities and determines how a change in one affects the other.
A.4.11 The student describes algebraically the relationship between quantities and answers questions about them, using various methods and tools.
A.4.12 The student analyzes the effects of the changes of coefficients and constants on the graphs of functions.
A.4.13 The student analyzes a relation to determine whether a direct or inverse variation exists and represents it algebraically and graphically, if possible.

Content Standard A.5: Technology

The student demonstrates the ability to use the tools of new technologies to solve algebraic problems.
Performance Standards
A.5.1 The student demonstrates the ability to use a calculator to solve problems and answer questions about real-world problems.
A.5.2 The student demonstrates the ability to use computer software to explore algebraic concepts.

Content Standard A.6: Statistics, Data Analysis, and Probability
The student demonstrates knowledge of basic skills, conceptual understanding, and problem-solving in statistics.
Performance Standard
A.6.1 The student demonstrates the ability, given a set of data points, to draw a line approximating a line of best fit, to find the equation of the line, and to use the graph and/or equation to make predictions.

Geometry

Mathematics is a language we use every day, often without knowing it. It builds and draws on conceptual understanding and skills and helps us make decisions and solve problems. This draft document tries to connect the notion of problem solving to conceptual understanding and skill development by embedding problem solving within the content strands at every grade. Accordingly, students are asked, for example, to:

- Ask relevant questions about problem situations.
- Decide between relevant and extraneous information.
- Choose appropriate operations tools and approaches to problem situations.
- Decide whether an exact or approximate answer is called for.
- Apply specific techniques in new situations.
- Explain, check, justify, prove, and judge the reasonableness of results.
- Create new approaches and connect knowledge and understanding in new ways.

Content Standard G.1: Problem Solving

The student demonstrates problem-solving ability by using geometric concepts and skills to solve problems for which specific and detailed steps toward solution are not provided. In the process, the student demonstrates mastery of all three elements of finding a solution: formulation, implementation, and conclusion.
Performance Standards
G.1.1 The student represents and solves a variety of textbook and real-life problems involving perimeter, circumference, area, volume, lateral area, and surface area, and non-routine geometric figures.
G.1.2 The student represents and solves a variety of problems involving classic construction, transformations, and coordinate geometry.

Content Standard G.2: Geometric Concepts

The student selects and uses appropriate units, tools, and degrees of accuracy to solve problems involving geometric figures and measures.
Performance Standards
G.2.1 The student knows, uses, derives formulas for, and solves problems involving perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures
G.2.2 The student describes how changes in the dimensions of an object affect the object’s peri-meter, area, and volume (e.g., tripling the radius of a sphere multiplies its volume by 27).
G.2.3 The student finds and uses measures of sides and interior and exterior angles of triangles and polygons to classify figures (e.g., as *isosceles*, *obtuse*, *convex*, *regular)* and to solve problems (e.g., to determine the number of degrees in a central angle of a regular polygon).
G.2.4 The student describes the relationships between vertical angles, angles that are supplementary and complementary, and angles formed when parallel lines are cut by a transversal and expresses and uses these to find missing angle measures in such systems of anges.
G.2.5 The student uses the Pythagorean Theorem, its converse, properties of special right triangles (e.g., sides in the ratio 3-4-5, angles of 30-60-90 degrees), and right triangle trigonometry to find missing information about triangles.

G.2.6 The student develops and implements a plan for obtaining indirect measures using similarity, proportional reasoning and trigonometric ratios.
G.2.7 The student compares, contrasts, classifies, and solves problems involving quadrilaterals (square, rhombus, rectangle, parallelogram, trapezoid, kite, cyclic on the basis of their definitions and properties (e.g., opposite sides, consecutive angles, diagonals).
G.2.8 The student applies the properties of angles, arcs, chords, radii, tangents, and secants to solve problems involving circles.
G.2.9 The student applies the triangle inequality properties (given information concerning the lengths of sides and/or measures of angles) to determine whether a triangle exists and the relative position of the sides and angles.
G.2.10 The student compares, contrasts, classifies and solves problems about solid geometric figures, including objects appearing in the real world.

Content Standard G.3: Communication and Reasoning

The student demonstrates the ability to reason geometrically and uses appropriate language and symbols to communicate this conceptual understanding of geometric ideas.
Performance Standards
G.3.1 The student constructs and judges the validity of a logical argument consisting of a set of premises and a conclusion. The student will:
• Identify the converse, inverse, and contrapositive of a conditional statement.
• Translate a short verbal argument into symbolic form.
• Diagram arguments involving quantifiers *(all, no, none, some)* using Venn diagrams.
• Use valid forms of deductive reasoning, including the law of syllogism.
• Use counterexamples to disprove a statement.
G.3.2 The student demonstrates an understanding of Euclidean geometry as an axiomatic system and of the nature of proof, by identifying and giving examples of undefined terms, axioms, theorems, inductive and deductive reasoning.
G.3.3 Using coordinate geometry and deductive, inductive, algebraic and/or transformational arguments, the student proves the Pythagorean theorem and proves theorems for the following:
• Parallel lines
• Congruent polygons
• Similar polygons
• Properties of quadrilaterals
• Properties of circles
G.3.4 Using a compass and straightedge, the student constructs the following basic construc-tions and uses combinations of them to create more complex constructions (center of a circle, tangents to circles, etc.).
• A line segment congruent to a given line segment
• The bisector of a line segment
• A perpendicular to a given line from a point not on the line
• A perpendicular to a given line at a point on the line
• The bisector of a given angle
• An angle congruent to a given angle

Content Standard G.4: Spatial Manipulation and Visualization

The student demonstrates an ability to visualize, manipulate, and describe objects, paths, and regions in two-dimensional and three-dimensional space.
Performance Standards
G.4.1 The student uses geometric language and diagrams to describe the intersection of two planes or a plane and a solid object (the cross section it cuts).
G.4.2 The student builds a three-dimensional model from a two-dimensional drawing and draws a two-dimensional representation of a three-dimensional object (e.g., nets, prisms, cones, pyramids).

Content Standard G.5: Technology

The student demonstrates the ability to use the tools of new technologies to solve geometric problems.
Performance Standards
G.5.1 The student demonstrates the ability to use a scientific calculator to solve geometric problems and answer questions about real-world problems.

G.5.2 The student demonstrates the ability to use computer software to explore geometric concepts and make conjectures.

Intermediate Algebra

Mathematics is a language we use every day, often without knowing it. It builds and draws on conceptual understanding and skills and helps us make decisions and solve problems. This draft document tries to connect the notion of problem solving to conceptual understanding and skill development by embedding problem solving within the content strands at every grade. Accordingly, students are asked, for example, to:

• Ask relevant questions about problem situations.
• Decide between relevant and extraneous information.
• Choose appropriate operations tools and approaches to problem situations.
• Decide whether an exact or approximate answer is called for.
• Apply specific techniques in new situations.
• Explain, check, justify, prove, and judge the reasonableness of results.
• Create new approaches and connect knowledge and understanding in new ways.

Content Standard IA.1: Problem Solving

The student demonstrates problem-solving ability by using algebraic concepts and skills to solve problems for which specific and detailed steps toward solution are not provided. In the process, the student demonstrates mastery of all three elements of finding a solution: formulation, implementation, and conclusion.
Performance Standard
IA.1.1 The student represents and solves problems involving systems of equations, systems of inequalities, matrices, functions and relations, conic sections, sequences, series, probability, statistics, and complex numbers.

Content Standard IA.2: Symbol and Symbol Manipulation

The student simplifies expressions and solves problems in symbolic form.
Performance Standards
IA.2.1 The student adds, subtracts, multiplies, divides, reduces, and evaluates rational expressions.
IA.2.2 The student adds, subtracts, multiplies, divides, and simplifies expressions containing radicals and fractional exponents
IA.2.3 The student solves quadratic, rational, radical, absolute value, and factorable polynomial equations in one variable and non-linear systems of equations in two variables.
IA.2.4 The student selects, justifies, and applies an appropriate algebraic technique (factoring, completing the square, quadratic formula) to solve a quadratic equation over the set of complex numbers and interprets the results graphically.
IA.2.5 The student formulates and solves problems involving joint and combined variations.
IA.2.6 The student use factoring techniques (e.g., common factor, difference of cubes, grouping) and synthetic division to solve factorable polynomial equations of degree four or less.
IA.2.7 The student understands and is able to explain the relationship among the coefficients, factors, roots, and *x*-intercepts of polynomial functions.
IA.2.8 The student compares, contrasts, and extends arithmetic and geometric patterns of growth and uses them to make predictions.
IA.2.9 The student applies the properties of arithmetic and geometric sequences and series to solve problems, including writing the first *n* terms, finding the *nth* term, and evaluating summation formulas.
IA.2.10 The student finds the sum of finite arithmetic and geometric sequences and infinite geometric sequences with $|r| < 1$, and uses and

interprets • notation.
IA.2.11 The student uses the Binomial Theorem to determine the *kth* term in $(a + b)^n$ and recognizes the relationship between the coefficients and Pascal's triangle.
IA.2.12 The student adds, subtracts, and multiplies (scalar product) matrices and finds the determinate and inverse of a matrix, when possible.
IA.2.13 The student understand the use of matrices to organize information and performs simple operations by representing and solving systems of two linear equations in two variables.
IA.2.14 The student solves linear equations and inequalities involving absolute value (e.g.,
$|3x – 4| \le 5$) and quadratic inequalities (e.g., $x2 – 7x –1 \ge -7$) and graphs their solutions on a number line.
IA.2.15 The student solves linear-quadratic and quadratic-quadratic systems of equations and inequalities algebraically and graphically.
IA.2.16 The student converts between logarithmic and exponential forms and uses the Laws of Logarithms to manipulate expressions.

Content Standard IA.3: Communication and Reasoning

The student demonstrates the ability to reason algebraically and uses appropriate language and symbols to communicate this conceptual understanding of algebraic ideas.
Performance Standards
IA.3.1 Using oral and written language, the student justifies steps used in simplifying expressions and solving equations and inequalities over the real and complex numbers. Justifications will include the use of concrete objects, graphs, tables, formulae, equations, and the properties of complex numbers.
IA.3.2 The student explains and critiques the process of a survey or experiment; describes how that process might have contributed to or influenced the results (e.g., the student might discuss reliability of sampling procedures, bias, missing or incorrect information); and describes misuses of statistical or numerical data.

Content Standard IA.4: Functions and Relations

The student extends his or her understanding of standard functions and relations to include conic sections, as well as quadratic, exponential, logarithmic, rational, radical, absolute value functions and factorable polynomials.
Performance Standards
IA.4.1 Given information about the figures a, the student identifies and writes equations for circles, parabolas, ellipses, and hyperbolas whose axes are parallel to the *x* and *y* axes.
IA.4.2 Given an equation in standard form, the student determines the center and radius of a circle; the center, vertices, foci, and axes of an ellipse or hyperbola; and the vertex, axis of symmetry, focus, and directrix of a parabola and uses this information to graph the figure.
IA.4.3 The student identifies a general quadratic equation in two variables (no *xy* term) as a circle, a parabola, an ellipse, or a hyperbola and writes its equation in standard form.
IA.4.4 The student determines the domain, range, and zeros of standard functions and relations and the inverse of a standard function.
IA.4.5 The student represents a standard function as a table of values, an equation, or a graph and translates among these representations.
IA.4.6 The student demonstrates and explains the effect that changing a parameter has on the graph of a function (e.g., the student might describe the effect on the graph of $y = x2 + 6x + C$ as
C grows from –5 to 5).
IA.4.7 The student determines the composition of two standard functions, including the necessary restrictions on the domain.
IA.4.8 The student relates arithmetic and geometric sequences to linear and exponential functions and expresses them in explicit and recursive form.
IA.4.9 The student determines which type of function (including step and piecewise) best models a situation, writes an equation noting the constraints on the domain of the function imposed by the situation, and uses this equation to answer questions about the situation.
IA.4.10 The student determines and interprets the meaning of the maximum or minimum values of a quadratic function in terms of applied, real-world situations.

Content Standard IA.5: Technology

The student demonstrates the ability to use the tools of new technologies to solve algebraic problems.
Performance Standards
IA.5.1 The student demonstrates the ability to use a graphing calculator to solve algebraic problems and answer questions about real-world problems.

IA.5.2 The student demonstrates the ability to use computer software to explore algebraic concepts.

Content Standard IA.6: Statistics, Data Analysis, and Probability

The student demonstrates knowledge of basic skills, conceptual understanding and problem solving ability in the areas of application, measurement, collection of data, analysis, making conclusions, and the defense of conclusions, in statistics and probability.
Performance Standards
IA.6.1 The student analyzes and interprets real-world data using graphs (frequency distributions, histograms, scatter plots) and analysis tools (estimated and calculator- or computer-generated regression lines, correlation coefficients, and residuals).
IA.6.2 The student applies curve-fitting techniques to data assesses the goodness-of-fit of a regression line or curve and its usefulness as a model for the data, and makes predictions by interpolating or extrapolating from the data or its graph.
IA.6.3 The student chooses an appropriate probability model (replacement, non-replacement, combination, permutation) and uses it to arrive at a theoretical probability for a simple or compound chance event.

Advanced Algebra and Trigonometry

Mathematics is a language we use every day, often without knowing it. It builds and draws on conceptual understanding and skills and helps us make decisions and solve problems. This draft document tries to connect the notion of problem solving to conceptual understanding and skill development by embedding problem solving within the content strands at every grade. Accordingly, students are asked, for example, to:

- Ask relevant questions about problem situations.
- Decide between relevant and extraneous information.
- Choose appropriate operations tools and approaches to problem situations.
- Decide whether an exact or approximate answer is called for.
- Apply specific techniques in new situations.
- Explain, check, justify, prove, and judge the reasonableness of results.
- Create new approaches and connect knowledge and understanding in new ways.

Content Standard AT.1: Problem Solving

The student demonstrates problem-solving ability by using algebraic concepts and skills to solve problems for which specific and detailed steps toward solution are not provided. In the process, the student demonstrates mastery of all three elements of finding a solution: formulation, implementation, and conclusion.
Performance Standards
AT.1.1 The student represents and solves problems involving trigonometry, vectors, matrices, logarithms, and exponential functions.
AT.1.2 The student analyzes and solves problems involving periodic phenomena (e.g., biological rhythms, sound waves, tidal variations).

Content Standard AT.2: Symbol and Symbol Manipulation

The student simplifies expressions and solves problems in symbolic form.
Performance Standards

AT. 2.1 The student solves quadratic, rational, logarithmic, exponential, radical, absolute value, and factorable polynomial equations in one variable and non-linear systems of equations in two variables.
AT.2.2 The student uses parametric equations to model and solve application problems and to write equivalent equations by eliminating the parameter, paying special attention to any restrictions on the variables.
AT.2.3 The student adds, subtracts, finds scalar products, dot products, and norms of vectors, noting the field properties which apply.
AT.2.4 The student determines, interprets, and uses a unit directional vector, perpendicular components, and norms to express vectors in the coordinate plane.
AT.2.5 The student graphs and interprets complex numbers as vectors and in polar form.
AT.2.6 The student finds the exact values of the trigonometric functions of multiples of 30∞•(•/6) and 45∞ (•/4), and, given standard values of trigonometric functions, finds the angle measure.
AT.2.7 The student proves basic trigonometric identities and makes substitutions using the basic identities.
AT.2.8 The student solves trigonometric equations that include both infinite solutions and restricted domain solutions and solves basic trigonometric inequalities.
AT.2.9 The student demonstrates understanding of the addition, half angle, double angle, sum to product and product to sum formulae and can use them.

Content Standard AT.3: Communication and Reasoning

The student demonstrates the ability to reason algebraically and uses appropriate language and symbols to communicate this conceptual understanding of algebraic ideas.
Performance Standards
AT.3.1 The student finds a formula for the sum of number patterns (e.g., first n consecutive even integers, k consecutive square numbers beginning with 16) and, using mathematical induction, proves the formula.
AT.3.2 The student writes and uses recursive formulas to express iterative patterns of change, including those of exponential growth and decay.

Content Standard AT.4: Functions and Relations

The student extends his or her understanding of standard functions and relations to include conic sections, as well as analytical geometry, exponential and logarithmic functions, and trigonometric functions.
Performance Standards
AT.4.1 The student determines the zeros, *y*-intercepts, end behavior, and symmetry of polynomial functions and graphs the functions.
AT.4.2 The student determines the zeros, asymptotes, *y*-intercepts, end behavior, and symmetry of rational functions and graphs the functions.
AT.4.3 Given the graph of a function, the student graphs its inverse.
AT.4.4 The student determines and graphs the sum, difference, and composition of two or more functions and the composition of a function with itself and determines the domain and range of the resultant function.
AT.4.5 The student determines the inverse of an algebraic or trigonometric function given as an equation, a graph, or a set of ordered pairs; determines its domain and range; and discusses the relationships among these representations.
AT.4.6 The student knows the definitions of standards functions and their reciprocals and can graph them.
AT.4.7 The student applies transformations to the graph of a basic function (e.g., trigonometric functions) and predicts and analyzes the results on the graph of the function.
AT.4.8 The student graphs polar equations (e.g., roses, limniscates), analyzes the results of parameter changes on the graphs, and classifies the equations according to their graphs.
AT.4.9 The student understands and explains the relationship between triangle trigonometry and the unit circle/wrapping function approach to trigonometry.
AT.4.10 Given the value of one trigonometric function, the student finds the values of other trigonometric functions.
AT.4.11 The student identifies key characteristics (e.g., domain, range, amplitude, period, phase shift, and vertical shift) of trigonometric functions and graphs the functions and their inverses.
AT.4.12 The student graphs and analyzes step and piecewise defined functions.
AT.4.13 The student defines and applies the properties of limits of functions, including infinite sequences, series, and the slope of the tangent to a curve of a point.

Content Standard AT.5: Technology

The student demonstrates the ability to use the tools of new technologies to solve algebraic and trigonometric problems.
Performance Standards
AT.5.1 The student demonstrates the ability to use a graphing calculator to solve advanced algebraic and trigonometric problems and answer questions about real-world problems.
AT.5.2 The student demonstrates the ability to use computer software to explore advanced algebraic and trigonometric concepts.

Content Standard AT.6: Measurement and Geometry

The student understands and uses periodic function, trigonometric relationships, and vectors to represent and answer questions about quantities.
Performance Standards
AT.6.1 The student uses similarity, right triangles, the Law of Sines, and the Law of Cosines to determine measurements of objects that are difficult to measure directly.
AT.6.2 The student draws a system of vectors and finds the resultant graphically.
AT.6.3 The student determines the area of a triangle when the value of trigonometric functions are needed to determine the altitude.
AT.6.4 The student identifies, creates, and solves practical problems, using a system of vectors and their horizontal and vertical components.

Do you live in a *new-new math* city?

Here are some of the places we have heard about.

- **Atascadero** -- The School board of Atascadero Unified has formally adopted the new version of Silver-Burdett as Math textbooks for K-6. The parents who worked on this felt a great sense of relief as Mathland was so close to being adopted just a few months ago. We want to thank Mathmetically Correct for giving us all the up to date news and articles on your website and other valuable information. For the school districts who are still struggling with these fuzzy math, we just want to let you know that persistance does pay off when you bombard your school board with hard evidence and grim results from present and past students. Our job may still not be done. Although we have brought back traditional math as a choice, IMP is still around. However, the message is getting through to Jr. High parents so that the signing up for IMP in high school is way down this coming year. We are hoping IMP will "die its own death". In the meanwhile, Glencoe is still our JR. High math. We have heard that it is not adequately preparing Jr. high school students into the traditional high school math sequence. Again, we thank you for helping us to make things happened. Other parents can send email to Concerned Parents of Atascadero.
- **Brea** -- We have formed a parent group in Brea called **BOLD (Brea Open Logical Debate)** . The Brea School Distrct (BOUSD) has jumped head first into the New Math curriculum having adopted Integrated Math at the Junior High and High Schools and Addison-Wesley for elementary schools. BOUSD has spent a lot of time and energy trying to sell this program, including holding "New-Math Nights" at every school in the district late last year. However, there are many parents that are not convinced and are fighting to regain some balance with the content and how the curriculum is taught.
- **Chico** -- We've named our Chico group, **"Access to Math"**, as we feel the district should allow parents the option of sending their children to schools that offer a traditional math curriculum, one that stresses the importance of mastering basic computational skills while also teaching math concepts. I guess the district is afraid of comparisons being made in test results. From what I see on the web and noting how many oppose fuzzy math, I feel that this year we can finally make some changes at the state level and begin to restore sanity to our state's schools. Good luck with your efforts.
- **Clovis** - I just got back from a meeting with the school Deputy Superintendent and other officials in our district. They are basically on my side. The teachers were supposed to be using the new math as a supplement to the current math text book not the other way around, which some were doing. They looked at my son's portfolio and saw how much writing and little math was being done. This concerned them greatly and thet are going to make sure it does not happen elsewhere in the district. They thanked me greatly and invited me to attend their monthly meetings that discuss the curriculums used/proposed in the district. It looks like things will turn out satisfactory. NOTE: CUSD parents feel free to E-mail me if you are experiencing problems with the math curriculum (or any other curriculum). rorabaug@cybergate.com
- **Davis** -- Our group, **PACE (Parents Advocating for Children's Education)**, expects to be involved with curriculum issues across all school subjects, although currently math concerns are foremost on our minds. For the last three years, the Davis Junior and Senior High Schools have taught Algebra I using a new-new math approach, Creative Publications, amidst growing complaints by parents and math professionals that this approach is not meeting the needs of many of the students. Recently, the Davis Board of Education has directed the school administration to establish a committee for the purpose of creating an Alternative (ie. more traditional) Algebra I course, using a more traditional textbook. Parents will be able to choose which course their child will attend this coming fall. Unfortunately, just a few months ago the School Board voted to adopt Mathland as the program that will be used in K-6 in the fall and the materials have recently been purchased at great expense. Mathland - also a Creative Publications product- has created quite a controversy within the Davis commmunity as well. We expect more and more fallout as additional parents get exposed to the new program. Many parents, whose children are involved in the "pilot" usage of Mathland in some Davis classrooms, have already been seeking out our organization with their concerns and horror stories. We would especially like to hear from other groups who have had experience with these two programs. Our snail address is PACE, 417 Mace Blvd., Suite J-250, Davis, CA 95616, Fax (916) 757-3010, or contact PACE by e-mail
- **El Segundo** -- Great to see you and others getting organized. We are just getting started here. We're in the process of forcing the district into creating a Math Advisory council made up of Teachers & Parents. We are "swallowing" hard on CPM for the first year. We are a very small district and cannot afford alternatives. Keep up the good work. Hopefully we can continue to share our information.
- **Escondido** -- The **Parents for Math Choice** group was successful in the reinstitution of traditional math courses. Middle school choice was recently voted in. Escondido now has a choice of traditional math programs from 7th grade forward. (The high schools put traditional back as a choice last year and 65% of students have selected this option.) You can e-mail Parents for Math Choice. For more information see the Update on Math in Escondido
- **Hemet**
  - -- CP Math (CPM) was piloted 4 years ago in this district. What's interesting is that no assessments have been done to see if it is effective. We will be looking into reducing, at the very least, the number of offerings. For issues about school to work and school to career programs, please see our web site.
  - -- The Governing Board in Hemet Unified School District has approved a policy on grading that contains the following: *"The Governing Board believes that grades serve a valuable instructional purpose by helping students and parents/guardians*

identify the student's areas of strengths and those areas needing improvement. Parents/guardians and students have the right to receive course grades that represent an accurate evaluation of the student's achievement. Group activities, projects and assignments are important in the classroom learning process, however, ***group testing may not be used in an individual student's grade.***

- **La Canada** -- We have formed **La Canada Math Advisory Group**. Our short term goal is to study math curriculum and make suggestions to school board. We will be taking a survey of parents about the new math program and new textbooks - Houghton Mifflin. We are a new group and have much to accomplish. Our e-mail address is LCMath@aol.com and phone number is 818-252-2938. We welcome suggestions and new members. Here are preliminary results of survey sent to parents of fifth grades students at Palm Crest Elementary. Over 60% received outside tutoring, Over 50% of parents reported not liking the new textbooks whereas only about 16% said they liked the books. Only 16% of parents liked the use of cooperative learning. Over 80% of parents either did not like or had no opinon regarding heterogenous groups. Parent comments include: *New texbooks are creative and engaging but explorations of how to apply concepts are sometimes unclear. *Higher order thinking skills better but kids can't apply them if they can't do the basic functions easily, accurately and quickly. *Memorization and drill are as essential to math as they are to music, hitting a baseball and spelling. *I understand the philosophy of heterogeneous grouping but in reality I suspect this retards the advanced student, fails to focus needed help on the student who is still developing, and creates just as much risk of stigma as the old system - kids tease as much if they are grouped together as they do apart. *I understand and value the theory. But, none of my four kids has learned the basics; all have had to learn from flashcards and Kumon. *New textbook looks like a coloring book; need drill and practice and problem solving. *Math disaster in 3rd grade - whole math *Very difficult for parents to tutor their children without a parent guide. *Instructions are not very clear.
  Vocal parents of fifth graders challenged the school administration re the policy of heterogenous grouping of students and begining in the next quarter the top 30 students basically with a 95 CTBS test score in math or better will be in a separate class. This may help matters for some kids but the same math book and teaching methods i.e. cooperative learning, a poorly written math textbook and a new teacher probably won't change things that much. We will be having a site council meeting on November 20 to talk about the math curriculum.
- **Lake Elsinore** -- Thought she caught "Dummy Virus" -- Thank you & God for finding out instead it is the New New Math. We have gone back to the ABEKA program. But cannot say how long it will take to undo what they have done in 1 1/2 years to our child in the Riverside County School District. Thanks for cluing us in. Keep up the good work.
- **Livermore** -- We are from Livermore California and we currently have Mathland and CPM. Besides these two evils we are also fighting the destruction of a traditional math program at East Ave Middle School. Teachers and parents have protested to an unresponsive school board and superintendent.
- **Los Angeles** -- My administrator bought us Mathland without telling us. One day the boxes just showed up in the class. I had heard about some schools adopting it entirely and how well it was working. I took the TE home to read and thought I might try it starting with the fractions unit since that was where I was with my students. I had to drop using it within a week, because, knowing the math curriculum for fourth and fifth grades as I do, I felt that the Mathland program did not go into fractions at this grade level at the depth needed. We don't have math texts. I am using old ones from another school that I brought with me to my new school for "reference". I create worksheets and homework problems based on this old text. Mathland may be alive and well in LA Unified but I can't use it and feel that I am doing my job as an educator.

  See also A Comparison of the LAUSD Math Standards and the California Math Standards
- **Manteca** -- Manteca Unified School District went toward the CPM curriculum a few years ago. Many teachers are starkly against the program, and there have been many reports that the teachers are being muzzled from speaking bad about it. All High School math teachers in the district have been forced through CPM training and further, they have been phasing out the choice of traditional mathematics. But what makes matters worse is the fact that the Science department is complaining about the amount of low-skilled students in the area of mathematics. Test scores in math have also dropped. Fortunately the teachers that do speak out have alienated many students to be against the curriculum, even if they didn't have a anti-CPM teacher they realize how much more complex it has made their class and how boring and incomprehensible it has been.
- **Napa** -- "If you're a teacher who has really high expectations and you want your kids to learn and do things, these materials don't meet my expectation levels. They don't meet my parents' expectation levels. And I think the kids are bored after a while." [San Francisco Chronicle]
- **Nevada City** -- The Grass Valley School District (K-6), serving western Nevada County, California, adopted MathLand for all grades starting Fall'95. MathLand is incredibly lacking in appropriate arithmetic skills practice. Even my first grader is bored. To quote a GATE sixth grader neighbor of mine, "MathLand sucks!". They have yet to adopt any standardized test to track the performance of the program, and there is no organized parents group questioning the changes.
- **Norwalk** -- I have read with great interest your web site concerning mathematics in California. I whole heartedly agree with you. My 2 boys attend Norwak-La Mirada School Distict in Los Angeles County and I am very concerned that they are receiving less than an adequate mathematical education. They are in the 3rd and 1st grade and I can already see the lack of mathematical skills at that age.
- **Novato** -- Education Research Network is a group that is trying to provide a broad range of background information useful to parents concerned about educational issues. Right now they are particularly interested in reading and in legislative issues. You can email the Novato branch of Education Research Network
- **Orange** -- News from Orange Unified--the Board of Education adopted the new California math standards (and the language standards) by a 7-0 vote July 9th, 1998. There was some discussion about how much money this is going to cost the district but there was no real

dissent. Our standards advisory committee is thrilled, but we realized this is only the first step. We are now putting together a comprehensive plan regarding what needs to happen next.

- **Orange Update** April 1999--the Orange Unified School District voted to permit math choice in response to numerous parental complaints about the integrated math program which began in 1995 in OUSD. Parents appeared at a number of board meetings to request a traditional mathematics track (Algebra I, Geometry, and Algebra II) to provide an option to the only available math curriculum--integrated math. The Board approved a dual track program with traditional math as the default program. They also required that informed consent be given in advance to those parents who would be enrolling their children in integrated math. Administrators stated that the dual track would be unworkable and that most parents would elect traditional math anyway. Integrated math is now being phased out and the traditional track is being phased in.
- **Palm Desert** -- Desert Sands Unified School District recently implemented "Mathlands" in all elementary grades, based on pilot programs of very short duration. I am still looking for the data which supports such an all-inclusive approach and shows an actual measurement of student success with this program. We have already tried "Math Their Way", "Renaissance Math" and CPM (in the middle and high schools) - test scores are way down. Got another bandwagon we can get on?
- **Palmdale** -- On the Westside of Palmdale we're going through the same thing as the rest of California, and also it seems the rest of the country. This district is hell-bent on piloting and adopting the new math programs, Mathland and etc. A couple of us went to great lengths to inform parents about what the district is trying to do -- front page news articles, 700 flyers stating all the facts, and copies of the negative info from this very web site sent to all the members of the school board -- but to no avail, nobody listened, nobody cared. Well a few of us are tired of fighting a battle that these parents out here don't seem to care about, so we've decided homeschooling is the best answer for us. I'm sorry for all the other kids with lethargic parents. The kids are going to be the ones who will suffer from this ridiculous math curriculum. As far as I see it, the only way to guarantee your child a good education is to homeschool or a go to a good private school.
- **Palo Alto** -- The reaction to the introduction of *Fuzzy Math* in Palo Alto is well documented by a group called **HOLD, Honest Open Logical Debate** on math reform. HOLD sprang up as a result of parents concern over issues that mirror those arising in San Diego today.
- **Petaluma** -- Thank you, thank you, thank you... for providing the information we need to let parents know what is going on during math time. Unfortunately, our district has adopted *Mathland*. What a mess! Many parents have no idea what garbage is being taught to their children. Keep up the good work. The minority groups like ours here in Petaluma need all the back up we can get in our attempts to inform the parents in our district. Please see the Web Site
- **Redondo Beach** -- Our elementary and middle schools were up for math text adoption this year. The teachers volunteered to pilot Quest 2000, Math Every Day, Gateway, Glencoe Interactive, Anytime Math, and Houghton Mifflin. Thanks to Mathematically Correct, HOLD, etc., and our neighboring district of Torrance, the adoption of math textbooks for K-6 was put off. It was by information provided by you and others that the adoption committee was convinced to hold off. Considering the recent developments at the State we now feel the odds have changed considerably towards finding a decent textbook. However, I'm finding out that the word "balance" has a different meaning for the different "players." We are now in our second school year of piloting and evaluating material. It's hard to believe but one of our schools is actually piloting Saxon! The downside is we now have two schools instead of one piloting Quest 2000.
- **Ridgecrest** -- I went to the HS open house. I was amazed when the teacher said there would be no textbook for the class and group study during class would be the majority of classwork. It turns out that Burroughs HS here has changed over to College Preparatory Mathematics: Change from Within (CPM1) - UC Davis' Fuzzy Math. When I started to help my daughter with her homework I really became concerned about this bookless algebra class. She showed me her guess-and-check approach to solving problems. I found she wasn't being taught any basic math theorems. We as parents were instructed not to teach 'foil'. Instead, they are taught to expanding quadratic equations by visualizing them as the sum of component rectangular areas. A cute idea I admit, but this area approach to the problem is an even bigger memory crutch than 'foil'. Group test scores are determined by rolling dice to see which group grade will be recorded for all group members. Homework is not corrected and graded, but merely tallied.
- **Riverside**
  - -- Great page!!This is so wonderful--all the resources we'll need to wow our non-thinking friends. Keep up the good work. I am looking forward to reading through all the info you have provided, but truly this is like an encyclopedia or a chronology of our downward plunge into new-new math. Thank you so much!! Send email to Citizens United for Education
  - -- *CPM: Impeding the learning process ...* reported in a high school newspaper
- **Sacramento**
  - -- Outstanding web site! Because my wife works in our public school district, we knew this "new new math" was a crock, but we didn't know just how bad until we found your site. Thanks for the excellent work.
  - -- Education Research Network is a group that is trying to provide a broad range of background information useful to parents concerned about educational issues. Right now they are particularly interested in reading and in legislative issues. You can email the Sacramento branch of Education Research Network
- **San Diego**
  - -- I heard that you are composing a letter to be sent to the SDCS board requesting that they hold off on their adoptions. I would like my name added to that letter. I am currently teaching "remedial" math to college students who can't pass the entry level math exam (and these classes are VERY FULL). I hope that SDCS will move more slowly into this very "fuzzy" new approach to teaching mathematics. My fear is that my classes will get even more crowded with students who have never mastered the algebraic skills necessary for college level math, science, and statistics.
  - -- I want to express my concern over the **Addison Wesley Focus on Algebra** text currently being used for Beginning Algebra

at the 9th grade level. It seems to have been mostly ignored on this web page. I teach General Math Studies (it is the reteaching of high school algebra to college students who can't pass basic math competency tests). Having majored in mathematics and taught secondary mathematics for many years I feel qualified to make the following statements. My son is a "victim" of the new-new math. In the 8th grade he was subjected to the CPM course for advanced 8th grade math. At the time I had grave concerns about the "lack" of Algebra he was actually learning. I expressed my concern to the school and was mostly dismissed as an hysterical parent. He is now repeating beginning Algebra in high school. [Editorial note: Many parents feel this is a good alternative to insure that a solid grounding in algebra is achieved. They could be wrong.] At first I was hopeful that he would recieve some traditional Algebra, but not so. He is using the text mentioned above which, although it is more "traditional" than the CPM program, is very poorly written and lacking in many areas. Correct mathematical vocabulary is sacrificed for 'fuzzier' statements. Discovery is encouraged through problem solving but formulas and algorithms are rarely summarized or formalized. I tutor several students using this text who complain of lack of examples, so that when they are away from an instructor they have very little or nothing to fall back on for explanation. Finally the subject matter covered in the text seems to be incomplete. Emphasis on linear equations and related problem solving seems to occupy most of the text, while more difficult concepts, such as working with rational polynomial expressions, are de-emphasized.

- **San Gabriel Valley** --
- Advocates for Better Education addresses the educational concerns of San Gabriel Valley parents and teachers; provides them with a forum for information, discussion, and support; and calls for action to improve education in the public schools.
- **San Juan Unified** -- The charter school that my 6th grader attends in the San Juan Unified District (Sacramento Co., CA) adopted Mathlands for its rapid learner program (self-contained GATE class). Before this they were testing mathlands and so my daughter had a "mathlands" 4th grade. She and her fellow GATE classmates are now behind in math and many are being tutored in math and spelling. I am tutoring her in math,at home, and several have moved their kids to other schools. The teachers have moved so far away from direct instruction that many parents feel their "GATE" kids are not working ahead a year but are actually behind!! What a crime to take the brightest learners and equip them with the weakest tools!!
- **San Pedro** -- My child attends 7th street elementary school in San Pedro, part of LAUSD. So far this year, the class is doing math 3 days out of 5 each week. On the fourth day they do games, puzzles, or whatever is entertaining but not math. On the 5th day the children who are passing math get to watch a video while the failing kids have to sit in the classroom and make up a weeks worth of work. This three-fifths program does not work for me. The principal defends this by saying that the school scored high on last years tests. She is neglecting the fact that "high" is really quite low on a world standard. Meanwhile I am supplementing my child's math program with Saxon Math, which is the only program that seems to really be teaching to realistic standards.
- **Santa Barbara**
  - -- We are currently publishing our first newsletter examining educational issues in Santa Barbara county. Our first issue will focus on the new new math and it's disastrous implications for our children. In addition there will be a statistical analysis of our CTBS scores. Our group is called **QeD Advocates for Excellence in the Public Schools**.
  - -- The report on Trouble in *MathLand* comes as a result of a meeting in Santa Barbara.
- **Simi Valley** -- Good News! Simi Valley Unified School District will officially offer both (traditional) Algebra 1, Geometry & Algebra 2 pathways, as well as Integrated 1, 2, & 3 at all four of our High Schools and all three of our middle schools beginning in September 1997. **CITIZENS FOR TRUTH IN EDUCATION**, 690-A Los Angeles Avenue #232, Simi Valley, CA 93065, or e-mail Citizens for Truth in Education
- **South Pasadena** - According to reports received at Mathematically Correct, the school district was considering two reform math programs for elementary school in this district. After being supplied with information from Mathematically Correct and the BOLD group from Brea, they re-opened their adoption and were able to apply for a waiver to avoid the use of Framework math texts. Congrats from Mathematically Correct.
- **Stockton** -- Stockton Unified School District has adopted Mathland as of 1996-1997 school year for K-6. The text selection committee members say it was the best of the programs offered to districts by the State, but those in high places seem unusually fanatic about forcing in the new program. Teachers are being coerced to change whether or not they had previously demonstrated successful, rigorous teaching, drawing upon a variety of proven pedagogic techniques. Senior Administrators have sent memos to staff insisting upon the implementation of Mathland. Administrators have removed old math text books so teachers could not fall back on these and would be forced into Mathland exclusively. Many teachers - especially veterans who've seen many failed fads and those with college math majors/minors or math-related college majors - have been secretly critical of the program for its downplaying of teaching standard algorithms and lack of structure, and some are trying to keep up with the basics by sending home their own supplemental practice dittos. However, administrators are forcing the issue by inspecting classrooms and focusing on Mathland in bi-annual evaluations. Standardized testing will change next year to a new program, so results of implementing this program will be obscured.
- **Torrance** -- May is the first anniversary of the **Concerned Parents of Torrance** group's first meeting. In that time we have forced the Torrance Unified School District to offer traditional Algebra, Geometry, and Algebra II classes at all four of our high schools and all the middle schools that offer Algebra classes. We elected two new School Board Members who are parents with children in the school system. These new School Board Members will not rubber stamp any curriculum the administration puts before them. We have also gotten parent involvement in the selection process for a K-8 curriculum for math. Parents have networked across the district and the information on the math materials has gotten out to parents. We continue to have watchdog groups attend all School Board meetings, all Math Steering Committee meetings, and all Parent Ad Hoc math material adoption meetings. After one year we are still meeting once a month with any interested parents to update on what the district is doing. [Note: The introduction of reform math was met with

considerable resistance from parents. They collected about 1,000 parent signatures and were thus able to get traditional math back at the high school level.]

- **Vacaville** -- We in Vacaville have been attacked by the "bug" as well. I teach mathematics ... forced to teach CPM Algebra I. The most significant thing I remember was a "newsletter" from the CPM who-ever. At the top was something to the effect: "Isn't giving group tests great? You have 28 students and have to only grade seven tests!! What will you do with your spare time?" I felt that summed up their whole approach.
- **Visalia**
  - -- Due to misgivings and misinformation, a recently held mathematics study session was merely an orchestrated "praise session" for CPM. This was not intended by either the interim Superintendent or the President of the Trustees. Therefore, they are in the process of rescheduling a new specil meeting which will be mediated in the methods used for "interest based negotiations" for union disputes. It should be interesting to say the least. It would help the cause if at least one parent other than myself showed up and spoke out.
  - -- We have one child, 1st grade, in the Visalia Unified School District, CA. Visalia District adopted the NEW MATH district K thru 12 at the start of the 95-96 school year. At our elementary school our teachers have been given the freedom to teach traditionally for 1/2 of the time, however we think this will change if parents do not stand up and present the facts about this program!
- **Vista**
  - -- My 14 old daughter is attending Vista High School in the Vista Unified School District. She was put into IMP math because she didn't take Algebra in Middle School. She won't be able to take Algebra because she is in IMP math. Comments included "this is stupid", "waste of my time", "makes no sense" etc.. These were the comments I can publish, others were more "colorful". We are trying to inform the public of this disastrous "math". It is so bad the students don't even "feel good" about it.
  - -- "Parents rebelled when they found out their textbooks were trying to make math easier to swallow by allowing group work. One fourth-grade textbook told teachers: 'Your job is ... not to judge the rightness and wrongness of each student's answer. Let those determinations come from the class ... Avoid showing any verbal or nonverbal signs of approval... '" [Investor's Business Daily]

And from out of state

- **Alaska** -- I am a teacher in Anchorage, Alaska. I am in a school which is 'piloting' MathLand. The teachers in my building have been concerned about the disjointedness of some aspects of the program as well as the lack of computational practice. There are some wonderful activities for teaching concepts, but that is not enough. After making some noises we were given supplemental dittos for computational practice, but this practice is still quite disjointed.
- **Arizona** -- Parents in Arizona are urged to contact Arizona Parents for Traditional Education. This group has lots of information for your area.
- **Florida** -- A Florida Parent Speaks about UCSMP
- **Georgia** -- In Fayette County, Georgia, a group of parents is currently forming to address the use of "Everyday Mathematics" in our schools. We are called Concerned Parents for a Better Fayette Education, and welcome e-mailed responses. There are many here who believe that our children would be better served by a return to the basics in mathematical instruction, i.e. math facts and computational skills--addition, subtraction, multiplication, and division. There are many parents here who are dissatisfied with the confusion level and lack of mastery inherent in the "Everyday Mathematics" program. The group has received many responses from parents in the community which indicate an inordinate amount of children being pulled out of the public schools, in favor of home schooling and area private schools. As a group, our first order of business will be to conduct a county wide survey to get an accurate assessment of the county's views on this math program. If there is majority opinion, we will proceed to the Board of Education with our survey results, and use this consensus to call for a change in the mathematics program. We have a county curriculum coordinator here who uses our high IOWA scores as evidence that "Everyday Mathematics" is a successful way to instruct children. However, these scores fail to consider the number of hours spent at home with parents who are reteaching math, as well as an inordinate amount of children being tutored in math, with some estimates as high as 50% and greater by the high school level. Furthermore, many students in this county have transferred here from other school systems who were not using New New Math texts, and may be skewing overall scores and actually showing a high result based on a more tradtional math program. The bottom line is that many of the parents in this group are far more concerned with their child's ability to consistently solve mathematical problems with an accurate result, than they are with test scores. Twenty years from now, which will be more important? Concerned Parents will hold their first public meeting on May 20, 1998 at the county public library in Fayetteville, Georgia, at 7:00 p.m. What started as a letter to the editor in our local newspaper, may well turn out to be the beginning of a "grass roots" movement to return the basics to public education in our county.
- **Hawaii** -- We left California because of the abysmal educational curriculum -- especially math -- but are finding the infection has spread into Hawaii also. What can teachers do when the textbooks are pure mush? My very bright 4th grader came home with a note from his math teacher, so I took a look at the work. I could not make head nor tail of it! I didn't have a clue what they wanted! And I have a MS in Mechanical Engineering from UC and have taught college level math! The questions were vague and had more to do with espousing politically correct social opinions than any academic subject. At that point I began homeschooling. sigh...
- **Illinois** -- Our classroom teacher this year asked us to allow our son to work with the school psychologist because he "values correct and

complete answers too much." When we questioned her more closely, she admitted that "feeling good about the answers given in class is sometimes more important than the right answer, even at the expense of the correct answer." Of course, we refused, but the fact remains that we are beginning to see teachers, who because of these "philosophies" of education, want to bar knowledge and academic achievement from our schools. It makes you wonder why the schools exist in the first place. I have been amazed at the number of parents in our district who share our experience. Thank you for your efforts and willingness to speak out. We can only hope that the message is heard. Our group is called **C.A.R.E. (Citizens Advocating Responsible Education)**.

- **Louisiana** -- See a new website for views from Louisiana.
- **Massachusetts**
  - -- Our schools in **Winchester** have decided to use the University of Chicago Math curriculum. I have a child in the 6th grade who has taken the field test program since 1st. They have decided to totally buy into this program and their high level math class in seventh grade will now use the same curriculum as the mid level class did the previous year. The program clearly does not challenge top level math students.
  - -- **Reading**: Concerned Parents of Reading, Massachusetts was formed by a group of parents with concerns regarding the use of the University of Chicago School Math Program (UCSMP) in the Reading Elementary School (K-4). The program seems to lack balance. Computational Skills are downgraded, testing is deemed irrelevant and students seem to spend excessive time writing about but not practicing mathematics.

    Update. The "Chicago Math" program is now used in K-5 and is expanding into the 6th grade next year. After 6 months of fighting for a standardized exam,The Stanford "9" was given in Fall of 1997 to the 5th graders. In spite of falling percentile and stanine ranks from the previous five years of Stanford 8 results (1993-1997), the school administrators ignored the results and created their own reality concerning the test scores. The "expert" from Psychological Corporation imported to discuss the results confused the school committee and parents so that they believed the students were actually doing better! He mistakenly transformed Stanford 9 to Stanford 8 data which he later admitted was undoable and incorrect. Many parents are privately tutoring their children so the decline in test scores is not as bad as it would be if tutoring was not done.
- **Michigan**
  - -- **Southern Michigan** -- Our relatively small school in southern Michigan will offer ONLY "core math" in the high school next year. Our senior daughter has escaped the curriculum ... hers is the last class to take the traditional sequence, having had algebra in 8th grade and she has just begun calculus. However, I have great concern for our 7th grader and 1st grader. The 13-year-old was VERY slow learning to read (thanks to an exclusive whole language approach), but had great arithmetic skills...until "integrated math" was introduced in 5th grade...the book was ALL reading...so he now couldn't do either (and this is supposed to INCREASE math confidence??) He is now enrolled in parochial school, where, miracle of miracles, he has learned reading, writing AND arithmetic skills!! They are doing a pre-algebra book...and he feels wonderful about himself. (Kids KNOW when a curriculum is being "dumbed down.") Unfortunately, his school only goes through 8th grade.
  - -- **Wayland Michigan** -- The Wayland Union school system is phasing in the University of Chicago School Mathematics Program and, in my opinion, with disastrous effects. Every parent that I know is upset about the program. Their children are struggling and not really learning to apply anything. Rather than mastering the basics of mathematics and building on them, they are introducing concepts that the children are just not ready to deal with. This is resulting in a lack of enthusiasm for the program on the part of the children and a growing sense that they are giving up because of the high level information that they cannot understand properly without the prerequisite basics.
- **Minnesota** -- Our group is called H.O.P.E. (Help Our Public Education), which is a citizen alliance committed to the advancement of scholastic achievement, parental involvement, and public accountability in education. We thank you for your work, energy and dedication to the best instructional practices available for students and teachers, indeed for our society.
- **Nevada** -- Great. After this idiocy fails miserably in California, it will be adopted by the schools here in Nevada. Something to look forward to.
- **New Mexico --** I am part of the **New Mexico Parents Commission** (NMPC), a coalition of parents and organizations concerned about the Education Standards being proposed in New Mexico. The proposed NM Standards advocate doing away with learning/memorizing multiplication tables and accepting as correct all answers that students can justify (such as 13 = 7). It's an up-hill battle. Parents have been completely left out of the process and are woefully uninformed. Glad I found you. People can contact us by email or write to NMPC, P.O. Box 53926, Albuquerque, NM, 87153-3926
- **New York**
  - **Upstate** -- I'm a high school math teacher in upstate New York. I have two kids, one in second grade and one will begin school in September. I am very concerned about fads and gimmicks in education, especially in English and Math. My school district began a program called "Math Their Way." Have you ever heard of it? It de-emphasizes basic skills and emphasizes group work and using manipulatives; it sounds a lot like what is going on in California.
  - **CUNY** -- Please keep up the excellent job that you are doing in disseminating information about this new-new math nonsense. This approach will merely lower standards even further (if that is possible). Parents need to be advised as to what is going on here -- and why.
- **Ohio** -- I'm a high school math teacher, and I find myself in complete agreement with your statements. I feel like a smothering cloud of doubt towards my own views on math teaching has been replaced with a breath of fresh air that clears my head, makes me think, hey,

maybe I'm NOT wrong about the USCMP books and the misuse of calculators, etc. Thanks for providing a list of obviously expert supporters.

- **Oregon**
  - -- Where I teach in Oregon we have completly destroyed a fine math program. The arguments used for change are the same arguments used across this nation. Where are the desenting voices? I am glad to finally see that there are a few of us left who feel that the mathematics we have been teaching for years is good. Yes improvement is needed but where is the evidence to show the changes made have any validity? Sadly for my students we seem to have designed a program for which no one will be prepared mathematically to do anything except offer an opion. My God don't we already have too many doing that.
  - -- Cottage Grove -- The efforts you have made in bringing better math to your schools has given some of our teachers the courage to just say no to IMP math and Applied Math. **We are getting our traditional math back** next year with IMP becoming an opt in program. Many thanks to all of you. See the Update on IMP in Cottage Grove
  - -- Portland -- Thank you for taking the time to articulate the concerns of parents, educators, and professionals regarding the state of mathematics education in America and for laying out the framework for building it back up to and perhaps beyond it's former glory. It is interesting that the principles used mid-century in this country as you noted are still valid and that those principles were in full bloom during the education of the engineers, technicians, scientists, etc., who put us on the moon and made this country number one in the world technologically. We are still riding their coattails. Ron Lermo, Boeing Commercial Airplane Group
- **U.S. Armed Forces** -- [Several overseas parents in the Department of Defense forces have written in to complain about the math programs their children receive. Here is one example.] I am the parent of a first grader in a DoD school in Japan. I really appreciate what you have done on your web page. The Mathland controversy raged for a while but has pretty much died down here, mostly because there hasn't been anything we parents could do to change the status quo. Unfortunately parents have no power in this system. The best we can do is annoy the bureaucrats while our children's brains rot. But thanks again for all the work you've done on behalf of all our children.
- **Texas** -- Everyday Math was introduced in some schools (elementary and middle) last year and in all of the elementary and middle schools districtwide this year. I teach sixth grade math and we are very frustrated by the program! There are approximately 61,000 students in our district that are being cheated out of QUALITY math instruction. By the time the students come to sixth grade, most of them are at a disadvantage already because they have not mastered the basic skills. Now, we're thrusting them into this program and they are completely lost and frustrated. The "trainers" that are supposed to be helping us through implementation are little help because they are in Chicago and their students have been in the program. We are now expected to teach 3rd, 4th, 5th, AND 6th grade work to get them up to par for seventh! Not to mention get them ready for our state exam. What a joke!
- **Vermont** -- New-new math (called Visual or Conceptual Math) here in Vermont is alive and on the march across our schools. Driven by our (appointed) State Board of Education and driven by the NCTM standards (which are never communicated to parents), it is being established as the sole path in many school districts. In our local district (Essex Junction) many parents are becoming aware of the damage this is doing to their children and are trying to get abstract or traditional math education back in the system. When we do so, we are called "threatened", "unprogressive", "cultist", and told we need to be "educated".
- **Washington**
  - -- I have a daughter in the 1st. grade in Snoqualmie Elementary School, Snoqualmie Wash. The school just implemented a new math program called "Everyday Math", which introduces calculators in the 1st. grade. I went to the last PTA meeting and expressed my concern about this and was basically told to not worry, that there was good research that this program really teaches good math. I of course don't buy that for one minute. I have a meeting with the principal this week to talk with her about this and to tell her that I think this is highly irresponsible. If there are any parents in the Snoqualmie Valley (Washington State) who are concerned about this sort of nonsense, please contact me.
  - -- We have experienced the phenomenon of new-new math in the form of "The Edmonds Math Project" in the Edmonds, WA area. No texts, no basic skills, no timetable for learning mathematics, no coordination from grade to grade...and this is in elementary school! Whenever I would ask about the situation, I was given explanations based on the latest brain research (sited in a way which was supposed to stifle discussion and for most parents succeeded in that goal). I wrote two letters in disgust to the Superintendant and finally moved out of the district to salvage our son's education. Good Luck in your work.
  - -- As a high school math teacher in the State of Washington, I am very concerned with the current dictates of what is coming out of Olympia, our state education headquarters. I have been teaching long enough to have seen the disaster of the set-based math instruction of the '60's, and feel that the New-New Math is another pendulum swing that will be equally disasterous.
  - -- Tacoma -- I am a fifth grade teacher and currently use Everday Mathematics. It is a nightmare for many reasons, but mostly because the students don't learn the correct, generalizable strategies necessary to succeed in math and when a strategy is presented there are minimal opportunities for practice and review. I appreciate your efforts to improve the quality of math instruction. Hopefully, our school districts will soon utilize valid and reliable research before adopting and implementing unproven and unsuccessful programs. Again, I thank you.
- **Wisconsin**
  - -- I, too, doubt that there are any studies that have been done on the effectiveness of Everyday Math! Our children are the guiney pigs, once again, in this system. This is only one issue in a long list. I have been horrified over the years. Trying to wake up the public is a hard task, but I can't give up.
  - -- A very active organization called **PRESS** is covering issues on reform math and other educational developments for the entire state. Contact them via e-mail at presswis@execpc.com or see the new PRESS Web site

- **England** - There have been similar silly attempts in this country to water down the content of rigorous maths and to substitute the sort of nonsense that I see being introduced in California. The Internet could be a powerful weapon in the fight back that will have to be made to restore standards. My best wishes to you in your campaign. [Editor's note: See *Tackling the Mathematics Problem* from the London Math Society]
- **England and South Africa** - I have taught Primary school in South Africa for 25 years. Last year I spent a month teaching in schools in London, England. This was my first experience with "progressive education". Quite frankly I was appaled at their low standards. There are many reasons for these low standards but an important one was that classes were composed of pupils who were grouped and worked in groups. I am used to whole class teaching with the odd bit of group work thrown in for variety. Let me tell you that whole class teaching is the only way to go because it works. It has been tested through the ages. In Britain there is a growing realisation that schools must return to whole class teaching. Unfortunately in South Africa with the new regime in power schools are being encouraged in "progressive education" in the form of Outcome Based Education. When will they ever learn!
- **Canada**
    - -- I am a parent in Toronto,Ontario. While we ,as yet,have not adopted the *New-New Math*, I feel it is only a matter of time. What we have is almost worse: we have NO program. Several years ago our school district went through a witch hunt and many schools were forced to burn- I kid you not- the older math texts that were potentially insulting to immigrants, the texts that had not enough pictures and the ones that had too many computational questions. Now no teacher in our past school has a complete set of texts because it would make the class too dependent on one way of learning! In our latest annual district report the educrats were so very proud of a grade 4 class that had taken a unit (generally about six weeks) and done a story problem based on a nursery rhyme- 31 students created half a dozen word problems in six weeks. Amazing. It makes one feel as if one is losing touch with reality, I am happy to say that after exploring your website that I am not alone.
    - -- I am a Grade 9 French Immersion student at Astral Drive Junior High School (near Halifax, Nova Scotia, Canada). Various schools in the Halifax Regional School Board have adopted the French version of the IMP Math program (IMPACTS MATHEMATIQUES) for ninth-grade French Immersion students as a pilot project. Essentially, there wasn't enough money to buy replacement texts for the old program (Actimath) and/or the old texts are no longer being published, so it was decided to go with the pilot project, which is being provided free of charge. We are not pleased with this math program, to say the least... And it looks like it may be adopted across the board, French and Core, for grades 9 through 12, unless we can stop it first.

Drop us a note to tell us your story or add your city to the list.

May 30, 2001

NYC HOLD
Honest Open Logical Debate on math reform

A Consortium of Concerned Parents, Educators, Mathematicians and Scientists

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NEW FRONT IN NYC MATH WARS

NEW YORK, NY - Mathematicians and scientists at New York University, City University, and Harvard will speak out against the controversial new math programs taught in New York City Schools, at a NYC HOLD parent hosted math forum on Wednesday, June 6th, Tishman Auditorium in Vanderbilt Hall, NYU School of Law, 40 Washington Square South, at 7pm. The forum will begin with a brief panel presentation, followed by panel-member response to parent questions and comments from the floor.

For the first time, NYC parents will meet to voice questions and concerns about their children's new math programs, asking for explanations and support from the university experts. Changes in how and what math is taught in NYC schools are part of a national reform effort which began over a decade ago. NYU mathematicians agree with math educators that improvements in how math is taught in US schools are critical, but warn the new programs lack important mathematical content and will fail to adequately prepare students for a broad range of college-level courses and majors.

NYC professors' concerns are shared by the vast majority of their colleagues across the country. Last year, over 200 of the nation's top mathematicians and scientists, including seven Nobel laureates and Fields Medal winners signed an open letter of protest, published in the Washington Post, urging US Secretary of Education Richard W. Riley to withdraw his earlier endorsement of ten of the experimental math programs.

The "new-new math", as its frequently called, echoes a previous, failed "new math" reform. The current NYC controversy is the latest struggle in the national "math wars", raging over the past ten years The programs were first tried in California in the early nineties, and dropped by the state several years later, after test scores plummeted and remedial math courses for incoming freshmen in the state university system sharply rose.

In NYC, organized parent and teacher opposition to the new math programs began in District 2, one of the city's best; and now extends to District 3, District 10 and District 15. Bronx high school teachers have organized to express opposition to next years' requirement they use only the Interactive Mathematics Project (IMP), an experimental high school math program. The teachers worry the program lacks important mathematical content necessary to prepare large numbers of their students for the Regents A exam and college level coursework.

The new programs, many without student texts, are based on a "constructivist" teaching philosophy, which discourages teachers from teaching mathematical rules and procedures. Instead, teachers guide students, through group activities, to their own "discovery" of personal solutions. Students are encouraged to seek help from each other, rather than from the teacher.

Students use pictures, beads, blocks, and coins to compute, and are discouraged from using the standard operations, such as column addition and subtraction. To measure angles, bent straws serve in place of protractors. Strips of paper, rather than rulers, are used to measure and to learn fractions. Memorization and practice are considered unnecessary; instead, students engage in activities such as skip counting, regrouping into friendly numbers, estimation exercises, games and class discussion. Knowing math facts, such as multiplication tables holds less importance.

In the later elementary grades and in middle school, as students approach algebra, "personal" strategies remain the preferred method. Students are required to keep math journals with written narratives of steps to solutions and essays on their "feelings" about math.

Parents worry the new programs do not effectively develop accuracy and fluency in mathematical procedures, developed over many hundreds of years, such as column addition, multiplication of two digit numbers, long division, the division of fractions, and procedures for solving algebraic equations.

Many opponents of the new math programs assert they ultimately fail to teach either important mathematical concepts, or precision and fluency. After a recent tour of elite District 2 middle and high school math classes, NYU Professor Alan Siegel described the new programs as "a cartoon-version of math". He has studied the recent trend in US math education reform, comparing the new programs with those used in the far better performing countries of Singapore and Japan.

Private tutors and tutoring institutions in NYC have seen a marked rise in their business since the new math programs began. Julie Tay, Director of Wossing, a tutoring school in lower Manhattan, expressed particular concern for the large new immigrant Chinese community in District 2. She sees the district's new programs failing to develop basic skills; and, together with the heavy emphasis on writing and talking about math, and the absence of textbooks, she worries, particularly compromise the chances for success among new immigrant English language learners. Immigrant parents are left unable to assist their children at home, and are reluctant or unable to explain their concerns to the school.

District 2 is the recipient of a $3.5 million grant to study three of the experimental new math programs -- Investigations in Number Data and Space (TERC), Connected Mathematics Project (CMP), and Mathematics: Modeling Our World (ARISE) -- the programs are mandated in all District 2 schools. Teachers are prohibited from supplementing with traditional materials or approaches, such as textbooks or explicit instruction in rules and procedures

Over the last several years, parents have repeatedly expressed their concern in letters and testimony to district and central board officials; only to have their concerns dismissed or ignored. At a recent District 2 school board meeting, one elementary teacher defended TERC, explaining more time was necessary to teach the "big ideas", and less time spent on basic skills. Another teacher stated teaching long division (absent from TERC and CMP) was too difficult and time consuming, and that today the skill was no longer necessary.

District 2 parent, and one forum organizer, Denise Haffenden, likens the administrators who are implementing the experimental curriculum to "medical salesman" who come into the operating room to advise doctors. "Where are their math degrees?" she asks.

Parents' concerns with the experimental nature of the reform are shared by the NYU professors. NYU research physicist Professor Bas Braams stated, "A practicing scientist might think that mathematics education reform would probably be guided by a respected body of research into what works and what does not. Sadly, this is not the way things are done.... the research is carried out by the same people that implement the reform ... This leads to the pernicious kind of advocacy research, where the people that are evaluating the outcome of a reform activity have a great deal of personal prestige and also future funding at stake in the results of their evaluation. The resulting research is so tainted from the start as to make it completely ignorable."

Last spring, NYU mathematicians and scientists began meeting with a group of concerned District 2 parents in preparation for a Community School Board 2 sponsored math forum, where community concerns were to be discussed. Since then, the school board cancelled the plans indefinitely, and District 2 officials refuse to hold any district wide forums. Parent organizers hope the June event will educate and mobilize concerned parents.

NYC HOLD members hope for the eventual development of collaboration between NYC educators and university mathematicians and scientists in the selection and development of mathematics programs, standards and assessments for city schools. Mid-summer, the NYC HOLD Web site will be launched: www.nychold.org & www.nychold,com

The forum panel will include: Professor Stanley Ocken, CCNY; Professor Emeritus Ralph Raimi, University of Rochester; Professor Wilfried Schmid, Harvard; Professor Bas Braams, Professor Charles Newman, Professor Sylvain Cappell, Professor Fred Greenleaf, Professor Jonathan Goodman, Department of Mathematics, Courant Institute, NYU; and Professor Alan Siegel, Department of Computer Science, Courant Institute, NYU.

Email contacts

Elizabeth Carson
Denise Haffenden

Content Review of CPM Mathematics

Wayne Bishop
Department of Mathematics and Computer Sciences
California State University, Los Angeles

NOTE: CPM withdrew its application to California so this report is not based on its formal submission but, instead, on the document that CPM supplies as part of the Teacher's Version entitled, "Correlation of CPM Mathematics 1, 2nd ed. (Algebra 1, v. 6.0) and the California Mathematics Standards", hereafter, "Correlation". Although Professor Bishop was a member of both the 1999 and 2001 state adoption cycle Content Review Panels, any official role as a CRP member ended with the conclusion of the 2001 cycle so this report is that of an experienced private citizen, not an official CRP review. Nonetheless, the criteria used herein were developed from the state criteria that Professor Bishop used for the official reviews of the 2001 adoption cycle. He is, however, more than happy to testify informally, by legal deposition, or in person, as to the quality and consistency of this report in comparison with those which he formally helped to prepare.

Overall Summary

With regard to mathematics content, this program does not sufficiently address the content standards and applicable evaluation criteria to be recommended for adoption.

In summary, most of the program is below the specified standards level and there is too much of an assumption that work will be done in teams. Although the publisher claims that all standards are met, several are clearly not met and several more identified herein as met are, in fact, not adequately met. Finally, there is a systemic misconception as to what is meant by logical argument in mathematics. If a statement looks to be true, students are told to put it into their "Tool Kit", then to be available in all settings of study and assessment thereafter.

Evaluation of Content Criteria

Criterion	Met Criterion
1. The content supports teaching the mathematics standards at each grade level (as detailed, discussed, and prioritized in Chapters 2 and 3 of the framework).	NO
2. Mathematical terms are defined and used appropriately, precisely, and accurately.	NO
3. Concepts and procedures are explained and are accompanied by examples to reinforce the lessons.	NO
4. Opportunities for both mental and written calculations are provided.	NO
5. Many types of problems are provided: those that help develop a concept, those that provide practice in learning a skill, those that apply previously learned concepts and skills to new situations, those that are mathematically interesting and challenging, and those that require proofs.	NO

6. Ample practice is provided with both routine calculations and more involved multi-step procedures in order to foster the automatic use of these procedures and to foster the development of mathematical understanding, which is described in Chapters 1 and 4.	NO
7. Applications of mathematics are given when appropriate, both within mathematics and to problems arising from daily life. Applications must not dictate the scope and sequence of the mathematics program and the use of brand names and logos should be avoided. When the mathematics is understood, one can teach students how to apply it.	NO
8. Selected solved examples and strategies for solving various classes of problems are provided.	NO
9. Materials must be written for individual study as well as for classroom instruction and for practice outside the classroom.	NO
10. Mathematical discussions are brought to closure. Discussion of a mathematical concept, once initiated, should be completed.	NO
11. All formulas and theorems appropriate for the grade level should be proved, and reasons should be given when an important proof is not proved.	NO
12. Topics cover broad levels of difficulty. Materials must address mathematical content from the standards well beyond a minimal level of competence.	NO
13. Attention and emphasis differ across the standards in accordance with (1) the emphasis given to standards in Chapter 3; and (2) the inherent complexity and difficulty of a given standard.	NO
14. Optional activities, advanced problems, discretionary activities, enrichment activities, and supplemental activities or examples are clearly identified and are easily accessible to teachers and students alike.	NO
15. A substantial majority of the material relates directly to the mathematics standards for each grade level, although standards from earlier grades may be reinforced. The foundation for the mastery of later standards should be built at each grade level.	NO
16. An overwhelming majority of the submission is devoted directly to mathematics. Extraneous topics that are not tied to meeting or exceeding the standards, or to the goals of the framework, are kept to a minimum; and extraneous material is not in conflict with the standards. Any non-mathematical content must be clearly relevant to mathematics. Mathematical content can include applications, worked problems, problem sets, and line drawings that represent and clarify the process of abstraction.	NO
17. Factually accurate material is provided.	YES
18. Materials drawn from other subject-matter areas are scholarly and accurate in relation to that other subject-matter area. For example, if a mathematics program includes an example related to science, the scientific references must be scholarly and accurate.	YES
19. Regular opportunities are provided for students to demonstrate mathematical reasoning. Such demonstrations may take a variety of forms, but they should always focus on logical reasoning, such as showing steps in calculations or giving oral and written explanations of how to solve a particular problem.	NO
20. Homework assignments are provided beyond grade three (they are optional prior to grade three).	NO

Notes on Individual Criteria

1. The idea of the shortfall here is explained in more detail in the Additional Comments at the end of the review. From the Correlation, a sufficient percentage of the CA Algebra Standards are addressed in some form as to ostensibly meet Criterion 1 but there is much less present than CPM indicates. In regard to some standards, more than three-fourths of the indicated citations are stretched beyond the limit of what the standards writers clearly had in mind.

2. CPM-1 is deliberately constructivist in regard to such things. It is unfortunate as well because, if a student has not accurately built his Tool Kit, there will be severe difficulties since there is no glossary or even clearly stated terms.

4. Some might disagree with this assessment because too much written work is often required. The problem is that it is often a misdirected effort that does not provide sufficient mental and written calculation of a genuinely algebraic nature - for example, a Guess and Check table of values in

word problems. More opportunity for standard "by-type" approaches is needed and more use and confirmation of standard skills such as arithmetic or rational function..

7. See #4 above. The program delights in major "daily life" problems except that the forest is lost for its trees. The entire Unit 7 "Big Race" is an example, and the introductory section of Unit 12 entitled "Problems Solving with Distance, Rate, and Time." It is almost beyond belief that students could then never have seen d = rt but it is true. The authors hold such an anti-"by type" bias that it happened; e.g., no such items are mentioned in the Assessment Handbook for either Team or Individual tests.

8. See #7 above. Another example is the absence of I = Prt; there is not so much as a mention of the terms. Similarly with the ideas of direct and inverse proportionality that the Framework deliberately discusses in Chapter 3. The self-proclaimed goal of the program is simply not met and, ironically, somewhat by design.

9. The program has such a pedagogical bias toward group work that it is not clear what, if anything, is expected of students outside of the classroom environment and includes such little direct instruction that it would be extremely difficult for a student who had to miss class to fill in the gaps. There is an accompanying Parent Guide but it is not clear that all parents would have a copy and, beyond that, it really is not much help.

11. Properties of exponents are shortchanged. The Pythagorean Theorem is just given (9: 12) when algebraic proofs are easily available, the quadratic formula is used without proof for a couple of chapters before a proof is given that only a leap of insight would call a proof.

12. Nothing close.

13. There is almost no emphasis given to standards of any kind, let alone the ideas of Chapter 3 of the Framework. For example, the second subheading is "Basic Skills for Algebra 1" and includes Standards 4.0-7.0, 9.0, and 15.0. A glance at the list in Criterion 1 shows that these basic skills are inadequately developed in CPM-1.

14. There is a great deal of irrelevancy, especially in Volume 1, but these are not supplemental.

15. The entire Volume 1, so the first half the course, would be better left in the closet. Almost anything mathematical is Grade 7, if not below, yet the time requirements are huge. For example, Unit 3:1 is a silly "Algebra Walk", literally, a human graphing exercise that is at the Grade 5 standard, AF 1.4 and 1.5, yet would take an entire class period to get organized, go outside, an conduct the exercise.

19. Regular opportunity is present but, reiterating Criterion 1, Standard 24.0 is not met. The entire course confuses heuristics and inductive reasoning, one form of mathematical reasoning, with logical argumentation.

20. It is not clear, even from the Teacher's Version that says "Homework begins here," what is to be homework and what is to be done as work in class as a team.

Standard by Standard Evaluation

Standard	Evaluation
Standard 1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable: 1.1 Students use properties of numbers to demonstrate whether assertions are true or false.	Met.
Standard 2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.	Not met, no fractional exponents and properties of exponential expressions with the same base are not confirmed until Unit 10: 40 and 43.

Standard 3.0 Students solve equations and inequalities involving absolute values.	Very weakly met, only the simplest of absolute value equations.
Standard 4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as 3(2x - 5) + 4(x - 2) = 12.	Met, but "cups and tiles" all the way through Volume 1!
Standard 5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.	Met.
Standard 6.0 Students graph a linear equation and compute the x- and y-intercepts (e.g., graph 2x + 6y = 4). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by 2x + 6y < 4).	Weakly met in 12: 99 ff but weakly assessed and not in the Two-Year Final at all.
Standard 7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.	Met.
Standard 8.0 Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.	Weakly met. 12: 110, 120 meet the perpendicular specification but they are not assessed or used regularly enough to be confirmed.
Standard 9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.	Met.
Standard 10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.	Met, but inadequate. No division of polynomials except simplification.
Standard 11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.	Met, but inadequate. Essentially no perfect square trinomials but 13: 79 hints at it.
Standard 12.0 Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.	Met, but too many are already in factored form and the skills are barely assessed.
Standard 13.0 Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.	Met, but too many are already in factored form and the skills are barely assessed.
Standard 14.0 Students solve a quadratic equation by factoring or completing the square.	Inadequate. There are quite a few by factoring so "or" is satisfied. (See #19).
Standard 15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.	Not met. There is too much of a program bias against "by type."
Standard 16.0 Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.	Inadequate.
Standard 17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.	Inadequate.
Standard 18.0 Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.	Inadequate.
Standard 19.0 Students know the quadratic formula and are familiar with its proof by completing the square.	Not met. The standard is to "know", while it is always available in the student's Tool Kit and the proof is weak since completing the square is weak and not assessed.
Standard 20.0 Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.	Weakly met and not assessed, the last topic of the course.
Standard 21.0 Students graph quadratic functions and know that their roots are the x-intercepts.	Met, but weakly, more systematic methods are needed.

Standard 22.0 Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.	Weakly met
Standard 23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.	Very weakly met. 13: 77 purports to address it but most won't see the relation.
Standard 24.0 Students use and know simple aspects of a logical argument: 24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each. 24.2 Students identify the hypothesis and conclusion in logical deduction. 24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.	Not met. The entire course confuses heuristics with logical argumentation.
Standard 25.0 Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements: 25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions. 25.2 Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step. 25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.	Weakly met.

Comments Regarding Assessment Philosophy

Clear indication of the fact that the CPM program is not serious about students meeting the CA Mathematics Content Standards for Algebra follows from looking critically at their own words about, and examples of, assessment. For example, from the CPM Assessment Handbook, Math 1, "A good first step is trying group tests. [P. 3]" "We recommend for this course that you don't give the usual type of in-process quiz that evaluates students' mastery of skills while they're still in the early stages of learning them. [P. 9]"

Why the de-emphasis on early skill mastery? It is philosophical, "When we test the students mastery of skills too early, the students focus is diverted from understanding where and how the skill is to be used."

As shown below, CPM has carried this philosophy, that does have some element of validity, beyond logic to a point that lack of mastery of skills - ever - still allows students to get good grades in the course without having developed skill mastery. For example, the model final included in the CPM Outline for a Two-Year Program has no mention of parallel or perpendicular lines, no items that require an equation given two points or given one point and the slope, one simple item that implies factoring (solve $x^2 + 6x + 8 = 0$) but no simplification of rational expressions, no mention of completing the square or quadratic formula, etc.

A strong student working at the level of the CA Grade 7 standards, i.e., a good pre-algebra background, would do very well on this final with no formal course, let alone two years of "algebra"! The proposed Team Final does address some of these in a minimal way but the two addition of rational expressions items are very easy and all but one of the expressions in the two quotient items are already in factored form, etc. Again, there is no mention, or use of, slopes in regard to parallel or perpendicular lines, completing the square, the quadratic formula, etc. So to argue, as CPM does, that the CA standards are met, just a bit delayed until mastery has had time to sink in, is nonsense. They are not assessed because they have not been mastered by sufficiently many students.

To put this in perspective, the reviewer's daughter is in a school that uses one of the California approved texts at the appropriate grade level. Three-quarters of the way through her sixth grade program she was presented the first-year final of the CPM two-year program and had no trouble setting up the percentage word problems ("43 is what percent of 125", etc.) in mathematical equation form out-loud as she read them. When she was shown the title, "Algebra - Part 1, Final Exam", she laughed and then excused herself, "I'm sorry, but it's kind of funny." Excluding the CPM specialized "Diamond Problems" that supposedly lead to eventual factoring (of which there are none, even on the Part I Team Final), she would have been able to do nearly every item correctly, and will be able to do all of them by the middle of seventh grade per the California Standards for that grade.

Indicative is the outrageous presentation of the only real word problem, "Solve the problem and write an equation. You may do this in either order. If you do not need a guess and check chart to solve the problem use it to define your variables." That is, use algebra to "solve" it or don't. A table of trials is perfectly OK half-way through algebra, in fact its inclusion is mandated even for those students perfectly capable of solving the problem entirely algebraically.

Looking more deeply at the assessment philosophy, "the emphasis should be on the mathematical thinking evident in the work and on what the student knows, not on what the student does not know.[P. 9]" Several pages of the Assessment Handbook are devoted to scoring holistically. "Holistic scoring means just writing the score by the problem 0, 1, 2, 3, or 4 and **not making corrections** on the students' papers. [P. 9, Bold is original.]" Still, the language, including that of the portfolios and the journals is sufficiently imprecise as to allow the possibility of clear, objective, individual student evaluation, "Assessment includes testing basic knowledge and skills, but it encompasses much more."

Comments Regarding Research Support

The reality of this program is that the standards are not met and genuine assessment would quickly confirm that fact. The evidence and testing, both globally for all CPM students and locally as a teacher tries to assess a student's knowledge, are entirely inadequate and the conclusions of studies as described in the Teacher's Version is not nearly as conclusive as the writers imply.

For example, the first paragraph of the page entitled "Research Summary, Comparison of CPM and Traditional Students" is in regard to use of the CSU/UC Mathematics Diagnostic Testing Program (MDTP), data from eight schools that purport to verify that students in seven of the eight learned more in CPM-1 than in their traditional counterparts. Ignoring the fact that "traditional" is not defined and the schools are not named so it is impossible to see exactly what CPM-1 was being compared against, this study did not use the MDTP. It used only 20 of the available 50 MDTP Elementary Algebra items. Two word problems specifically designed for CPM evaluation were also included in the test. Key components of a traditional Algebra I course which are largely or completely absent from CPM -1 were omitted. The following tables indicate the breakdown of the original MDTP items into its subscales as well the distribution of MDTP questions used in this study:

MDTP Subscale	# Items in MDTP	# Items in CPM Test
Linear & Quadratic Equations	13	5
Arithmetic	6	5
Geometry	7	5
Graphs	6	5
Rational Expressions	5	0
Exponents and Square Roots	6	0
Polynomials	7	0

Further down the Research Summary page is "CPM End-of-Year Assessment". The very nature of the description is indicative of the CPM approach, "we gave two questions to ..." Two questions, even with "presentations of complete solutions", is not the kind of algebra assessment most people envision when they read the subtitle.

Another measure of CPM "success" is the state SAT-9 scores, in which a page of data purports to prove that CPM schools are more successful than "their peers who use other curriculum materials," but there is so much missing as to make the data almost meaningless. As a start, the SAT-9 is not algebra! Of course, there are some exercises, ratio and proportion problems for example, that lend themselves to nice algebraic representation, but it is not algebra at the level of then President Clinton's assessment, "Algebra is algebra!"

A comparison of the California Algebra Standards Test would be useful data, but even that (which CPM chose not to publish) would be comparing CPM-1 students against a far less homogeneous group, some using an even more aggressively "reform" curriculum. Beyond that, some schools use CPM for regular classes and a more traditional program for more advanced ones. That could be taken as evidence, probably supportable, that CPM is preferable to "general math" but hardly an argument for using it in place of a traditional college preparatory curriculum as its name would imply.

Finally, CPM cannot be trusted to give us an honest picture. The "MDTP" study that they continue to use is a clear indicator of that fact with its 20 of the 50 MDTP items. Since the public lacks the names of the CPM schools in the CPM summary sheet, it is impossible to do a quick comparison of SES factors, for example, to see if most of the CPM schools in these counties might have had a head start even before any choice of mathematics curriculum.

Comments Regarding Assessments in CPM

A much clearer vision of how far short CPM falls on more traditional end-of-course assessments is contained within their own Teacher's Version and, most explicitly of all, within their "Outline for a Two-Year Program", the guide to teachers for setting up the same program but over a two-year, less-demanding schedule referred to above. This document supplements the Teacher's Version guides for constructing unit tests and tells what the designers really have in mind for verified competence. It is far off the algebra standards of California or, beyond that, of any other set of standards for algebra.

Here is one way that the assessment materials are designed to appear to be sufficiently demanding of standards-level competence when they are not. After Unit 3, the Assessment Handbook itself does not have model exams (they are in the Two-Year Program guide), but it does have item banks, by unit, along with instructions for constructing the tests themselves. Indicative in these instructions is "If you give an individual test ..." That is, even the act of having an individual, bottom-line assessment can not be taken for granted in a CPM environment. Going onward with the quote, "it would be best to make this a very short test."

Most indicative of all, however, is the test bank itself. The instructions recommend, "no more than one question of any type," which would be reasonable advice if the items in each set were, in fact, of the same type. That is obviously not the case, the sets are constructed so that it is possible to avoid confirmation of the ideas involved at the level that a cursory look could imply.

Standards Representation in CPM Test Items

The test-bank items are not numbered (so as to make it easy to omit the item entirely) but some representative examples are the following from the indicated unit with item number counting from the first item in that unit looking at every reference given for the particular standard in the Correlation for the CA Standards for Algebra I .

Standard 8 Students understand the concepts of parallel and perpendicular lines and how these slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

> Unit 7: 49, and 73 These words are not mentioned in these references, nor in the assessment-bank, Team or Individual. Ref. 7:83 looks at parallel for slope of 2/3 but not in general.
>
> Unit 11: 101 This item does have students note that the slope of one specific line is -5/3 when the original line was 3/5 and that

the lines are perpendicular, but without verification other than they look like it. Students are to "Record your observation in Your tool kit." These words are not mentioned in the Study Team Questions (Team) but in #2 of the Individual Test (Individual), parallel is to be recognized by slope, without graphing. No student constructed equations are expected except when given two points.

Unit 12: 110, 122 Both of these meet the standard for perpendicular slope. Neither parallel nor perpendicular is mentioned in either the Team or Individual test banks.

Unit 13: 28, 61, 102 Items 28 and 102 meet the standard and 61 is borderline.

Standard 11. Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Units 0-8: diamond problems 0: 3 ff, 2: 53, 78, 4:52, 6:70, 82, 96 All of these references from Volume 1 are so far off the standard as to be open to a charge of lying. For example, the last, 6: 96 consists of 11 exercises of multiplying a monomial or binomial times a binomial, not the reverse. Granted, converting expressions in factored form into un-factored form is helpful but it is does not begin to address, let alone meet, the standard.

Unit 8: 2-4, 10, 12 These are only preliminaries to factoring; e.g., 4 is only recognizing if an expression is written in any kind of factored form, 10 and 12 are tile-pushing with the factored forms already given. 8: 18 finally is an actual factoring but only with algebra tiles, 19 and 20 tie factoring into the earlier "generic rectangle" extension thereof, 31 is factoring out monomial common factors. 8: 50-51 does treat the difference of two squares and 8: 52 is ten mixed factoring problems, including one of them. 8: 57-63 meet the standards along with 70-71 and 77-80. The Unit 8 Team #1 has six expressions to be factored, including one difference of two squares and #3 uses factoring to solve quadratic equations in one variable, none of which is a difference of two squares, exactly as in the Individual, four quadratic equations to be solved, three that still need to be factored, two that are trinomial, and none that are the difference of two squares. In the Two-Year Program, that includes models of actual tests, not "select from the following," the Unit 8 Individual Test includes six factoring items, five with assistance and one stand alone. There is one difference of two squares item, but not "by type" but only by coincidence. The test includes no quadratic equations to be solved.

Unit 9: 90 does FOIL factoring (not in the CA Standards) and this is the only reference in the Correlation but, in fact, they missed some, 9: 21, 42, etc. However, the Team Questions only include two factoring exercises and one quadratic equation to be solved. The Individual test includes a choice of three or two factoring items, one of which includes a difference of two squares, and block of five equations to solve that includes one quadratic. By contrast, the Two-Year Unit 9 Mid-Unit Individual Test (there is no unit test because of the up-coming First Semester Test, contains no factoring items nor quadratic equations to be solved. The First Semester Individual Final has two, $x^2 - 7x + 12$ and $x^2 + 5x$. There are no difference of squares items and no perfect square trinomials.

Unit 10: 1-3 ff, 17 Recurring exercises do confirm the ideas. The quadratic formula is simply given in 10: 86 so it is not clear whether or not factoring will continue in solving quadratic equations. In regard to assessment, several Team and Individual items have radical expressions or decimal approximations indicating that they are not to be factored since completing the square is not introduced until three units later. Somewhat surprisingly, given the end-of-course exams, the Two-Year Unit 10 Individual Test does have several factoring problems including the advice to look for difference of two squares and a simplification of a rational expression that requires factoring both numerator and denominator (See CA Standard 11).

Unit 11-13 practice in homework 13:79 hints at perfect square trinomials but this standard is not met under almost any level of generosity. The Two-Year Part 2 end-of-course Individual Final Exam is the most indicative, here. Instead of demonstrating that students finally have mastered these ideas, there is exactly one factoring exercise, #6b) Solve: $x^2 + 6x + 8 = 0$. There are no simplification of rational expressions, let alone multiplication or division of them.

Standard 12. Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to lowest terms.

There are some, but see the last line of the Standard 11 remarks above. Since the assessment specifications allow for picking and choosing - they're deliberately not numbered - it is impossible to say to what extent the program expects individual student competence.

Standard 13. Students add, subtract multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

There are some, but see the last line of the Standard 11 remarks above. Since the assessment specifications allow for picking and choosing - they're deliberately not numbered - it is impossible to say to what extent the program expects individual student competence. Indicative of how far short the program is of the intended standard, there is one example given in the 1999 Mathematics Framework to demonstrate the intent of this standard: Solve for x and give a reason for each step:
$2 / (3x + 1) + 2 = 2/3$. There is no equation of this level of difficulty to be solved in the two volumes.

Standard 14. Students solve a quadratic equation by factoring or completing the square.

The same remarks apply; completing the square is an afterthought at the very end of the book, Unit 13: 67, and only with the CPM insistence on an overuse of so-called "algebra tiles" belying the problem with an odd or fractional middle term, and students are simply not expected to use it. In fact, the disclaimer at the beginning of the Unit 13 Individual Test admits as much, "We really do not expect many students to begin to master the topics in Unit 13. So, we provide a little extra assistance so we can still test them on these topics," followed by inclusion of the quadratic formula with no items that require completing the square, not even with an even middle term and a pile of algebra tile. The Part 2 Individual Final is the most indicative, of course, neither is ever needed. The one quadratic equation is already in standard form and factors easily.

Standard 15. Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

From the Correlation comment one would expect this standard to be well met, "Word problems and investigations are at the core of the CPM program. Students regularly solve word problems without categorizing them "by type." It is the second sentence that belies the first. Nearly all of the Unit 4-9 exercises are below the level of the algebra standards, many at the 6th grade standards, just far wordier and often with a great deal of irrelevant fluff. A good example is Unit 7, the entire "Big Race" premise. It is a complicated d = rt exercise without saying so but therefore not on standard at all, but a year or two off. Even among the last exercises that the Correlation indicates, Unit 13: 1, 12-14, 52, 62, 72, 78, 101, only 62, 78, and 101 meet the intended standard and nothing of the kind appears on the Two-Year Final. The only genuine word problem leads immediately to a pair of simultaneous linear equations. That qualifies but only as a small part of the intent of the standard. More ordinary problems are described in words and that is to be commended, but that is not an acceptable excuse for avoiding the others.

Other standards are far wide of the mark as well, and by design. CA Standard 24 is claimed to be met in the Correlation by lots of "Explain your answer..." and there is much of that in CPM-1. Most of them are not close to what the standard says and means. The books can be opened almost anywhere to see examples but using one that they chose:

Unit 10: 33 The item consist of three parts, solving a quadratic equation in standard form by factoring, graphing the corresponding function with these points as zeros and finally the supposed logical argumentation, "How are (a) and (b) related?" This not an unreasonable exercise, even good. The problem is that the informal argumentation of the exercise has nothing to do with the listed standards:

Standard 24. Students use and know simple aspects of logical argument:

24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.

24.2 Students identify the hypothesis and conclusion in logical deduction.
24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

Or another example on the same theme, 10: 96. Again, the exercise is reasonable; it is asking students to expound on the connection between the discriminant term of the quadratic formula and factorability. In fact, it would meet 24.1 if were it a bit more direct, something like, "Argue that if b^2 - 4ac is a perfect square, then ..." instead of "Explain what the values of sqrt(b^2 - 4ac) tells you about factorability of the polynomial?" Few students would recognize that this new tool constitutes proof that the polynomial can or cannot be factored; they will not distinguish it from an earlier "logical argument" 10: 74 prior to the introduction of the quadratic formula, "Is the expression $x^2 + 6x + 2$ factorable? Explain your answer." Even assuming knowledge of uniqueness of factorization (which should get acknowledged in a formal setting), lacking the rational roots theorem, the ability to complete the square that (unconscionably!) is not introduced until three chapters later, or the quadratic formula, it is not at all clear what a "good" explanation would be! "I tried everything I could think of!" perhaps?

Final Observations Regarding The Curriculum Materials

In spite of being too far off the algebra standards to warrant state approval but on the positive side, the books themselves have, or rather, Volume 2 has some distinct advantages over more traditional texts. The regular, mixed review of exercises is excellent. So is requiring students to present sensible explanations in support of their conclusions. If the authors of the Teacher Version introduction materials were to abandon their strong allegiance to what's often known as "Authentic Assessment," i.e., team tests, journals, portfolios, and individual observation and get back to lower case authentic assessment that algebra teachers everywhere have traditionally done, the program would be greatly enhanced. Yes, there should be more "by type" word problems and they should be part of the regular mixed review exercises, more formal logical argument, actual use of completion of the square in a way that students are expected to be able to use it, etc., but that is not the worst problem. The worst problem is pedagogical.

Volume 2, in the hands of a competent traditional algebra teacher, could be reasonably effective; short of the standards, but effective. If all of it, through Unit 13, is covered and if individual students do all of the exercises, they will master enough of the skills to be considered competent at the level of algebra 1, capable of going forward successfully in mathematics that depends on these concepts and skills. The problem is, and it is convicted as fatal by the included Individual Tests in the Two-Year Program guide, there is no assurance - nor any genuine effort to confirm - that this is what will happen. A little group work from time-to-time is fine, having stronger students help weaker ones master the material likewise, but that is not the described philosophy; in fact, it is completely the reverse. Similar is the work with algebra tiles. A little at the beginning? Sure, why not. Plastic toys instead of mental manipulation far into the course is very different. But, although serious, the worst parts are not the leisure introduction to factoring, that does have supportable logic, or too much graphing calculators, or being too far off the standards. They are two; one is pretending that everyone is mastering algebra while letting strong students carry weak or lazy ones, and the other is the heavy-handed, non-algebra, time-wasting in Volume 1.

The latter problem is associated with the first pedagogically and philosophically although the supporters would phrase it differently, of course. From the Introduction and Overview, "Telling has little to do with promoting learning - students must construct their own understanding." In a sense, this is true since we do not learn by "direct download" but neither do we learn much mathematics by activity-based insight. Being told wrongly by a convincing fellow student, a situation common in un-led team settings, is far worse than being told what is correct by a competent teacher. Yet, "The daily activities in this course will require much more work in study teams and much less introduction and explanation of ideas by the teacher." Much of Volume 1 actually detracts from developing algebraic competence. Almost all of the mathematical content is at the level of the Grade 7 standards or below, e.g., the equations to be solved all are, but the activities are still very time consuming and sometimes frustrating. The worst of all, however, is not teaching the power of algebra itself. Unit 4: 123 is TOOL KIT CHECK UP and it is mandated that it contain Guess and Check tables, and "cups and tiles to model solving equations." This is not algebra and it is not *college preparatory math*, no matter what it calls itself. Eventually, Volume 2 starts teaching some algebra but it is too little and too late.

Excerpts from

Poor Performance Review

by Ralph A. Raimi
Professor Emeritus of Mathematics
University of Rochester

The Washington Times
Sunday, April 1, 2001

MSPAP is an examination system that Maryland has been using for almost a decade to assess student performance. It consists of a set of "performance tasks" given all Maryland school children at the levels of grades 3, 5, and 8.

We spent days studying copies of all the exams given in the past few years, with actual student answers and scoring data selected for each file to show typical student performance - and graders' performance in assessing the quality of the answers.

The system was of no value for its announced purpose. In most subjects, the tasks did not come close to testing for knowledge of the things asked for in Maryland's own standards ... the grading was inconsistent, with correct answers often not recognized as such by some of the graders, and incorrect answers given full credit. The questions themselves were often incoherent, trivial or based on erroneous understanding of the subject matter ...

School mathematics education in the United States has recently fallen prey to some sadly ineffective and mischievous notions. To oppose them is not to advocate return to some mythical past golden age. The state of mathematics education has never been good, and reform of some sort is always in the wind. But it does not follow that those of us who as mathematicians (and citizens) find current dogma mistaken are seeking to return to some earlier error.

Maryland's MSPAP is fatally flawed, and it is time for Maryland to realize it has made a mistake, and to repair it.

A New Mission for NCTM
- Save Our Schools -

IN THE LOOMING NATIONAL ELECTION WHERE EDUCATION IS ONE OF THE MAJOR ISSUES, THE ROLE OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS MAY BE CRUCIALLY IMPORTANT. IF THE NCTM CONFIRMS THE NEW POSITIONS, TAKEN AT THEIR MEETING IN CHICAGO LAST APRIL, THEY WILL REGAIN LOST CREDIBILITY AND CONTRIBUTE SUBSTANTIALLY TO THE SURVIVAL OF PUBLIC EDUCATION IN AMERICA.

By Frank B. Allen
Professor of Mathematics Emeritus, Elmhurst College
National Advisor for Mathematically Correct
and former President of NCTM

At their meeting in Chicago on last April, the National Council of Teachers of Mathematics (NCTM) unveiled, with much fanfare, a revision of their three "Standards" reports, the first of which was published in 1989. This revision, labeled "Principles and Standards for School Mathematics", was an attempt by NCTM to allay the parental concerns and cope with the intense opposition triggered by the original reports. This attempt resulted in some startling reversals of some of the crucially important policies which had been widely applied in mathematics classrooms throughout the nation in compliance with the earlier "standards" reports. Two examples are especially noteworthy. The original reports (1) minimized the importance of correct answers and emphasized the "critical thinking" the student must do in order to get any answer, correct or not, and (2) minimized the importance of memorizing number facts and acquiring facility in performing the fundamental operations of arithmetic.

The NCTM's reversals of policy on these and other matters were widely publicized in the national press. For example, in the New York Times for April 13, 2000, under the heading Math Teachers Back Return of Education in Basic Skills, we find an article by Anemona Hartocollis, in which the following statements appear.

> In an important about-face, the nation's most influential group of mathematics teachers announced yesterday that it was recommending, in essence, that the arithmetic be put back into mathematics, urging teachers to emphasize the fundamentals of computation rather than focus on concepts and reasoning.
>
> A decade ago, the National Council of Teachers of Mathematics galvanized math educators from kindergarten through high school by preaching that it was more important for students to understand how they arrived at an answer, rather than the answer itself. In the process, they de-emphasized such basic computational skills as memorization of multiplication tables. As their proposals were put in place by hundreds of school boards, parents and even some teachers and university mathematicians began to rebel.
>
> Yesterday, after being on the defensive for years, the council issued a revision at its national convention here that critics and even some supporters of the old curriculum said was a retreat. While not abandoning its original agenda, the council added strong language to its groundbreaking 1989 standards, emphasizing accuracy, efficiency and basic skills like memorizing the multiplication tables.
>
> The message, said Joan Ferrini-Mundy, chairwoman of the committee to revise the standards, is: "Get the right answer." Releasing the revised standards at a news conference here, during a convention attended by 18,000 math teachers, Ms. Ferrini-Mundy said that students must be fluent in arithmetic computation, use efficient and accurate methods, and understand those methods. They should know their basic addition, subtraction, multiplication and division combinations as well as understand

them.

These statements clearly indicate that the NCTM is beginning to free itself from domination by the Education Establishment (EE) where, according to professors of education, all memory is "rote", all facts are "mere" and teachers are no longer expositors who explain and transmit the cultural heritage of the race to the next generation.

Our graduate schools of education, which dominate the EE, have become arcane realms where professors promulgate theories which have no foundation in either research or experience and which they themselves have never applied in the classroom. James Taub and John Leo give us a glimpse of the "thinking" that prevails there.

Taub, a New York Times writer, states: "I was invited to participate in a round-table discussion of school reform at Harvard ... The prevailing attitude around the table was that schools were far too preoccupied with *assuring academic skills*, that *we've gone berserk in this country with standardized testing*, that we need to cultivate *emotional intelligence*." (Teachers too *preoccupied* with academic skills? Is your doctor too preoccupied with healing?)

John Leo notes in the US News and World Report for 1/11/99, "A study by the Public Agenda Group found that only 7 percent of education professors think teachers should be conveyors of knowledge, 92 percent believe teachers should just *enable students to learn on their own*".

The Path to Redemption

The NCTM must not be dominated by the EE whose destructive policies include endorsement of:

1. "Inventive spelling"- the spelling equivalent of not caring about correct answers in mathematics.

2. The "whole language" approach to the teaching of reading which produces non-readers and thus heavily impairs the student's ability to learn mathematics or anything else for that matter. Further investigation led me to the astonishing discovery that the avowed purpose of the WL movement is to teach children how to be disruptive agents in our society,not how to read! The following statement appears in the paper The Political Agenda of Whole Language vs. Phonics by professor Patrick Groff (Education Consumers Network): "In the Whole Language Catalog, edited by WL co-founder Kenneth Goodman (and often elsewhere), leaders of the WL movement declare firmly that an essential purpose of reading instruction is to teach children how to instigate and carry out social class conflict."

3. "Fuzzy math," based on the old NCTM Standards which have caused the present turmoil in school mathematics and are now partially recinded.

4. "Inclusion", the "one size fits all" doctrine which destroys the carefully designed provisions for meeting the needs of all students in school populations that display wide variations in ability and training,

5. "School to Work" for ALL students which reduces the time devoted to academic subjects, and

6. Group testing and subjective, "authentic" assessment procedures that destroy the validity of course grades.

These policies, left unchecked, will destroy public education in America. They have already damaged the learning of English, Science and History (Social Studies) even more than they have damaged the learning of mathematics. Indeed, the controversy now raging in school mathematics is just one battle of the EE's many-fronted War on Learning.

I spent 35 years (1929-42 and 1946-68), teaching mathematics in the public high schools of Illinois. I very much want to see public education survive this war. It has, for many years, provided the only hope for kids who were as poor and disadvantaged as I was when I attended public schools. The only way it can survive is for the power of the EE to be broken. The only way this can happen is for some strong organization to openly rebel against EE policies. This would dispel the aura of invincibility that the EE now enjoys. It would also trigger immediate and powerful support from concerned parents, employers and college admission officials throughout the land.

I strongly urge the NCTM to take this course. To move in this direction and confirm its commitment to the redemptive course set by Ms. Ferrini-Mundy, the NCTM should publicly endorse the following ten statements each of which is completely consistent with the Council's revised position.

1. In order for our students to acquire mastery of the fundamental operations of arithmetic, as recommended by Ferrini-Mundy, which provide the indispensable foundation on which the understanding of algebra is built, we follow the lead of countries that outscored us on the objective tests administered by the Third International Math and Science Study (TIMMS) and ban the use of calculators in grades K-5.

2. Deficiencies in the mathematical preparation of teachers is, by far, our most serious problem and little improvement is possible until this disgraceful condition is remedied. ("Thirty eight percent of our high school math teachers don't have even a minor in mathematics" Gail Burrell).

3. The teaching of mathematics is a highly individualistic art and teachers must be free to use any methods that they believe will enable their students to do well on objective standardized tests. Their options should not be limited by supervisors. For example, the prescription of cooperative learning in the classroom on the grounds that it parallels the procedures employed by research teams in industry, is based on a complete misunderstanding of the latter process. Actually, the two situations are so different that no valid parallels can be drawn. The use of cooperative learning groups should be only one of many options available to teachers.

4. Properly constructed objective, standardized tests provide valid measures of student achievement in school mathematics. We must not try to avoid accountability by downgrading the objective standardized tests on which it is based. The extensive and disgraceful cheating currently elicited by these so called high stakes tests must be eliminated, not by eliminating the tests but by restoring integrity to the testing process.

5. A good assessment system must be highly objective and well understood by the students. It preserves the integrity of individual course grades and contains a strong diagnostic component which enables the teacher to adjust instruction to the needs of the class and assign individually prescribed remedial work when necessary.

6. "Critical thinking" in mathematics is a disciplined process which requires a factual base and a language which furnishes a logic-oriented vocabulary. (See Language and the Learning of Mathematics.) It is misleading to suggest that critical thinking fosters "Higher thinking skills" when the language necessary for acquiring and using those skills is NOT provided. The alleged dichotomy between "getting the right answer" and "critical thinking" is false. There is nothing about "getting the right answer" which implies any deficiency in the thinking process.

7. Parents who complain to school officials about policies applied in mathematics classrooms are entitled to factual, jargon-free replies and must never be subjected to personal attack.

8. The conditions under which mathematics teachers work must be substantially improved. They must have parental and administrative support and the respect and pay due a skilled professional who continues to study his subject after having completed a rigorous training program. They must have time during the school day to plan presentations, grade student work and consult with colleagues. This may require a drastic reallocation of resources.

9. We will apply only those research results that have real validity, can be replicated by responsible investigators and have control groups where appropriate. We will never use biased or flawed research results to justify failed policies.

10. The slogan "Drill and Kill" is replaced by the maxim that has served students well for centuries, "Practice and Learn".

If the NCTM follows this course it will again deserve to be called The National Council of TEACHERS of Mathematics.

In making the concessions noted above, the Council has taken a long step in the right direction. Now we must ask the NCTM to confirm its commitment to this redemptive course. This is urgently necessary (1) because administrators, supervisors and teachers need assurance that the NCTM is sincere in making this commitment and (2) because of the bizarre behavior of Lee V. Stiff, NCTM's newly elected president.

At the same Chicago meeting (April 12-15), where this retreat was announced, Prof. Stiff gave a strong defense of constructivism which provides the philosophical basis for the now partially repudiated false doctrines in the old NCTM Standards.. Two weeks later, he went to Los Angeles to urge the Board of Education to ignore the recommendation of its own expert panel which had recommended a strong mathematics program, and adopt one of the ten weak programs against which over 200 mathematicians had protested in a letter to the Secretary of Education. He did this on the racist grounds that Blacks and Hispanics cannot learn structured mathematics. Later in the summer, he went on a similar mission to Massachusetts where he lost. When asked about the lost generation of mathematical illiterates produced by nationwide application of now rescinded NCTM doctrines, he snapped "There was no such generation". Oh yes, there was, Prof. Stiff, although we would never have known this had we relied on NCTM's "authentic assessment". Nevertheless, we respect a man for standing up for his convictions. We must inquire about the nature and extent of the classroom experience which provides the basis for this conviction. We direct the same inquiry to each member of the writing team and to each member of NCTM's Board of Directors.

For many years this organization meant more to me than any other. I hope that I live to see the day when the NCTM is again recognized as the positive force in the teaching and learning of mathematics, which in 2000 helped us to save our schools.

Frank B. Allen

Professor of Mathematics Emeritus, Elmhurst College
and Past President (1962-64) of NCTM.

Foreword

What follows is the text of a speech presented by Frank B. Allen to the NCTM in 1988. To set this in historical perspective, it must be noted that the NCTM was then gearing up for the release of the Curriculum and Evaluation Standards for School Mathematics. It should also be noted that Professor Allen is a former president of the NCTM and is currently the National Advisor for Mathematically Correct. We are proud to present his comments here to shed light on the issues in mathematics education that are every bit as real today as they were when this speech was first given.

Paul Clopton
Mathematically Correct

Language and the Learning of Mathematics

A speech delivered at the NCTM Annual Meeting
Chicago, April 1988
by Frank B. Allen, Emeritus Professor of Mathematics
Elmhurst College

We think in terms of words. Yet today anyone who identifies lexical reasoning, based on gradually formalized natural language, as the key to the learning of mathematics, is filing a minority report. Only seven of the 515 sections in this program deal specifically with the importance of reading and writing in the study of school mathematics. The tendency to downplay this important idea has been dominant in mathematics education circles for over 20 years. Certainly there is, in current texts, much less emphasis on linguistic logic and the careful use of language in exposition, than there was in the early sixties when Max Beberman's University of Illinois program and the SMSG programs were flourishing.

A somewhat parallel situation exists in secondary school English. Nowadays, English teachers who seek to clarify meaning and facilitate intelligible communication by insisting on the enforcement of the accepted rules of grammar and syntax are fighting a losing battle and, worse, are patronized and ridiculed for being "uptight". Those of us who insist on the crucial importance of lexical reasoning and proof in the teaching and learning of secondary school mathematics are in the same boat even though this was once the dominant viewpoint.

We live in an age of ambiguity where it is far more profitable to be fluent and verbose than it is to be articulate. At such a time the learning of mathematics, which requires understanding of carefully worded expositions as well as appreciation of carefully drawn distinctions, is bound to suffer, and it is suffering. We are constantly being bombarded with evidence indicating that school mathematics in the United States operates at a level far below those found in most other industrialized nations (2) (3) (4) (5). Judging by the performance levels of the early sixties, school mathematics in the United States is in regression. Perhaps we should reassess some of the highly controversial doctrines which have, in recent years, gained a large measure of uncritical acceptance.

At any given time there are certain widely accepted viewpoints that are difficult to challenge. Unpopular ideas that question this prevailing orthodoxy are silenced or ignored even though their logic is simple and their validity clear.

In the present context the widely accepted viewpoints to which I refer are essentially these: If we emphasize the applications of school mathematics in a wide variety of problem solving situations two good things will happen; (1) the student will somehow learn the mathematics needed to solve these problems, and (2) seeing that mathematics is useful, he will be motivated to learn more mathematics. Surely this is the message of the Agenda for Action's recommendation 1.1 which recommends that the mathematics curriculum be organized, not around its own internal structure, but around problem solving. (1) To be sure this recommendation is carefully hedged and qualified so as to provide the writers with a measure of "plausible deniability". But the overall thrust toward problem solving and applications is unmistakable.

I come out of retirement to challenge both of these viewpoints -- because they run counter to my experience during 46 years in the mathematics classrooms of Illinois, first at the secondary and later at the college level -- and because I love school mathematics and still care about what is happening to it.

According to my experience, students must know the mathematics before they can apply it. Or to say it differently, they cannot apply mathematics they do not know. To expect them to learn mathematics in the process of applying it is preposterous. It is like trying to teach people to play water polo before they know how to swim.

Nor do I believe that students are necessarily motivated to study mathematics because it is useful. Most will concede that it is useful -- so are castor oil and other revolting forms of medication. Ultimate usefulness will not motivate study by a teenager -- and it should not. They subconsciously realize that the most miserable people in the world are those who are doing distasteful things just because they can make a living at it. Many of these are spending their lives as adjuncts to a computer.

The great mathematician, Marshall Stone, says, "I hold that utility alone is not a proper measure of value, and would even go so far as to say that it is, when strictly and shortsightedly applied, a dangerously false measure of value." (6)

Does the current emphasis on the application and problem solving aspects of school mathematics indicate that the utility criteria are being "strictly and shortsightedly applied"? I think it does. I think that utility is being overemphasized at the expense of certain intrinsic values of school mathematics which would serve the student better in the long run. We never hear anymore about the beauty of mathematics or about its structure and internal consistency -- or about mathematics as an ideal arena for the application of logic to the thinking process -- or indeed about any of the cultural values of mathematics that have been cherished by the race for generations.

All this is ignored by the sweeping recommendation that the mathematics curriculum should be organized around problem solving. In fact, this recommendation almost denies that school mathematics exists as a separate entity apart from its applications. It implies, moreover, that mathematics in and of itself is pretty dismal stuff which can interest no one unless it is attached to some supposedly interesting situation in the "real world". If we make this attachment maybe some of the interest derived from these external situations will "rub off" on the mathematics and thus render it more palatable to the student. This, I submit, is an example of the "strict and shortsighted application" of the utility criteria which Professor Stone deplored. It is moreover a bizarre position for an organization of mathematics teachers to take.

It is strange, indeed, when those of us who profess to be fascinated by the values that inhere in the subject matter of school mathematics find ourselves in the minority and on the defensive in a convocation of mathematics teachers.

You might as well know that was and still is my condition. I loved the structure of algebra and the classical theorems of geometry. My enthusiasm for these great ideas never flagged. It never occurred to me to be bored with them as I taught them to class after class over the years -- any more than it would occur to a musician to become bored with the great masterpieces of Brahms, Beethoven, and Bach.

Look back with me to what attracted us to mathematics in the first place. Was it not its structured character - - its internal consistency as a body of subject matter where answers can be checked and where results (theorems) can be established by applying the rules of logic without appeals to feelings, authority, or faith?

If we found these qualities to be attractive, challenging, and motivational why wouldn't our students find them so? I think they would -- if we still felt that way. Do we? Or are we overwhelmed by the prevailing doctrines that sweep all this aside?

If your enthusiasm for the inherent qualities of mathematics has withered under the flood of dogma emanating from the AASA and the ASCD (Association for Supervision and Curriculum Development) -- and, oh yes, the NCTM, let me try to rekindle it. These inherent qualities can be appreciated only when they are understood.

This brings me to my major thesis that natural language, gradually expanded to include symbolism and logic, is the key to both the learning of mathematics and its effective application to problem situations. And above all, the use of appropriate language is the key to making mathematics intelligible. Indeed, in a very real sense, mathematics is a language. Proficiency in this language can be acquired only by long and carefully supervised experience in using it in situations involving argument and proof.

Due to the current overemphasis on problem solving and applications, the student of school mathematics does not get nearly enough experience with the various aspects of proof.

There also seems to be a widespread belief that even the most elementary fundamentals of logic needed for mathematical discussion are too difficult for secondary school students. (Actually it is their omission that renders mathematics unintelligible.) This belief may stem from the fact that the abrupt introduction of proof in tenth grade geometry, without the language needed to render it understandable has led to frustrating and even traumatic experiences for both students and teachers.

Whatever the reason, proof in the applications-problem centered domain of school mathematics is postponed - - suppressed - - downgraded. (We won't have any proof questions on the exam, will we?) There is even strong support for the idea that we should not presume to do much with proof at the secondary level. In Professor Usiskin's new U of C math series, formal proof does not appear until the second semester of the tenth grade. This will sell a lot of books. And the progressive emasculation of American texts in school mathematics, which began in the late sixties, will continue apace! (Algebra without structure -- geometry with little or no proof!)

I ask you this question: Can we suppress proof without distorting school mathematics and seriously impairing the students ability to understand it? Mathematics is essentially a structured hierarchy of propositions forged by logic on a postulational base. For how long do we protect our students from the pedagogical consequences of this fact? And indeed, whom are we protecting? Those who advocate this language of logical discourse are seeking clarity of exposition rather than pretentious rigor in proof.

Also, due to our preoccupation with applications, there is not nearly enough time spent in deriving key propositions and theorems. When you and I say that we understand the theorem "the determinant of a square matrix is zero if and only if its rows are linearly dependent", we are saying that we understand how this theorem fits into a hierarchy of propositions -- and could, given time, derive it from first principles.

Why should our students be any different? To be sure, they are working at a more elementary level. But this nagging question remains: What basis do they have for understanding anything without seeing how it fits into a structure based on something?

Yet when high school students say they understand the formula for the cosine of a difference, they generally mean that they have memorized the formula and can apply it. They generally do not mean that they can derive it from first principles. Very often they do not realize that this essential dimension of understanding even exists; i.e., their understanding is deficient. So they get by on memory and facility until the cumulative effects of these deficiencies ultimately overwhelm them, and they leave mathematics in frustration and despair.

We can prevent this by equipping the student with the essentials of the language needed to understand mathematical reasoning. What are they? My answers are contained in the handout "The Language of Mathematical Exposition" which is summarized below. These are the distillation of the last 25 of my 46 years as a teacher of school and college mathematics during which time I literally lived with them and applied them daily in the classroom. They were the salient characteristics of three textbooks I co-authored which were published in 1964, 1966 and 1973 respectively.

> **Summary of the Language of Mathematical Exposition:** Set language including set builder notation. Quantification, including "Some", "All", "None" and "there exists". Negation and contradiction. Conditions under which conjunctions, disjunctions, and implications are true or false. Equivalent statements. The statement "P implies Q" is equivalent to its contrapositive "Not Q implies not P", whereas its converse "Q implies P" is only a conjecture whose truth value (T or F) must be investigated. A theorem having *n* premises in its hypothesis and a single conclusion has *n* (partial) contrapositives, each of which is obtained by exchanging a contradiction of the conclusion with a contradiction of one of the *n* premises. Each of these contrapositives is equivalent to the theorem. This theorem also has *n* (partial) converses, each of which is obtained by exchanging the conclusion with one of the *n* premises. Each of these converses is a conjecture whose truth value must be determined. Tautologies. Proof patterns. Valid arguments. Proof.[(1)]

I will spend little time on this list which is already well-known to you. Indeed, that may be one of our problems. As math majors, we know these so well that we tend to assume that our students know them too. I respectfully suggest that this assumption is unwarranted. Our students do not know them, and they will not unless they are given the necessary instruction. It is my position that understanding of this Language of Mathematical Exposition is absolutely essential for the successful study of school mathematics -- or any other subject where deductive reasoning is employed.

Understanding this language and facility in its use emerges over a 4 to 6 year period. It should not be abruptly introduced in the middle of 10th grade geometry. Some of it should be introduced in grades 6, 7 and 8, where very little is going on now. (In fact in some series the amount of duplication between texts for these grade levels is so great that it is actually difficult to arrange them in the proper order.)

I take issue with the idea that a problem has to be "practical" or look practical in order to be motivating and worthwhile. This idea, when taken as the basis for extensive classroom activities, is often found to be misleading, pretentious, and generally counter-productive. It is also time-consuming and often leaves little measurable residue in terms of the students understanding of basic ideas. Moreover it is simply not true. Some practical problems are a bore; some fanciful problems are not only more fun but are far more instructive.

There are those who believe that the most important use of problem situations in school mathematics is to ensure that the students can employ the formalized language of mathematics to demonstrate their understanding of the underlying theory. It is this ability and this understanding that will enable them to succeed in more advanced courses in mathematics and science. It is this ability and this understanding that will confer upon them the power to formulate and solve the problems they will encounter in these encounter advanced courses and in the real world.

There are essentially three proof formats available to us: essay, ledger (two column) and flow. Beginning students have great difficulty with the essay form and, worse, I have great difficulty analyzing their essay proofs in ways that will be helpful to them. For this reason it is generally agreed that, for the beginners, we need some kind of a pattern which will enable us to point out such errors as omissions, unwarranted assumptions and non-sequiturs. This accounts for the prevalence of ledger proofs in our geometry texts. But, as you supposed, I believe that the flow proof is far superior to the ledger proof in the all important matter of delineating the structure and thrust of a mathematical argument.

I also believe that the study of proof should begin in algebra rather than in geometry where troublesome incidence relations raise their ugly heads. (How much can we assume from the drawing?)

I conclude my remarks on flow proofs with some caveats and observations.

Don't confront the class with a completed flow proof except where you:

(1) ask for reasons; (2) ask for scrutiny to find mistakes; (3) ask for translation to the essay form. Otherwise build the proof with them in class. They will gradually learn with you to work forward from the premises (the search for necessary conclusions) and backward from the conclusion (the search for sufficient conditions) until a seamless sequence of implications extends from the hypothesis to the conclusion. This can be an exciting and challenging task - - but it is not easy. But to paraphrase the great teacher, R. H. Moore, "Mathematics properly taught is difficult." Flow proofs plow up unexpected difficulties. These difficult situations where misunderstandings lurk were there all the time. The flow proof merely exposes them so that they can be resolved.

I do not expect my students to go through life using flow proofs any more than English teachers expect their students to go through life diagraming sentences. My ultimate objective is to develop the students' ability to read and write essay proofs of the kind they will encounter in more advanced courses in mathematics.

Now that I have presented my language of exposition as essential to reasoning and proof in school mathematics, let us consider the consequences of neglecting lexical reasoning and proof.

These are catastrophic: alienation of our students (later the public); isolation of our subject; and failure to prepare our students for advanced study.

Due to our reluctance to use a handful of universally valid and easily understood logical principles, we are denying our students the opportunity to understand.

For too long have we sheltered our students from explanations deemed to be too difficult for them. When explanations are inadequate or missing entirely, students attribute their resulting lack of understanding to the idea that there is something abstruse and forbidding about mathematics which they will never be able to understand. The result may be permanent alienation from both mathematics and the mathematical sciences. This alienation not only cripples thousands of students, it cripples our economy and our defense posture as well.

Examples of public alienation to mathematics abound. It is the only subject I know where intelligent people will openly flaunt their ignorance. "So you're a math professor" they say; and then almost inevitably I wince when they add, "Well, I never was very good at mathematics."

The public's misapprehension of the nature of mathematics would be amusing if it were not so tragic. They think that we have a curious predilection for carrying out complicated numerical algorithms, and they are pretty sure that the advent of the computer has rendered us obsolete. In bridge playing groups they say, "You're the mathematician, you can keep score." This is like saying, "You are the archeologist. You can dig the post hole." We've all had to suffer through stories about absent-minded and wacky mathematicians -- stories whose impact is that to do the kind of reasoning required in mathematics, you don't have to be insulated from reality -- but it helps. My only weak rejoinder is that we don't know how flaky these characters would have been if they had not studied mathematics.

All of this indicates that mathematics is the object of widespread misunderstanding and hostility on the part of the public. Our PR is PU. [Dysmetria means aversion to or fear of mathematics. We have all seen evidence indicating that many high school students suffer from Acquired Incurable Dysmetria Syndrome.]

The notion, already too widely held, that one must have a "mathematical mind" in order to deal with the peculiar thinking required in mathematics has served to isolate our subject from the real world to an extent that we cannot possibly counteract by our current determined (and somewhat trendy) overemphasis on applications and problem solving. This "mathematical mind" syndrome has hurt us enormously. It leads to the mistaken notion that mathematics is easy for those with this mysterious native ability and beyond the reach of those who lack it, no matter how hard they work.

Using this language of exposition to develop an interplay between logic and mathematics, as suggested here, will clearly indicate to the student that the same logical principles apply in every field; that there is nothing esoteric or arcane about the thinking required in mathematics.

Another dire consequence of our downgrading of proof is that it distorts our subject in such a way that the early courses, as currently presented, give the student no idea of what mathematics is really like.

Problem solving, important as it is, does not provide adequate preparation. Many students who enjoy working the problems in high school math and even in calculus, and hence decide that they like mathematics and want to major in it, find that the nature of the subject changes abruptly when they encounter the proof courses which follow calculus. They are bewildered and dismayed in courses such as Abstract Algebra (Algebra Structures), Linear Algebra, Advanced Calculus, and Topology where proof is the name of the game. Their previous problem solving courses did not prepare them for this abrupt change in emphasis. Their rude awakening often comes so late that it is difficult to change majors. This is tragic. Is it not our function to prepare students to deal with proof at the college level?

My own experience as Professor of Math at Elmhurst College abundantly confirms the fact that the student's first collision with proof can be traumatic.

Our math majors were strong students who were required to complete four years beyond calculus from these one semester courses:

Algebraic Structures	Linear Algebra
Advanced Calculus	Differential Equations I
Differential Equations II	Math Statistics I
Math Statistics II	Topology
Fundamentals of Geometry	Complex Variables

[This sequence is about the same as Potsdam, NY, of which more later.]

Since they had virtually no experience with proof, some had great difficulty when they encountered Linear Algebra. The combination of new subject matter and the proof requirement staggered them. Since I had the same experience myself, I was sympathetic and by extra effort was able to help them pass the "proof barrier". I occasionally resorted to flow proofs. I am happy to say that most of them went on to become strong majors. But this is often not the case.

We must face up to the fact that serious deficiencies in the area of lexical reasoning are causing a general lowering of the levels of math competency among beginning college students. The remedial (precalculus) mathematics that must be offered in college now accounts for almost two-thirds of the college math enrollment in the United States (7). For emphasis, I reiterate that many potential math and science majors flounder in the higher level math classes because of their continuing inability to think in terms of the language of mathematics. Recognizing this, some colleges have tried to devise remedial courses dealing with the basic techniques of mathematical proof. Having worked on developing such a course for about eight years, Professor Susanna Epp of DePaul University (Chicago) has this to say. "During this period I came to realize that many of my students difficulties were much more profound than I had anticipated. Quite simply, my students and I spoke different languages. ...Very few of my students had an intuitive feel for the equivalence between a statement and its contrapositive or realized that a statement can be true and its converse false. Most students did not understand what it means for an if-then statement to be false, and many also were inconsistent about taking negations of "and" and "or" statements." (8) She goes on to list other deficiencies of the type which could and should have been remedied in high school by teaching the material on the handout.

Let us, therefore, resolve to realign our efforts in the teaching of school mathematics, even though this realignment is at variance with the recommendations of prestigious committees. We must not dissipate our energies in dealing with a bewildering array of specific problems. This prepares the student, at best, to deal with similar problems. No. We must focus on building proficiency in the formalized natural language required to apply the thinking process in mathematics. Only then can we open the channels of communication which will enable our students to profit by more advanced instruction. Only then can we prepare our students to cope effectively with problems which are today unforeseen and unforeseeable.

Now let me present an Agenda for Understanding which is intended for college capable(intending?) students of secondary school mathematics (grades 6-12).

1. The school mathematics curriculum must be organized around the internal structure of school mathematics. The idea that mathematics is a hierarchy of propositions forged by logic on a postulational base should begin to form in the student's mind about grade 9 and should be thoroughly established by grade 12. Then our students will know what mathematics is really like and they will be in a position to decide whether or not they want to continue. We will have been honest with them.

Will students be repelled by what may appear to some to be a rather austere and abstract portrayal of mathematics? No. On the contrary. The able ones are more likely to be repelled by a formless, flaccid curriculum in which no structure is discernible and which, as a consequence, provides no basis for understanding anything.

2. The teaching of mathematics should be regarded initially as an extension of the teaching of language. Our efforts to develop an awareness of the intimate relationship which exists between grammar, mathematics and logic should begin with games of " How do we know?" in the early grades (12) continue with the introduction of formal proof not later than grade 9 (perhaps with the aid of flow-diagrams) and culminate in the ability to read and write lucid essay proofs by grade 12.

3. We must try to take a more balanced view of the role of problem solving in school mathematics lest our preoccupation with it causes us to neglect the very mathematics that makes problem solving possible. On Page 153 of the NCTM publication "Curriculum Evaluation and Standards for School Mathematics" (9), now in draft form, the following edifying statement appears, "Students success in mathematical problem solving requires knowledge of mathematics." I was relieved to see this. Reading current literature I had begun to believe that all one needed to do was follow Polya's four problem solving stages, which as Jeremy Kilpatrick notes (10) have recently been rediscovered by trendy mathematics educators, most of whom are not problem solvers. At least I do not see them listed as either solvers or proposers in the problem sections of any of the several mathematical journals that I peruse each month. Sometimes I am almost overwhelmed by the amount of mathematics one must know in order to cope with these problems. If we do not know the definitions and theorems invoved in a given problem, Polya's four stages will not help us find a solution.

Don't misunderstand. I am not opposed to problem solving. I had large collections of problems that I used in the classroom. "Problems of rare charm and distinction" I called them. These, I thought, were challenging, ingenious, inherently fascinating and instructive -- and best of all I could work them. Such problems are the life blood of mathematics. But let us not fail to convey to our students that the body of mathematics is given structure and coherence by the bones and sinews supplied by definitions, postulates and proof.

We are mature, experienced and mathematically trained professionals and it is time for us to stop overreacting to the plaintive question "What good is all this?" Experience has shown that students lose interest in this question when they understand what is going on. The antidote to shortsighted, pragmatic demands for immediate utility is understanding of the kind that can be imparted by the linguistic approach here advocated -- not a fragmented curriculum organized around problem solving.

4. We must restore the emphasis on proof which was once a major feature of the college preparatory sequence. Formal proof, founded on this language of mathematical exposition, should begin in 9th grade algebra and should be the dominant theme in 10th grade geometry -- and thereafter. I do not object to an electronically activated, laboratory approach to the informal geometry of grades 6-8. But such experimental methods must not be allowed to supercede proof in the geometry of grade 10. Proof must remain the central theme of this geometry, as it has been for centuries. We must somehow cope with this surging aversion to proof which not only drastically reduces the proof content of geometry, but actually jeopardizes its position in the curriculum. Geometry at this level is not a laboratory subject.

Geometry with proof is the keystone of the secondary mathematics sequence. Geometry without a strong emphasis on proof offers little more than an unstructured collection of geomerric facts including many the student has already encountered in junior high school. Such a course is not worth the student's time.

Will not all this emphasis on proof make learning more difficult for the student? You bet. But education results from overcoming difficulties not from evading them. Urge your students to climb the mountain. Don't pretend that there is no mountain to climb. Besides, adolescents are not repelled by difficulty. Our students will find proof to be challenging, rewarding and even exciting provided we find it so.

5. It is time for us to reconsider our policies with respect to a national curriculum and nationwide testing. For a language to be effective it must be well-nigh universally applied. Accordingly, there is an urgent need for a mathematics sequence based on lexical reasoning, which can be (a) used universally for college capable students in grades 6-12 and (b) validated by a nationwide testing program. Not all college capable students would complete this sequence but all would take four years of mathematics in high school. For the best students the 12th grade course would be calculus, and the valuable parts of so-called discrete mathematics, implemented by matrices, vectors and computer techniques, would be in the algebra courses which precede calculus.

Our failure to provide such a program for college capable students makes us unique among the industrialized nations of the world and undoubtedly accounts for many of our failures, including the fact that foreign-born students are dominating our graduate level programs in mathematics and engineering (13).

Recommendation 6 in the Agenda for Action should not apply to college capable students. These students most emphatically do not need "A flexible curriculum with a greater range of options --- designed to accommodate their diverse needs." Diversity will come soon enough. In secondary school they need a well-defined program which develops their ability to read, write and think in the formalized, symbolic language of mathematics.

The benefits of such a program would be enormous. College instructors would know what to expect from students, depending on how far they had progressed in the sequence. But most important, it would clear the language channels that must be open if students are to learn or apply mathematics at the college level.

As we convene here in Chicago today a wave of dissatisfaction with education is sweeping over our country -- and this dissatisfaction applies with special force to school mathematics. Are we going to ignore this -- arrogantly continuing with policies that have been thoroughly discredited? I prefer to take this as a warning.

Because of some ill-considered turns we have made we are now on a collision course with disaster. It is time for us to get back on course. Let us strive for renewed confidence in the inherent values of our subject and in ourselves as expositors. We should be encouraged by the experience at Potsdam College, Potsdam, NY, where a traditional pure mathematics department contributes 20% of each graduating class as mathematics majors (about 200). (11) Recruiters from industry are literally standing in line to hire these math majors who they are convinced, have learned how to think in the language of mathematics.

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(1) The Agenda for Action - - Appendix to Changing School Mathematics - - A Responsive Process, National Council of Teachers of Mathematics, 1906 Association Drive, Reston, Virginia 22091, Copyright 1981.

(2) Steen, Lynn Arthur, <u>Smokestack Classrooms</u>, FOCUS (Newsletter of the Mathematical Association of America), March-April 1987.

(3) The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective. Curtis C. McKnight, et al.

Stipes Publications, 10-12 Chester Street, Champaign, IL 61820, 1987.

(4) Beck, Joan, "What can we learn from the successes of Japanese kids?", Chicago Tribune, February 9, 1987.

(5) "Math Instruction Doesn't Make the Grade", Sheldon L. Glashow, Virginia Pilot, April 7, 1986.

(6) Stone, M. H., "Mathematics and the Future of Science", Bulletin of the American Mathematical Society, Vol. 63, No. 2, March 1957, pp. 61-76.

(7) Steen, Lynn Arthur, Undergraduate Mathematics in China, FOCUS (Newsletter of the Mathematical Association of America), September-October, 1983.

(8) Epp, Susanna, The Logic of Teaching Calculus, Paper written for the Tulane/Sloan Foundation Conference on Calculus, 1986, subsequently published (1987) in Toward a Lean and Lively Calculus, Ronald G. Douglas, Editor, MAA 1529 Eighteenth St., NW Washington, D.C. 20036

(9) "Curriculum and Evaluation Standards for School Mathematics" -- Romberg et al -- NCTM 1987.

(10) Kilpatrick, Jeremy, "George Polya's Influence on Mathematics Education." Mathematics Magazine, Vol. 60, No. 5, December 1987. pp. 299-300.

(11) Poland, John, "A Modern Fairy Tale?" The American Mathematical Monthly1 March 1987, pp. 291-95.

(12) Lipman, Sharp, Oscanyan, "Philosophy of the Classroom", Universal Diversified Services, West Caldwell, NY, 07006, 1977.

(13) Study cited in an editorial in Chicago Tribune for February 17, 1988.

Note 1. A full version of "The language of Exposition" is available to anyone having access to Acrobat Reader by sending an E-Mail to franka@elmhurst.edu

Excerpts from the Wall Street Journal Editiorial of January 4, 2000

Math Wars

Reinventing math is an old tradition in this country. It has been around at least since the 1960s, when the inimitable Tom Lehrer mocked the New Math in Berkeley cafes.

Today the original New Math is old hat, but many folks in the education world are hawking yet another reform. It is known by names like "Connected Math," or "Everyday Math."

. . . today's New Math has powerful allies. Education Secretary Richard Riley and other Clintonites smile on it. Eight of the 10 curriculums recently recommended for nationwide use by an influential Education Department panel teach the New New Math.

Not that all members of the Academy are joining the movement. Within weeks of the Education Department findings, 200 mathematicians and scientists, including four Nobel Prize recipients and two winners of a prestigious math prize, the Fields Medal, published a letter in the Washington Post deploring the reforms.

... programs of the sort picked by the federal panel turn out to be horrifyingly short on basics.

Consider MathLand ... [It] does not teach standard arithmetic operations. No carrying and borrowing at the blackboard here.

MathLand also does away with textbooks--too hierarchical, we suppose. No chance therefore for anything as sane as systematic review.

Next comes Connected Math, another panel favorite. It too skips or glosses over crucial skills. Example: The division of fractions, an immutable prerequisite for algebra, is absent from its middle-school curriculum. In shutting the door to algebra, David Klein of Cal State Northridge points out ...

Everyday Math ensures juvenile dependency to calculators by endorsing their use from kindergarten. Rather than teach long division, the program devotes substantial time to that important area of math study, self-esteem.

And then move on to the main question: Why? The reason for the New New Math, as for many other curriculum reforms, is that teachers, school administrators and their unions are tired of

being blamed for statistical declines and poor student performances.

New Mathie and federal panel member Steven Leinwand explains: "It's time to recognize that, for many students, real mathematical power, on the one hand, and facility with multidigit, pencil-and-paper computational algorithms, on the other, are mutually exclusive." Or, as Professor Klein translates: "Underlying their programs is an assumption that minorities and women are too dumb to learn real mathematics."

New Math will take its casualties, especially among the poor, adding to the already mounting costs of the decline in national educational standards.

See the full article .

AN OPEN LETTER TO UNITED STATES SECRETARY OF EDUCATION, RICHARD RILEY

Dear Secretary Riley:

In early October of 1999, the United States Department of Education endorsed ten K-12 mathematics programs by describing them as "*exemplary*" or "*promising*." There are five programs in each category. The "*exemplary*" programs announced by the Department of Education are:

> Cognitive Tutor Algebra
> College Preparatory Mathematics (CPM)
> Connected Mathematics Program (CMP)
> Core-Plus Mathematics Project
> Interactive Mathematics Program (IMP)

The "*promising*" programs are:

> Everyday Mathematics
> MathLand
> Middle-school Mathematics through Applications Project (MMAP)
> Number Power
> The University of Chicago School Mathematics Project (UCSMP)

These mathematics programs are listed and described on the government web site: http://www.enc.org/ed/exemplary/

The Expert Panel that made the final decisions did not include active research mathematicians. Expert Panel members originally included former NSF Assistant Director, Luther Williams, and former President of the National Council of Teachers of Mathematics, Jack Price. A list of current Expert Panel members is given at: http://www.ed.gov/offices/OERI/ORAD/KAD/expert_panel/mathmemb.html

It is not likely that the mainstream views of practicing mathematicians and scientists were shared by those who designed the criteria for selection of "*exemplary*" and "*promising*" mathematics curricula. For example, the strong views about arithmetic algorithms expressed by one of the Expert Panel members, Steven Leinwand, are not widely held within the mathematics and scientific communities. In an article entitled, "*It's Time To Abandon Computational Algorithms*," published February 9, 1994, in Education Week on the Web, he wrote:

> "*It's time to recognize that, for many students, real mathematical power, on the one hand, and facility with multidigit, pencil-and-paper computational algorithms, on the other, are mutually exclusive. In fact, it's time to acknowledge that continuing to teach these skills to our students is not only unnecessary, but counterproductive and downright dangerous.*" (http://www.edweek.org/ew/1994/20lein.h13)

In sharp contrast, a committee of the American Mathematical Society (AMS), formed for the purpose of representing the views of the AMS to the National Council of Teachers of Mathematics, published a report which stressed the mathematical significance of the arithmetic algorithms, as well as addressing other mathematical issues. This report, published in the February 1998 issue of the Notices of the American Mathematical Society, includes the statement:

> "*We would like to emphasize that the standard algorithms of arithmetic are more than just 'ways to get the answer' -- that is, they have theoretical as well as practical significance. For one thing, all the algorithms of arithmetic are preparatory for algebra, since there are (again, not by accident, but by virtue of the construction of the decimal system) strong analogies*

between arithmetic of ordinary numbers and arithmetic of polynomials."

Even before the endorsements by the Department of Education were announced, mathematicians and scientists from leading universities had already expressed opposition to several of the programs listed above and had pointed out serious mathematical shortcomings in them. The following criticisms, while not exhaustive, illustrate the level of opposition to the Department of Education's recommended mathematics programs by respected scholars:

> Richard Askey, John Bascom Professor of Mathematics at the University of Wisconsin at Madison and a member of the National Academy of Sciences, pointed out in his paper, "*Good Intentions are not Enough*" that the grade 6-8 mathematics curriculum Connected Mathematics Programentirely omits the important topic of division of fractions. Professor Askey's paper was presented at the "Conference on Curriculum Wars: Alternative Approaches to Reading and Mathematics" held at Harvard University October 21 and 22, 1999. His paper also identifies other serious mathematical deficiencies of CMP.
>
> R. James Milgram, professor of mathematics at Stanford University, is the author of "*An Evaluation of CMP*," "*A Preliminary Analysis of SAT-I Mathematics Data for IMP Schools in California*," and "*Outcomes Analysis for Core Plus Students at Andover High School: One Year Later*." This latter paper is based on a statistical survey undertaken by Gregory Bachelis, professor of mathematics at Wayne State University. Each of these papers identifies serious shortcomings in the mathematics programs: CMP, Core-Plus, and IMP. Professor Milgram's papers are posted at: ftp://math.stanford.edu/pub/papers/milgram/
>
> Martin Scharlemann, while chairman of the Department of Mathematics at the University of California at Santa Barbara, wrote an open letter deeply critical of the K-6 curriculum MathLand, identified as "*promising*" by the U. S. Department of Education. In his letter, Professor Scharlemann explains that the standard multiplication algorithm for numbers is not explained in MathLand. Specifically he states, "*Astonishing but true -- MathLand does not even mention to its students the standard method of doing multiplication.*" The letter is posted at: http://mathematicallycorrect.com/ml1.htm
>
> Betty Tsang, research physicist at Michigan State University, has posted detailed criticisms of the Connected Mathematics Project on her web site at: http://www.nscl.msu.edu/~tsang/CMP/cmp.html
>
> Hung-Hsi Wu, professor of mathematics at the University of California at Berkeley, has written a general critique of these recent curricula ("*The mathematics education reform: Why you should be concerned and what you can do*", American Mathematical Monthly 104(1997), 946-954) and a detailed review of one of the "*exemplary*" curricula, IMP ("*Review of Interactive Mathematics Program (IMP) at Berkeley High School*", http://www.math.berkeley.edu/~wu). He is concerned about the general lack of careful attention to mathematical substance in the newer offerings.

While we do not necessarily agree with each of the criticisms of the programs described above, given the serious nature of these criticisms by credible scholars, we believe that it is premature for the United States Government to recommend these ten mathematics programs to schools throughout the nation. We respectfully urge you to withdraw the entire list of "*exemplary*" and "*promising*" mathematics curricula, for further consideration, and to announce that withdrawal to the public. We further urge you to include well-respected mathematicians in any future evaluation of mathematics curricula conducted by the U.S. Department of Education. Until such a review has been made, we recommend that school districts not take the words "*exemplary*" and "*promising*" in their dictionary meanings, and exercise caution in choosing mathematics programs.

Sincerely,

David Klein
Professor of Mathematics
California State University, Northridge

Richard Askey
John Bascom Professor of Mathematics
University of Wisconsin at Madison

R. James Milgram

Professor of Mathematics
Stanford University

Hung-Hsi Wu
Professor of Mathematics
University of California, Berkeley

Martin Scharlemann
Professor of Mathematics
University of California, Santa Barbara

Professor Betty Tsang
National Superconducting Cyclotron Laboratory
Michigan State University

The following endorsements are listed in alphabetical order.

William W. Adams
Professor of Mathematics
University of Maryland, College Park

Alejandro Adem
Professor & Chair
Department of Mathematics
University of Wisconsin-Madison

Max K. Agoston
Associate Professor
Department of Mathematics and Computer Science
San Jose State University

Henry L. Alder
Professor of Mathematics
University of California, Davis
Former member of the California Board of Education
Former President of the Mathematical Association of America

Kenneth Alexander
Professor of Mathematics
University of Southern California

Frank B. Allen
Professor of Mathematics Emeritus, Elmhurst College
Former President, National Council of Teachers of Mathematics

George E. Andrews
Evan Pugh Professor of Mathematics
Pennsylvania State University

Gregory F. Bachelis
Professor of Mathematics
Wayne State University

Michael Beeson
Professor of Mathematics and Computer Science
San Jose State University

George Biriuk
Professor of Mathmatics
California State University, Northridge

Wayne Bishop
Professor of Mathematics
California State University, Los Angeles

Gary J. Blanchard
Professor of Chemistry
Michigan State University

Charles C. Blatchley, Chair
Department of Physics
Pittsburg State University

Michael N. Bleicher
Professor Emeritus,
University of Wisconsin - Madison
Chair, Department of Mathematical Sciences
Clark Atlanta University

John C. Bowman
Vice-President
National Association of Professional Educators

Khristo N. Boyadzhiev
Professor of Mathematics
Ohio Northern University

Bart Braden
Professor of Mathematics
Northern Kentucky University

Stephen Breen
Associate Professor
Department of Mathematics
California State University, Northridge

David A. Buchsbaum
Professor of Mathematics, Emeritus
Brandeis University

Frank Burk
Professor of Mathematics
California State University, Chico

Ana Cristina Cadavid
Professor of Physics
California State University, Northridge

Gunnar Carlsson
Professor of Mathematics
Stanford University

Douglas Carnine
Professor of Education
University of Oregon
Director of the National Center to Improve the Tools of Educators

Mei-Chu Chang
Professor of Mathematics
University of California, Riverside

Sun-Yung Alice Chang
Professor of Mathematics
Princeton University and UCLA

Jeff Cheeger
Professor of Mathematics
Courant Institute, NYU

Orin Chein
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1979 Nobel Prize in Physics

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IRVision, Inc.
San Jose, California

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Scripps Institution of Oceanography
University of California, San Diego

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California State University, Dominguez Hills

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Professor of Mathematics
Mathematics Department
UC Berkeley

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Professor of Mathematics and
Senior Associate Dean
College of Arts and Sciences
Cornell University

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University of Southern California

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Rutgers, The State University of New Jersey

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Univerity of Pennsylvania

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Harvard University

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Superintendent of Public Education
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Department of Mathematics
Whittier College

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University of California at Berkeley

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Department of Mathematics
Washington University in St. Louis
St. Louis, Missouri

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Princeton University

Abel Klein
Professor of Mathematics
University of California, Irvine

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University of California at Davis

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Professor of Mathematics
Yale University

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University of Kansas
and Fellow, Kansas Center for Advanced Scientific Computing

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The Brookings Institution
Washington, DC

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Management Science and Engineering
Stanford University

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Former Member, National Science Board, 1973-1979

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Co-Director, Center for International Security and Arms Control
(Research) Professor
Department of Engineering-Economic Systems and Operations Research
Stanford University

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Stanford University

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Stanford University

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Professor of Mathematics
Michigan State University

John E. McCarthy
Professor of Mathematics
Washington University

Henry P. McKean
Professor of Mathematics
Courant Institute
New York University

Michael McKeown
Professor of Medical Science
Program in Molecular Biology, Cell Biology and Biochemistry
Brown University
Former Member - San Diego Unified Math Standards Committee
Former Member - Superintendent's Math Advisory Committee, San Diego
Co-Founder Mathematically Correct

Marc Mehlman
Associate Professor of Mathematics
University of Pittsburgh, Johnstown

Adrian L. Melott
Professor of Physics and Astronomy
University of Kansas

Aida Metzenberg
Assistant Professor of Biology
California State University, Northridge

Stan Metzenberg
Assistant Professor of Biology
California State University, Northridge

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Professor of Mathematics
California State University, Chico

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Cornell University

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Professor and Chair of Mathematics
Courant Institute of Mathematical Sciences
New York University

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Professor of Mathematics
Courant Institute, New York University

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Lee University
Cleveland, TN

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Caltech

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Professor of Mathematics
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Professor of Mathematics
University of Wisconsin at Madison

Peter Petersen
Undergraduate Vice Chair and Professor of Mathematics
Department of Mathematics
UCLA

Steven Pinker
Professor of Psychology
Department of Brain and Cognitive Sciences
Massachusetts Institute of Technology
Author of *How the Mind Works*

Jacek Polewczak
Professor of Mathematics
California State University, Northridge

Dr. Ned Price
Mathematics Department
Framingham State College
Framingham,Ma.

David Protas

Professor of Mathematics
California State University, Northridge

Ralph A. Raimi
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Professor and Chair of Mathematics
University of Rochester

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Assistant Professor of Mathematics
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Professor of Mathematics
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California State University, Northridge

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Professor of Mathematics
Clinical Assistant Professor of Dermatology
University of California, Irvine

Wilfried Schmid

Dwight Parker Robinson Professor of Mathematics
Harvard University

Dr. Martha Schwartz
Geophysicist
California Mathematics Framework Committee
Co-founder of Mathematically Correct

Albert Schwarz
Professor of Mathematics
University of California, Davis

Roger Shouse
Asst. Professor of Education Policy Studies
The Pennsylvania State University

Barry Simon
I.B.M. Professor of Mathematics and Theoretical Physics
Chair, Department of Mathematics
Caltech

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Case Western Reserve University

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Professor of Mathematics
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Department of Mathematics
California State University, Long Beach

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University of California at Davis

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Professor of Psychology
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College of Education
East Tennessee State University

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Deputy Commissioner for Academic Affairs and Planning
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Research Associate
Harvard Graduate School of Education

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Professor of Mathematics
University of California at Berkeley

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Professor of Mathematics
Rutgers University--New Brunswick

Steven Weinberg
Josey Regental Professor of Science
University of Texas at Austin
1979 Nobel Prize in Physics

Steven H. Weintraub
Professor of Mathematics
Louisiana State University

James E. West
Professor of Mathematics
Cornell University

Brian White
Professor of Mathematics
Stanford University

Professor Olof B. Widlund
Courant Institute of Mathematical Sciences
New York University

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Professor of Mathematics, Emeritus
University of Texas at Austin

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Drexel University

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Department of Computer and Information Sciences
University of North Florida

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Institute for Advanced Study at Princeton

Jon Wolfson

Professor of Mathematics
Michigan State University

Wei-Shih Yang
Professor of Mathematics
Temple University

Shing-Tung Yau
Professor of Mathematics
Harvard University

Recent Directions in San Diego Mathematics Education

by Paul Clopton

Background

State textbook adoptions are made on the basis of frameworks. The last mathematics adoption by the state was made under the guidelines of the 1992 Mathematics Framework. This was from a time before the state had mathematics standards at all. This framework had little or no emphasis on mathematical content and instead stressed an extreme version of discovery learning based on a radical constructivist view of learning. Textbooks were judged on their adherence to the pedagogical ideas of the framework. Not surprisingly, the textbooks adopted by the state under this framework were awful. The 1992 framework and the direction we were headed lead directly to the birth of Mathematically Correct.

It was clear that the state was lacking in direction relative to the content of mathematics education. A new California Mathematics Framework was drafted at the same time that the new California Mathematics Standards were developed.

Following the state adoption of the standards, it became evident that schools lacked the materials they needed to allow students to learn the required content. If nothing had been done, money to buy new textbooks aligned with the new framework wouldn't have been available for years. To correct this problem, AB2519 authorized $1 Billion in a special, one time allocation, split over several years, so that schools could buy new materials to meet the standards. Textbooks aligned with the standards and authorized for purchase with AB2519 funds were selected by the state for use in grades K-8, as is the custom in California for textbook adoptions. Because the state standards now describe algebra as a grade 8 course, algebra I textbooks are, for the first time, subject to the state textbook approval process.

The Past

Members of Mathematically Correct served on the San Diego Math Standards Committee for many months. The committee produced high level Mathematics Standards that were unanimously approved by the Board. The "math wars" issues were actually resolved and everyone was in agreement about the direction to take.

District adoption committees for mathematics were ongoing at this time. These were run by administrators, but the members included teachers and parents. The recommendations of the committees were passed on to the board which ruled in public on the recommendation. This is the typical adoption method in use throughout the state. Those committees recently began to make reasonable decisions. Prior committees had not, and the evidence suggests that this was caused by administrators, not by teachers.

Then, the new Superintendent came in and the Standards were ignored. The district lost funding for the USI grant for lack of attention to mathematics and science, which was the truth.

Next, a draft document circulated around the district math advisory panel. It indicated that Mr. Alvarado had decided that the textbooks approved by the state under AB2519 were not going to be used in San Diego. Instead, the district would seek alternative funding to get around the state guidelines.

When questioned a little too publicly, the draft document was quickly changed to wording that was more vague on this point to obscure the truth.

While the state and district standards continued to be ignored in San Diego, the Blueprint was drafted by the district. Following this the so-called San Diego Math Framework was developed. These documents together are essentially worthless as they provide no direction. The standards and frameworks provided by the state are vastly superior. Nonetheless, these local documents are important to the administrators as they can claim that there actions are aligned with policy (it is hard to find actions not aligned with these policy statements), and continue as if their ideas are already approved.

The Present

Indeed, the district has now adopted textbooks for the lowest achieving schools that are not approved by the state, and HAS done so without a public hearing, without public involvement, and with no board action.

District administrators stressed Everyday Mathematics in their "informational" presentation to the board. They noted that this program was approved by the state. However, it was approved only under the 1992 framework, not as part of AB2519. In fact, the 1995 edition was approved under the old framework. The district wants to buy the 1999 edition. The publishers applied for approval of the 1999 edition under AB2519, but the review committee found the material to be inadequate. The curriculum commission and the state Department of Education worked together to inform the publisher of the changes that would be required to be considered adequate for AB2519. The publisher withdrew from the adoption process, never making the changes needed.

So, we know that this program is not aligned with the state standards. This fact has not been disclosed by the district to the board. The board members were mislead by the administrators.

However, it is quite likely that the Everyday Math program highlighted by the administrators is actually the best of the programs they adopted without a board vote. When districts in Texas were looking at textbooks for adoption, we did a review of several programs. Since the standards in Texas were not as high (or as clear) as ours were, we used the San Diego standards as the benchmarks for our evaluations. We gave Everyday Mathematics grades of C and C- in our review. We also happened to review one of the other programs in the new San Diego list, Connected Mathematics. The program received an **F** rating. It is simply awful. It is completely and totally unrealistic to think that this program comes anywhere close to meeting either the state or the district math standards.

It is obvious that the district is planning to used dumbed-down mathematics in the focus schools. They are taking the approach we have fought so hard to avoid - lowing expectations while claiming otherwise.

The Future

In March, April, May, and June, we are likely to see evaluations made by the district of the success of their methods. They will issue glowing reports of their accomplishments in the focus schools. At this point, they will try to use their own evaluations to press for more dumbed-down mathematics in non-focus schools in San Diego. When the truth comes out with the state test results next summer, it will be too late.

Mathematically Correct is not in the business of endorsing political candidates. A rare exception has been made in the support of Frances O'Neill Zimmerman for her support of mathematics education in San Diego. Regarding this problem, she has said:

> ... the Board of Education ... now has a 3:2 rubber stamp majority voting yes for anything that is proposed. At present, I am one of two members looking skeptically at a weak proposal for a new math framework which has been severely criticized as empty by mathematicians from Stanford and Berkeley...

The *3:2 rubber stamp majority* allowed the district administrators prescribe watered-down math for the children in San Diego's focus schools without so much as a vote or a hearing. Until this situation changes, there is little hope for any real improvement.

Mathematically Correct
Mathematics Program Reviews for Grades 2, 5, and 7

by
Paul Clopton, Erica McKeown, Michael McKeown, and Jamie Clopton

Sponsored by
Education Connection of Texas

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The Authors

The four authors of this report are cofounders of Mathematically Correct, an advocacy group for the improvement of mathematics education in America's schools. Mathematically Correct was founded in Southern California in 1995 by parents concerned about the weakness of the mathematics programs available to students in California. The group is now widely supported by parents, educators, mathematicians, and just plain folk around the country.

The Sponsor

This report would not have been possible without the gracious and dedicated support provided by **Education Connection of Texas**.

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Education Connection of Texas is located at 9323 Bowen Drive, San Antonio, Texas 78250. The telephone number is (210) 523-0743.

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Table of Contents

Next

Initial Review, Draft San Diego City Schools K-12 Mathematics Framework

by Michael McKeown
June 7, 2000

Background:

An effective educational program needs to have clearly defined goals as to what students should know, understand and be able to do, as well as a plan that effectively guides instruction to the levels defined by the standards.

Late in 1997 California adopted a comprehensive set of Mathematics Standards. Early in 1998, San Diego City Schools adopted a comprehensive set of Mathematics Standards largely aligned with the California Standards. Both documents contain substantial detail about what students should know, understand and be able to do at every grade, but they lack any prescription as to how students should be taught.

California has since adopted a State Mathematics Framework that discusses possible instructional strategies linked to the state standards at each grade. The Framework recognizes that some students are behind the level of the standards and contains material and suggestions to focus instruction at those topics of greatest importance to preparing lagging elementary students to succeed in Algebra as eighth graders.

In the two years since the adoption of the California and San Diego Mathematics Standards, San Diego City Schools has failed to develop a systematic program to educate San Diego students to the level of the standards. Indeed, it has been the expressed policy of the district and the Institute for Learning that efforts would be concentrated on "literacy" rather than math. On June 6, 2000, the Institute for Learning released the first public draft of the San Diego City Schools K-12 Mathematics Framework.

Description of the Draft Mathematics Framework

The Framework presented to the pubic consists of 6 pages: A title page, an introductory page, one page entitled "Mathematical Content" and two pages entitled "Mathematical Process."

The introduction asserts alignment of the Framework to the district standards and also asserts the following:

- The elements of the framework also are convergent with those of the Mathematics Framework for California Public Schools, the Standards in Mathematics for California High School Graduates [NB, this is not the California Mathematics Standards] and the 2000 NCTM Principles and Standards for School

Mathematics.

- The framework will serve as a guide for building and implementing a comprehensive curriculum that is coherent across all grades.

- The... Framework establishes a system to link and align district mathematics standards, professional development, instructional practice and curricular materials.

The Mathematics Content section is one page long and is divided into four strands, each presented as single short paragraph: Number Sense and Operations, Functions and Algebra, Measurement and Geometry, and Data Analysis, Statistics and Probability. No item contains reference to specific grade levels or courses, nor does any item refer, except in the broadest sense, to any understanding, knowledge or skill expected of students.

The Mathematics Process section is divided into 7 strands, each represented by one short paragraph: Quantitative Literacy, Computational Fluency, Problem Solving, Using Representations, Using Reason and Proof, Communicating, and Making Connections. None of these topics are specifically related to classroom practice in any grade or course, nor are any of these linked to specific applications or skills.

Critique

As a document to guide student success in gaining the understanding, knowledge and skills described in the California or San Diego Mathematics Standards, this framework is a failure. The goals stated are broad and far reaching platitudes, e.g. "students can engage in tasks for which the solution is not known in advance" and "analyze situations in mathematical terms," but lack any detail regarding exactly what should be covered at any year and how it should be presented. Only the complete lack of specificity amid the overarching statements of this framework supplies any justification for the claim that this document aligns or converges with any other standards or framework document. Like an inkblot test, someone can attach whatever meaning he desires to the elements of the framework.

This complete lack of detail renders the document void of practical meaning. The material in this framework is sufficiently shallow and lacking in specificity as to make it unusable as a guide to classroom instruction, teacher training, or parental aid to students. There are as many classroom practices consistent with this framework as there are teachers, or there are as few as Institute for Learning staff declare in private meetings. A second grade teacher trying to develop in his students understanding of the how and why of multidigit addition and subtraction and use of that knowledge in problem solving receives no guidance from this document. A fourth grade teacher trying to help her students catch up to the level of the standards and prepare them for algebra in grade 8 receives no guidance from this document. A fifth grade teacher looking for help in figuring out an optimal order to present various topics receives no guidance from this document. An algebra teacher trying to help her students understand the quadratic formula, its derivation and use receives no guidance from this document. A pre-calculus teacher trying to prepare his students for college-level math receives no guidance from this document. A committee trying to decide among textbooks receives no guidance from this document.

The broad, vague nature of this framework removes significant accountability and public review of district instructional practices and the practices of the Institute for Learning. This document represents the only opportunity for the board, the public and teachers to discuss openly the nature of mathematics instruction in the district and to set policy on mathematics instruction. Once this framework is approved as district policy on math instruction, the actual details of teacher training and instructional practice are removed from public scrutiny. Framework-aligned instructional practice will become whatever the Institute for Learning trainers say it is. Teachers will have no document to look to for guidance prior to review by Institute for Learning personnel, and no clear defining document against which to appeal a negative review.

Recommendations

This document is sufficiently flawed that it should be rejected outright by the Board of Education. The board then has two options. 1) The board can specify that a new framework will be written and that it should contain grade by grade descriptions of various potentially effective instructional practices that relate to mastery of the standards. In addition, the new framework should contain descriptions of strategies for remediating those students who have fallen behind, with special emphasis on preparing elementary students to be prepared to pass algebra in grade 8 and emphasis on strategies for those students who need more substantial interventions. Once such a document is developed, it should then be carefully examined to determine if the array of instructional practices are likely to be effective. Only then should the framework be accepted. 2) The board could vote immediately to adopt the California Mathematics Framework as the San Diego City Schools Mathematics Framework. This second option has the significant advantage that it allows the district to adopt a document meeting all of the criteria outlined above immediately, without further waste of district funds or time. This option is strongly recommended.

DRAFT
IN PROCESS

San Diego City Schools

Institute for Learning

MATHEMATICS DEPARTMENT

K-12 Mathematics Framework

- Introduction
- Mathematics Content
- Mathematics Processes

Note: The content in this document was distributed as a draft released by San Diego City Schools on June 6, 2000.

Introduction

The San Diego City Schools K-12 Mathematics Framework is a document designed to improve student achievement. The framework will serve as a guide for building and implementing a comprehensive curriculum that is coherent across grades, focused on high-quality mathematics and expects excellence in the teaching and learning of mathematics.

Built upon district standards, [1] the framework is organized into two categories:

1. Mathematical content – the conceptual strands of mathematics.

2. Mathematical processes – the tools and habits of mind people use when solving problems.

The 1996 NAEP results demonstrate that many students are able to compute, but cannot solve problems. The San Diego City Schools Mathematics Framework includes the breadth and depth of content students need to understand mathematics, so that they can compute and solve problems. Students who learn the mathematics put forth in the framework will be resourceful and flexible problem solvers who can effectively apply mathematical knowledge. They will be able to represent mathematical knowledge in a variety of ways to meet the demands of higher education and the world of work.

The San Diego City Schools Mathematics Framework establishes a system to link and align district mathematics standards, professional development, instructional practice, and curricular materials. The framework focuses and organizes mathematics content and instructional practice, leading the way to improved mathematics learning for all students in the San Diego City Schools.

[1]The elements of the framework also are convergent with those of the *Mathematics Framework for California Public Schools*, the *Standards in Mathematics for California High School Graduates* and the *2000 NCTM Principles and Standards for School Mathematics*.

Mathematical Content
1. Number Sense and Operations Students develop number sense as they learn to think and reason flexibly, make sound numerical judgments and are able to judge what's numerically reasonable in various situations. When faced with a situation that calls for numerical calculations, students need to be able to choose the correct operations, decide on the numbers to use, do the necessary calculation, and then appropriately interpret the results. Development of a deep and fundamental understanding of numbers and operations continues throughout the grades.
2. Functions and Algebra Algebra emphasizes relationships among quantities, including functions, ways of representing mathematical relationships and the analysis of change. Instructional programs designed to provide ongoing experiences with patterns lead to an understanding of function. Experiences with numbers and their properties lay the foundation for later work with symbols and algebraic expressions. Viewing functions and algebra as a strand in the curriculum at all grades builds the solid foundation of understanding and experience that prepares students for more complex work in algebra in the middle grades and high school.
3. Measurement and Geometry Measurement and geometry provide students ways to interpret and understand their physical environment. They provide tools for the study of other topics in mathematics and science. Across the grades, students build an understanding of units of measure and learn to apply appropriate techniques, tools and formulas. They learn about geometric shapes and structures and how to analyze their characteristics and relationships. An important aspect of geometric thinking is spatial visualization. School experiences provide opportunities for students to build and manipulate mental representations of two- and three-dimensional objects. They work with concrete models, drawing and dynamic computer software. Visualization, spatial reasoning and geometric modeling are strategies that help students analyze and solve problems
4. Data Analysis, Statistics and Probability Data analysis, statistics, and probability develop students' ability to understand, analyze and evaluate statistical information, an important skill for an informed citizen, employee, and consumer. School experiences provide students opportunities to formulate questions, collect, organize and display relevant data. They learn methods of analyzing data and ways of making inferences and drawing conclusions. Students learn basic concepts and applications of probability, with the emphasis on the way that probability and statistics are related.

Mathematical Processes
1. Quantitative Literacy Students who understand numbers and the number system make sense of them, use them when solving problems, and recognize reasonable results. They can break numbers apart (for example, they can see the number 24 and know that it is 2 tens and 4 ones and also two sets of 12). Quantitatively literate students use appropriate numbers as referents (such as 10, 2, 5), solve problems using the operations and their relationships and apply their understanding of the base-ten system.
2. Computational Fluency Computational fluency is essential. Students with computational fluency have the ability to use a variety of efficient procedures to perform calculations that produce accurate results. They use mental calculations, estimation, and paper and pencil calculations using mathematically accurate algorithms. In addition, students use calculators appropriately. Calculators should be set aside with the instructional focus is on developing or practicing computational algorithms.
3. Problem Solving Problem solving means that students can engage in tasks for which the solution is not known in advance. Problem solving is both a goal and a means of learning mathematics. Problem solving helps students learn to deal with unfamiliar situations and develop habits of persistence. It requires that students explore, make conjectures and question one another. Students who are adept at problem solving can analyze situations in mathematical terms. Throughout the grades, students develop and expand a more complex set of tools such as using diagrams, looking for patterns or working backwards, and they learn to monitor and adjust strategies as they solve a problem.
4. Using Representations Representations enable students to organize, record, and communicate mathematical ideas to themselves and to others. Students who understand representations can select, apply, and translate them when solving problems. Students learn and use conventional forms of mathematical symbolism, and develop the ability to represent their own mathematical ideas in ways that make sense and clearly communicate them to others. Forms of representation such as diagrams, graphical displays, and symbolic expressions are acquired as powerful and useful tools for learning and doing mathematics and for communicating the results of problem solving, reasoning, and proving or verifying results.
5. Using Reason and Proof School experiences in reasoning and proof build from early experiences using inductive arguments to later work with deductive argument. Throughout secondary school, students deepen their understanding and use of mathematical proofs. K-12 experiences with reasoning and proof help students develop the skills necessary to make and investigate mathematical conjectures, and evaluate mathematical and other arguments.

6. Communicating Communication is an essential part of mathematics and mathematics instruction. Through communication, students organize and consolidate their mathematical thinking. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others, they learn to be clear and convincing. Discussions about mathematical ideas help students learn to analyze and evaluate the mathematical thinking and strategies of others. Communicating clearly, whether orally or in written form, requires that students use the language of mathematics to express mathematical ideas precisely.
7. Making Connections Focusing on the connections among mathematical ideas helps students understand that mathematics is a coherent whole, rather than a collection of isolated skills and arbitrary rules. Students who use connections recognize how ideas in different areas are related, and are able to use insights gained in one context to verify conjectures in another. Students who make connections see and experience the interplay among mathematical topics, between mathematics and other subjects, and between mathematics and their own interests.

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New-New Math in Santa Monica

The Santa Monica-Malibu Unified School District (SMMUSD), has had its share of problems but there have been few issues quite as egregious as the "new-new" math curriculum mistakes. The curriculum problems in Santa Monica are complicated by an insidious form of discrimination where students in the more affluent neighborhood schools have the benefit of a State approved, State content aligned curriculum and the schools in less affluent southern part of the District have MathLand and CPM, neither of which is State approved for educational content. In July of this year the SAT-9 results were released for the third year in a row- in every year by every measure, regardless of what type of group we compared, ie, LEP, non-LEP, low SES, ethnicity, etc., it was apparent the schools using State approved, State aligned math curriculum had high average student scores, sometimes as much as 60% higher than the other schools in the District.

Troubled that the District could be discriminating by not allowing all students access to the curriculum which demonstrated results, we looked further into this problem and discussed the issues with other local parents, many of whom knew nothing about the math curriculum their children were using, let alone that it was not State approved for educational content. We were not content to drop this problem which seemed to mushroom at each turn. When the local newspaper ran some articles which quoted administrators falsely claiming that all the texts used in the District were State approved, we knew we had to correct the record. With nearly one-sixth of the students in our elementary and middle schools in danger of retention at the end of June, 2000, giving parents complete and truthful information was an important step in helping children succeed. We formed a coalition of parents and other interested parties to address the issues at the District level, Santa Monicans Working for Equity and Excellence in Public Schools (SMWEEPS). Our group of five people has grown exponentially since the end of July; we now number more than one hundred. You can reach us at smweeps@aol.com. We welcome your participation.

A few parents in our group will be making brief speeches during the public comment section at the SMMUSD Board of Education meeting on Thursday, September 21, at 7 p.m. which will held in the City Council chambers. Parents are asking for:

1. establishment of a District level curriculum oversight committee that includes as many parents as educators and is subject to the Brown Act. Among many responsibilities, the committee should ensure that compelling evidence of success be in hand from unbiased, comprehensive and scientifically sound research when any curriculum is adopted in the District that differs from the State content approved list. This committee should be established no later than April, 2001.

2. a choice in curriculum for every student at every school when more than one curriculum is employed anywhere in the District no later than August, 2001. Parents must give informed consent if their child is enrolled in a course that is not supported by curriculum approved by State for educational content.

3. the establishmentof comprehensive and intensive math tutoring programs aligned to the State Content Standards no later than December 1st, 2000, for every student on every campus every day. -SMWEEPS 9-18-00

Post script-

A. After much urging from SMWEEPS and other unassociated community members, the Board of Education of the Santa Monica Malibu School District adopted the State Standards for Mathematics on February 15, 2001.

B. On April 26, 2001, the SMMUSD Board of Education adopted a new policy on curriculum selection which substantively supports the adopted standards and which includes parents and community in the decision making process.

Mathematically Correct

Mathematics Program Reviews and Information

- California Mathematics Program Adoptions for 2001 - The textbooks judged as best aligned with the California Standards and Framework in a regular adoption
- California Math Adoptions for 2000 - The textbooks judged as best aligned with the new California standards in a supplemental adoption
- California State Adopted Middle School Math Programs reviewed
- Singapore Math and Science Books now Available - For those who ask about these books, here is where you can get them
- School District Alert for Mathematics Textbook Selection
- Texas School Districts Reject "Fuzzy Math" Textbooks -- Major Defeat for Statewide Systemic Initiative
- A Comparision of Three K-6 Mathematics Programs: Sadlier, Saxon, and SRA McGraw-Hill, by David Klein and Jennifer Marple
- Mathematics Program Reviews for Grades 2, 5, and 7

  - Second Grade Programs
    - Harcourt Brace *Math Advantage*
    - McGraw-Hill School Division *Math in My World*
    - Saxon Publishers *Math 2: An Incremental Development*
    - Scott Foresman - Addison Wesley *Math Grade 2*
    - SRA/McGraw Hill *Math: Explorations and Applications*
    - Dale Seymour Publications *Investigations in Number, Data, and Space*
    - Everyday Learning *Everyday Mathematics*
    - Silver Burdett Ginn *Mathematics: The Path to Math Success*
  - Fifth Grade Programs
    - Scott Foresman Addison Wesley *Scott Foresman - Addison Wesley Math*
    - Harcourt Brace *Math Advantage*
    - McGraw-Hill *Math in My World*
    - Saxon Publishers *Math 65: An Incremental Development*
    - Silver Burdett Ginn *Silver Burdett Ginn Mathematics*
    - SRA/McGraw-Hill *SRA Math: Explorations and Applications*
    - Dale Seymour Publications *Investigations in Number, Data, and Space*
    - Everyday Learning Corporation *Everyday Mathematics*
  - Seventh Grade Programs
    - Dale Seymour Publications *Connected Mathematics Program*
    - Glencoe/McGraw-Hill *Mathematics: Applications and Connections, Course 2*

Harcourt Brace *Math Advantage Middle School II Preparation for Algebra*
McDougal Littell *Passport to Mathematics Book 2*
McDougal Littell *Math Thematics Book 2*
Prentice Hall *Middle Grades Math: Tools for Success Course 2*
Saxon Publishers *Math 87*
Scott Foresman - Addison Wesley *Middle School Math Course 2*
Pre-Algebra Level Texts Potentially Used in Seventh Grade
Glencoe/McGraw-Hill *Pre-Algebra, an Integrated Transition to Algebra and Geometry*
McDougal Littell *Passport to Algebra and Geometry*
Saxon Publishers *Algebra 1/2*

- Mathematically Correct Algebra 1 Reviews
  - Foundations of Algebra and Geometry
  - Algebra 1 (Foerster)
  - Focus on Algebra
  - UCSMP Algebra
  - Glencoe Algebra 1
  - HRW Algebra
  - McDougal Littell Algebra 1
  - Heath Algebra 1
  - Algebra: Structure and Method *(new)*
  - Prentice Hall Algebra
  - Cord Algebra 1
  - South-Western Algebra 1

- CPM
  - Content Review of CPM Mathematics
  - CPM: Impeding the learning process
  - Study Looks at Effectiveness of CPM vs Traditional Math
  - Both Programs Fail -- Is this CPM Success?
  - The Major Goal of CPM? -- It's Public Relations
  - A look at CPM "evidence" in their September newsletter
  - CPM "Geometry" Rejected by San Diego Teachers
  - Escondido Drops CPM, Re-Institutes Traditional Math
  - The Training Wheels *A Parable for Modern Times*
    Warning to CPM parents: This may produce strong emotional reactions.
  - Math method not working for some by Amita Sharma in the Riverside Press-Enterprise on Apr. 26, 1998, describes shut down of some CPM programs after high failure rates
  - A Letter to Kenilworth Parents
  - Schools chief sees problems, challenges by Guy Kovner in The Press Democrat

- Connected Mathematics Project (CMP)
  - Testimony of Susan Sarhady, U.S. House of Representatives Committee on Education and the Workforce, February 2, 2000
  - Connected Math Disconnects Parents, by Timothy P. Williams
  - Petition targets connected math: Parents object to curriculum, by Jonathan M. Bell, Plano Star-Courier, June 17, 1999
  - CMP at Okemos Middle Schools
  - Review of Grade 7 program
  - *Parents asking state agency to look into math program*,by Sandy Louey, The Dallas Morning News Mar. 16, 1999, tells of complaints about Connected Math
  - Connected Mathematics in Plano Independent School District
  - One visit does not tell the complete 'fuzzy math' tale, Montgomery Journal, Sept 1, 1999
  - PISD says 'No' to alternative math program, Kelli Conlan, Plano Star Courier, Apr. 2, 1999
  - Fuzzy' math: PISD parents appeal to state, by Kelli Conlan, Plano Star Courier, Mar. 4, 1999
  - MCPS officials have 'fuzzy' answers for math curriculum, by Robert Rosenfeld, the Montgomery Journal, June 14, 1999

- An Integraded High School Math Program: McDougall Littell

- Parent's Evaluation of the Arise Math Program

- Core Plus Mathematics Project (CPMP) / Contemporary Math in Context
  - Letter Regarding Contemporary Math in Context from Steven Krantz (pdf file), chair of the Mathematics Department at Washington University in St. Louis, Mo
  - There Are Integrated Programs, and There Are Integrated Programs, by Richard Askey
  - A sample list of errors in Core Plus Materials from Lawrence F. Gray, Professor of Mathematics at the University of Minnesota, Minneapolis, MN
  - How a new math program rose to the top
  - High School Students And Lab Rats
  - Outcomes Analysis for Core Plus Students at Andover High School: One Year Later, by R. James Milgram, Department of Mathematics, Stanford University
  - Math debate heats up, by Rusty Hoover, The Detroit News
  - Reform vs. Traditional Math Curricula: Preliminary report on a survey of the graduating classes of 1997 of Andover High School and Lahser High School, Bloomfield Hills, Michigan, concerning their high school math programs and how well these programs prepared them for college math, by Gregory F. Bachelis
  - *New math doesn't compute with students, survey finds*, by Tamara Audi, Detroit Free Press, March 19, 1999
  - Parents wary of new program for teaching math, By Nicole Bondi, The Detroit News

- Reform Calculus
  - What is Wrong With *Harvard Calculus*?
  - Calculus Reform--For the $Millions, by David Klein and Jerry Rosen in Notices of the AMS

- Algebra: Themes, Concepts, Tools
  - Review of Creative Publications' *Algebra: Themes, Concepts, Tools*
  - *The Pythagorean Theorem* by G. D. Chakerian and Kurt Kreith

- Addison Wesley
  - Brea Group Evaluates Addison-Wesley's *Quest 2000*
  - Addison-Wesley Math Programs in Arizona: X + Y = F
  - MTV Math Doesn't Add Up
  - 'Rain Forest' Algebra Course Teaches Everything but Algebra
  - More on Addison-Wesley *Focus on Algebra*, from Richard Askey
  - *Focus on Algebra* in the Congressional Record
  - Addison-Wesley's *Focus on Algebra* draws fire
  - Texas Adopts Textbook Rejected by Nation, by Chris Patterson, Nov. 1997

- Scott Foresman Addison Wesley *Math*, 4th Grade review by Kevin Killion

- A review of *Geometry: tools for a changing world* by David E. Joyce

- Palo Alto Parent Compares Three Reform Texts
  - Everyday Learning's *Everyday Math* (UCSMP)
  - Dale Seymour's *Investigations* (TERC)
  - Creative Publications' *MathLand*

- An Evaluation of Selected Mathematics Textbooks, by Wayne Bishop:
  - Sadlier-Oxford, Division of William Sadlier, Inc.
  - Everyday Mathematics, Everyday Learning Corporation (publishers of the materials of the University of Chicago School

Mathematics Project, UCSMP)
    - Connecting Math Concepts (CMC), SRA Division of Macmillan/McGraw Hill
    - Saxon Publishers

- MathLand
    - *SR board frowns on MathLand program*, by Robert Digitale, Santa Rosa Press Democrat, Oct. 15, 1998
      "*Santa Rosa school board members voiced strong displeasure ... with the results of the MathLand math program ... Three board members ... said their own children have been poorly served by MathLand ...*"
    - Math Gadfly Calls Math Faddists' Bluff, by Debra Saunders, SF Chronicle, Aug 7, 1998, comments on a MathLand school in LA
    - A Letter to the MathLand Folks
    - Atascadero picks Silver-Burdett over MathLand
    - Mathland Lunacy
    - Reviews from QED on *Mathland*
    - 'New, new math' has parents crying 'back to basics'
    - Can you believe this rationale?
    - Trouble in *MathLand* (updated 11/2/96)
    - Mathland in Davis
    - Petaluma parents battle MathLand
    - MathLand in San Francisco
    - *MathLand* in Stockton

- MathLand and Glencoe Interactive in the DoDDS
    - Four-Star Math Follies
    - The Stars and Stripes editorial (July 31, 1997) notes: *Two years ago DoDDS implemented the new-new math programs, MathLand in elementary school, and Interactive Mathematics in Jr. high. The first year of implementation was a complete disaster ...*
    - Parents' cry can subtract MathLand
    - Mathland and Glencoe Interactive in DoDDS
    - Mathematics Reform in Theory and Practice, and its Implications for DoD Students, by Denise McArthur

- *Reality Therapy* Founder Pushes for OBE in Minnesota

- The Core Knowledge Series

- UCSMP [also known as *Everyday Mathematics* and *Chicago Math*]
    - Evaluation of Everyday Mathematics
    - If math were a color . . . by Marcia Tsicouris
    - Concerned Parents of Reading sent a Letter to the Massachusetts Board of Education, Feb 13, 2000
    - Test Scores reported by Concerned Parents of Reading
    - A Florida Parent Speaks about UCSMP
    - Chicago math by Redyarrow
    - Suburban Chicago: Is Your Child Learning Math?
    - Officials: Calculators add up trouble, by David Rogers, Stoneham Sun, Nov 3, 1999
      Kennedy said the dependence [on calculators] *may be in part due to the textbook they use. The school system has been using the Chicago textbook series, which requires a calculator for proper instruction. "On the first page they tell what type of calculator they should have," said Kennedy.*
    - Teachers choose traditional text, by Jeanne Russell, San Antonio Express-News, Mar. 31, 1999

- IMP
    - An Overview of IMP Years 1 and 2, by Kim Mackey
    - *It doesn't add up: New math displays some disturbing numbers*, San Diego Union-Tribune, Jan 22, 1998
      "*Q. Why is a new-math textbook like a guinea pig?*"

"*A. Because a guinea pig doesn't come from Guinea and it isn't a pig.*"

- Review of the Interactive Mathematics Program (requires Adobe Reader)
- Comments on the California Math Wars
- IMP: Manifesto on an Experimental Concept Gone Awry
- IMP: A Student's View, with Comments
- SAT Scores and IMP in California
- IMP: A Teacher's Review for Parents
- IMP in Cottage Grove
- No more IMP, no more CPM in Escondido
- IMP Woes in Oregon

- Project Follow Through
  - Honest follow-through needed on this project
  - The Association for Direct Instruction

Mathematically Correct

National Issues

Page Index

Open Letter on the Department of Education's List of Programs

Members of Mathematically Correct discussed issues in mathematics education with Richard Riley, US Secretary of Education, subsequent to his call for a cease fire in the *Math Wars*. Unfortunately, little progress has resulted. Even more unfortunately, his Department of Education recently released a list that endorsed some of the weakest mathematics programs available in the world. Contrary to his call for peace, it would appear that the Secretary has taken up sides and provoked the issue.

As a result, a letter of protest was drafted and has now been endorsed by over 200 individuals, mostly professional mathematicians but including other noteworthy people in education and scientific fields. The letter was published in the Washington Post and Education week, and makes the objections to the actions of the Department of Education clear. The topic has now been a subject of discussion at a Congressional hearing.

- An Open Letter to Richard Riley

- Related Info:
  - Rebuttal to Johnny Lott's "Stalkers"
  - False Rumors in Minnesota
  - How a new math program rose to the top
  - Secretary Riley Reignites the Math Wars, by Bill Evers
  - Testimony to the House of Representatives regarding the Department of Education
  - Why the U.S. Department of Education's recommended math programs don't add up
  - Math Wars Excerpts from the Wall Street Journal Editiorial
    of January 4, 2000
  - Wall Street Journal editorial
  - Professors say math programs don't add up, American School Board Journal
  - Open Letter on Mathematics Curricula Ignites Debate, Notices of the American Mathematical Society
  - Open Letter on the CER web
  - No Such Thing As Malpractice In Edu-Land, by Debra Saunders, SF Chronicle
  - Hearing on The Federal Role in K-12 Mathematics Reform

- *Experts, Parents Fault Education's Math Curriculums*, The Washington Times
- Multiplying Math Woes, the Los Angeles Times
- Where's the Math?
- Academics Urge Riley To Reconsider Math Endorsements, by Debra Viadero, Education Week, Nov. 24, 1999
- Scholars' Plea on Math Praised, by William Sadlier Dinger, Education Week, Dec. 8, 1999
- Maybe Ms. Watkins needs to indulge in some critical thinking, by Ze'ev Wurman, Education Week, Dec. 8, 1999
- Experts attack math teaching programs, by Richard Lee Colvin, LA Times, Nov 17, 1999
- Scholars Weigh In Against Education Dept.'s Endorsement of 10 Math Programs for Children, by Julianne Basinger, The Chronicle of Higher Education
- Mathematicians dispute federal education experts, by Linda Seebach, Denver Rocky Mountain News, Nov. 28, 1999
- Division In The Math Ranks, by Rick Green, The Hartford Courant, Nov. 28, 1999
- County won't get 'fuzzy' grant: Officials planned to use $6M to expand NSF math programs, by Jennifer Jacobson, The Journal Newspapers, Dec. 9, 1999
- Toward a Cease-Fire in the Math Wars
- *Math Wars Heat Up in Washington*, NY Times, Nov. 27, 1999
- *Experts negative on math lessons: Federally endorsed programs just don't add up, ad tells public*, By Anjetta McQueen, the Houston Chronicle, Nov. 28, 1999
- *Scientists challenge math endorsements*, Orange County Register, Nov. 28, 1999
- *Leading mathematicians say new teaching techniques don't add up*, San Diego Union-Tribune, Nov. 28, 1999
- *Education Dept Defends Math Program*, The Associated Press, Feb. 2, 2000

Statements from Mathematically Correct

Here is an assortment of statements released by Mathematically Correct on various issues in mathematics education.

- Statements authored by Frank B. Allen, National Advisor to Mathematically Correct
  - A Program for Raising the Level of Student Achievement in Secondary School Mathematics
  - Repairing school mathematics in the US
  - Critique of NCTM Standards
  - Language and the Learning of Mathematics
  - Mathematics "Council" Loses Hard-Earned Credibiility
  - A New Mission for NCTM: Save Our Schools

- Statements on the proposed national mathematics test
  - Mathematically Correct Testimony on the Voluntary National Test Plan
  - Comments on the National 8th Grade Mathematics Test
  - Letter to President Clinton regarding the 8th Grade Mathematics Test

- Other statements on assessments
  - Statewide Mathematics Assessment in Texas
  - Don't Believe It
  - Mile Wide/Inch Deep Interpretatons
  - Evaluating Entry Level Mathematics Placement in the California State University System

- Lists of objectives
  - Toward a Cease-Fire in the Math Wars
  - The Ten Things
  - Mathematics Standards draft submitted the San Diego City Schools

On the NCTM Standards

The emergence of weak mathematics programs that are overtaking many classrooms in America was stimulated by the National Council of Teachers of Mathematics (NCTM). Their hallmark documents, called *Standards*, contain much of the philosophy behind the *Whole Math* movement. These documents do not provide content standards as the name might suggest. Instead, they focus on the promotion of their theories about how math should be taught.

- Some issues with the NCTM Standards
  - Critique of NCTM Standards, by Frank B. Allen
  - The Truth About The NCTM Standards
  - Professor Harold W. Stevenson on the NCTM Standards
  - HOLD's Suggestions on NCTM Standards
  - Withdraw Endorsement of NCTM Standards, by David Klein, Notices of the American Mathematical Society, March, 1997

NCTM is working on a revision of their Standards. To that end, they have sought comment from professional societies.

- Comments from the professional societies
  - MAA Report on the NCTM Revision
  - First Report from the MAA Task Force on the NCTM Standards
  - Second Report from the MAA Task Force on the NCTM Standards
  - AMS Comments on NCTM Standards Revision
  - AMS NCTM2000 Association Resource Group First report
  - The AMS and Mathematics Education: The Revision of the "NCTM Standards", by Roger Howe, Notices of the AMS, Feb 1998

- Related information
  - "I have never heard of it" -- Gail Burrill, NCTM President
  - Repairing school mathematics in the US
  - Toward a Cease-Fire in the Math Wars

International Comparisons

Considerable attention has been focused on mathematics education as a result of international comparisons, notably the Third International Mathematics and Science Study (TIMSS). These efforts generally point out the weakness of mathematics education in the United States. However, the results are often used in arguments is misleading ways. The following may help to clear up the international picture.

- TIMSS Information
  - A TIMSS Primer: Lessons and Implications for U.S. Education, by Harold W. Stevenson
  - Searching For The Truth About The TIMSS 4th Grade Math Test by Bill Quirk
  - Don't Believe It
  - Mile Wide/Inch Deep Interpretatons
  - Math Lessons from Japan: *The TIMSS and the Truth*
  - U.S. Teens Rank Low in World Tests: High school students dismal in math, science, by Nanette Asimov, SF Chronicle, Feb. 25, 1998

- Other international information
  - Singapore - 1999 exam syllabi
  - Singapore Math and Science Books now Available
  - US Math Standards Too Low
  - America -- Test your 12-year-olds: Math Problems from Japan
    see also American problems for 9th grade
  - Russian Mathematics for 6th graders

State Assessments

The statewide assessment system in Texas has used as evidence for achievement gains there. However, the gains are less impressive when the assessments are evaluated carefully.

- Texas Mathematics Education in Transition: An Analysis of the TAAS Shows The Need to Move from Minimum Competence to High Achievement
- Statewide Mathematics Assessment in Texas
- Adding it all up; What does it really mean to pass the TAAS?, by Shaila Dewan, Houston Press

For many years, California had no statewide test. Although the new state Standards and Framework were coming, something had to be done quickly to gain some sort of accountability in the state. As a result, the Stanford test (SAT9) was implemented statewide. By the second year of testing, the SAT9 tests were augmented with items written to address the new Standards explicitly. The statewide assessment program is known as STAR.

- Sample 1999 Augmented STAR Mathematics Items
- California STAR test results
- California STAR Math Scores by County
- STAR math test results for California, Los Angeles, and San Diego

Finally, we have information gained from Iowa:

- A Lesson on Norm-Referenced Testing from Ames, Iowa

The Mathematics Framework in Massachusetts

Fireworks went off when Massachusetts struggled to get mutliplication and division into their state mathematics framework.

> For a summary of the underlying causes of the conflict, see Math Wars in Massachusetts: The Battle Over The Mathematics Curriculum Frameworkby William G. Quirk, Ph.D.
>
> The outcome of the struggle, as summarized by the Massachusetts Department of Eduction, was a Conditional Endorsement
>
> Based on their experiences, the Concerned Parents of Reading sent a letter to the Massachusetts Board of Education.
>
> Details of the story of the Attack by Hyman Bass on the Massachusetts Deputy Commissioner of Education for her support of arithmetic.

The proposed National Voluntary Mathematics Test

Noting the inadequate achievement in school mathematics in the United States, President Clinton proposed a voluntary national test as a way to combat the problem. Examinations can be an effective way to stimulate achievement gains, but the devil is in the details. Clinton's plan got off to a bad start. The first committee working on it was full of fuzzy math supporters, and the initial plans were dismal.

The test design was shifted from the initial committee to the National Assessment Governing Board. The progress was often delayed and funding was restricted by congress. The idea is still not dead, but it has been in a prolonged coma.

Here are some of the items that detail the issues and history of the test plan:

- Comments and Testimony
  - Letter to President Clinton regarding the 8th Grade Mathematics Test
  - Mathematically Correct Testimony on the Voluntary National Test Plan
  - Testimony on the Voluntary National Test from Mary Damer
  - Statements to NAGB on the Voluntary National Test Plan from:
    - Wayne Bishop
    - Chester E. Finn, Jr.
  - Comments on the National 8th Grade Mathematics Test
  - Comments before The Mathematics Committee of The National Test Panel

- Media Coverage
  - Contractor Gets More Time To Write National Test Items, by David J. Hoff, Ed Week, August 6, 1998
  - Flunking the tests by Debra Saunders, SF Chronicle, Feb. 15, 1998
  - *Assessment Board Delays National Student Testing*, Washington Post, Jan 23, 1998
  - EXAM SCAM -The Latest Education Disaster: Whole Math
  - President Clinton's Mandate for Fuzzy Math, Lynne V. Cheney, The Wall Street Journal
  - *Clinton's Test Plan Under Fire From Many Directions*, By Peter Applebome, NY Times
  - *House Votes Down Clinton Plan for National Reading, Math Achievement Tests*, by Rene Sanchez, Washington Post
  - A Failing Grade for Clinton's National Standards, by Lynne Cheney, the Wall Street Journal
  - Riley Delays National Tests' Development, by Millicent Lawton, Education Week
  - *"On ... national reading and math tests ... President Clinton has apparently lost a test of wills with Congress .... Senator Ashcroft and more than 30 other Senators also say the eighth grade math test is full of what they call fuzzy math"*, CBS News, Nov. 5, 1997
  - *House Limits Clinton's National Testing Program*, by Jerry Gray, New York Times, Nov. 9, 1997

Other reports on math education

- Mathematics Equals Opportunity, U.S. Dept. of Education
- On Society, John Leo, U.S. News & World Report
- Sen. Byrd warns the U.S. Senate about Fuzzy Math
- No Excuses by Tyce Palmaffy in Policy Review, Jan-Feb 1998
- Review of *The Schools We Need & Why We Don't Have Them* by E.D. Hirsch, Jr.

MATH PROBLEMS
Why the U.S. Department of Education's recommended math programs don't add up

By David Klein

What constitutes a good K-12 mathematics program? Opinions differ. In October 1999, the U.S. Department of Education released a report designating 10 math programs as "exemplary" or "promising." The following month, I sent an open letter to Education Secretary Richard W. Riley urging him to withdraw the department's recommendations. The letter was coauthored by Richard Askey of the University of Wisconsin at Madison, R. James Milgram of Stanford University, and Hung-Hsi Wu of the University of California at Berkeley, along with more than 200 other cosigners. With financial backing from the Packard Humanities Institute, we published the letter as a full-page ad in the Washington Post on Nov. 18, 1999, with as many of the endorsers' names and affiliations as would fit on the page. Among them are many of the nation's most accomplished scientists and mathematicians. Department heads at more than a dozen universities--including Caltech, Stanford, and Yale--along with two former presidents of the Mathematical Association of America also added their names in support. With new endorsements since publication, there are now seven Nobel laureates and winners of the Fields Medal, the highest award in mathematics. The open letter was covered by several newspapers and journals, including American School Board Journal (February, page 16).

Although a clear majority of cosigners are mathematicians and scientists, it is sometimes overlooked that experienced education administrators at the state and national level, as well as educational psychologists and education researchers, also endorsed the letter. (A complete list is posted at http://www.mathematicallycorrect.com.)

University professors and public education leaders are not the only ones who have reservations about these programs. Thousands of parents and teachers across the nation seek alternatives to them, often in opposition to local school boards and superintendents. Mathematically Correct, an influential Internet-based parents' organization, came into existence several years ago because the children of the organization's founders had no alternative to the now "exemplary" program, College Preparatory Mathematics, or CPM. In Plano, Texas, 600 parents are suing the school district because of its exclusive use of the Connected Mathematics Project, or CMP, another "exemplary" program. I have received hundreds of requests for help by parents and teachers because of these and other programs now promoted by the Education Department (ED). In fact, it was such pleas for help that motivated me and my three coauthors to write the open letter.

Common problems

The mathematics programs criticized by the open letter have common features. For example, they tend to overemphasize data analysis and statistics, which typically appear year after year, with redundant presentations. The far more important areas of arithmetic and algebra are radically de-emphasized. Many of the so-called higher-order thinking projects are just aimless activities, and genuine illumination of important mathematical ideas is rare. There is a near obsession with calculators, and basic skills are given short shrift and sometimes even disparaged. Overall, these curricula are watered-down math programs. The same educational philosophy that gave rise to the whole-language approach to reading is part of ED's agenda for mathematics. Systematic development of skills and concepts is replaced by an unstructured "holism." In fact, during the mid-'90s, supporters of programs like these referred to their approach as "whole math."

Disagreements over math curricula are often portrayed as "basic skills versus conceptual understanding." Scientists and mathematicians, including many who signed the open letter to Secretary Riley, are described as advocates of basic skills, while professional educators are counted as proponents of conceptual understanding. Ironically, such a portrayal ignores the deep conceptual understanding of mathematics held by so many mathematicians. But more important, the notion that conceptual understanding in mathematics can be separated from precision and fluency in the execution of basic skills is just plain wrong.

In other domains of human activity, such as athletics or music, the dependence of high levels of performance on requisite skills goes unchallenged. A novice cannot hope to achieve mastery in the martial arts without first learning basic katas or exercises in movement. A violinist who has not mastered elementary bowing techniques and vibrato has no hope of evoking the emotions of an audience through sonorous tones and elegant phrasing. Arguably the most hierarchical of human endeavors, mathematics also depends on sequential mastery of basic skills.

The standard algorithms

The standard algorithms for arithmetic (that is, the standard procedures for addition, subtraction, multiplication, and division of numbers) are missing or abridged in ED's recommended elementary school curricula. These omissions are inconsistent with the mainstream views of mathematicians.

In our open letter to Secretary Riley, we included an excerpt from a committee report published in the February 1998 Notices of the American Mathematical Society. The committee was appointed by the American Mathematical Society to advise the National Council of Teachers of Mathematics (NCTM). Part of its report discusses the standard algorithms of arithmetic. "We would like to emphasize that the standard algorithms of arithmetic are more than just 'ways to get the answer'--that is, they have theoretical as well as practical significance," the report states. "For one thing, all the algorithms of arithmetic are preparatory for algebra, since there are (again, not by accident, but by virtue of the construction of the decimal system) strong analogies between arithmetic of ordinary numbers and arithmetic of polynomials."

This statement deserves elaboration. How could the standard algorithms of arithmetic be related to algebra? For concreteness, consider the meaning in terms of place value of 572:

$$572 = 5\,(10^2) + 7(10) + 2$$

Now compare the right side of this equation to the polynomial,

$$5x^2 + 7x + 2.$$

The two are identical when $x = 10$. This connection between whole numbers and polynomials is general and extends to arithmetic operations. Addition, subtraction, multiplication, and division of polynomials is fundamentally the same as for whole numbers. In arithmetic, extra steps such as "regrouping" are needed since $x = 10$ allows for simplifications. The standard algorithms incorporate both the polynomial operations and the extra steps to account for the specific value, $x = 10$. Facility with the standard operations of arithmetic, together with an understanding of why these algorithms work, is important preparation for algebra.

The standard long division algorithm is particularly shortchanged by the "promising" curricula. It is preparatory for division of polynomials and, at the college level, division of "power series," a useful technique in calculus and differential equations. The standard long division algorithm is also needed for a middle school topic. It is fundamental to an understanding of the difference between rational and irrational numbers, an indisputable example of conceptual understanding. It is essential to understand that rational numbers (that is, ratios of whole numbers like 3/4) and their negatives have decimal representations that exhibit recurring patterns. For example: 1/3 = .333..., where the ellipses indicate that the numeral 3 repeats forever. Likewise, 1/2 = .500... and 611/4950 = .12343434....

In the last equation, the digits 34 are repeated without end, and the repeating block in the decimal for 1/2 consists only of the digit for zero. It is a general fact that all rational numbers have repeating blocks of numerals in their decimal representations, and this can be understood and deduced by students who have mastered the standard long division algorithm. However, this important result does not follow easily from other "nonstandard" division algorithms featured by some of ED's model curricula.

A different but still elementary argument is required to show the converse--that any decimal with a repeating block is equal to a fraction. Once this is understood, students are prepared to understand the meaning of the term "irrational number." Irrational numbers are the numbers represented by infinite decimals without repeating blocks. In California, seventh-grade students are expected to understand this.

It is worth emphasizing that calculators are utterly useless in this context, not only in establishing the general principles, but even in logically verifying the equations. This is partly because calculator screens cannot display infinite decimals, but more important, calculators cannot reason. The "exemplary" middle school curriculum CMP nevertheless ignores the conceptual issues, bypassing the long division algorithm and substituting calculators and faulty inductive reasoning instead.

Steven Leinwand of the Connecticut Department of Education was a member of the expert panel that made final decisions on ED's "exemplary" and "promising" math curricula. He was also a member of the advisory boards for two programs found to be "exemplary" by the panel: CMP and the Interactive Mathematics Program. In a Feb. 9, 1994, article in Education Week, he wrote: "It's time to recognize that, for many students, real mathematical power, on the one hand, and facility with multidigit, pencil-and-paper computational algorithms, on the other, are mutually exclusive. In fact, it's time to acknowledge that continuing to teach these skills to our students is not only unnecessary, but counterproductive and downright dangerous."

Mr. Leinwand's influential opinions are diametrically opposed to the mainstream views of practicing scientists and mathematicians, as well as the general public, but they have found fertile soil in the government's "promising" and "exemplary" curricula.

Calculators

According to the Third International Mathematics and Science Study, or TIMSS, the use of calculators in U.S. fourth-grade mathematics classes is about twice the international average. Teachers of 39 percent of U.S. students report that students use calculators at least once or twice a week. In six of the seven top-scoring nations, on the other hand, teachers of 85 percent or more of the students report that students never use calculators in class.

Even at the eighth-grade level, the majority of students from three of the top five scoring nations in the TIMSS study (Belgium, Korea, and Japan) never or rarely use calculators in math classes. In Singapore, which is also among the top five scoring countries, students do not use calculators until the seventh grade. Among the lower achieving nations, however, the majority of students from 10 of the 11 nations with scores below the international average--including the United States--use calculators almost every day or several times a week.

Of course, this negative correlation of calculator usage with achievement in mathematics does not imply a causal relationship. There are many variables that contribute to achievement in mathematics. On the other hand, it is foolhardy to ignore the problems caused by calculators in schools. In a Sept. 17, 1999, Los Angeles Times editorial titled "L.A.'s Math Program Just Doesn't Add Up," Milgram and I recommended that calculators not be used at all in grades K-5 and only sparingly in higher grades. Certainly there are isolated, beneficial uses for calculators, such as calculating compound interest, a seventh-grade topic in California. Science classes benefit from the use of calculators because it is necessary to deal with whatever numbers nature gives us, but conceptual understanding in mathematics is often best facilitated through the use of simple numbers. Moreover, fraction arithmetic, an important prerequisite for algebra, is easily undermined by the use of calculators.

Specific shortcomings

A number of the programs on ED's list have specific shortcomings--many involving use of calculators. For example, a "promising" curriculum called Everyday Mathematics says calculators are "an integral part of Kindergarten Everyday Mathematics" and urges the use of calculators to teach kindergarten students how to count. There are no textbooks in this K-6 curriculum, and even if the program were otherwise sound, this is a serious shortcoming. The standard algorithm for multiplying two numbers has no more status or prominence than an Ancient Egyptian algorithm presented in one of the teacher's manuals. Students are never required to use the standard long division algorithm in this curriculum, or even the standard algorithm for multiplication.

Calculator use is also ubiquitous in the "exemplary" middle school program CMP. A unit devoted to discovering algorithms to add, subtract, and multiply fractions ("Bits and Pieces II") gives the inappropriate instruction, "Use your calculator whenever you need it." These topics are poorly developed, and division of fractions is not covered at all. A quiz for seventh-grade CMP students asks them to find the "slope" and "y-intercept" of the equation $10 = x - 2.5$, and the teacher's manual explains that this equation is a special case of the linear equation $y = x - 2.5$, when $y = 10$, and concludes that the slope is therefore 1 and the y-intercept is -2.5. This is not only false, but is so mathematically unsound as to undermine the authority of classroom teachers who know better.

College Preparatory Math (CPM), a high school program, also requires students to use calculators almost daily. The principal technique in this series is the so-called guess-and-check method, which encourages repeated guessing of answers over the systematic development of standard mathematical techniques. Because of the availability of calculators that can solve equations, the introduction to the series explains that CPM puts low emphasis on symbol manipulation and that CPM differs from traditional mathematics courses both in the mathematics that is taught and how it is taught. In one section, students watch a candle burn down for an hour while measuring its length versus the time and then plotting the results. In a related activity, students spend a whole class period on the athletic field making human coordinate graphs. These activities are typical of the time sacrificed to simple ideas that can be understood more efficiently through direct explanation. But in CPM, direct instruction is systematically discouraged in favor of group work. Teachers are told that as "rules of thumb," they should "never carry or grab a writing implement" and they should "usually respond with a question." Algebra tiles are used frequently, and the important distributive property is poorly presented and underemphasized.

Another program, Number Power--a "promising" curriculum for grades K-6--was submitted to the California State Board of Education for adoption in California. Two Stanford University mathematics professors serving on the state's Content Review Panel wrote a report on the program that is now a public document. Number Power, they wrote, "is meant as a partial program to supplement a regular basic program. There is a strong emphasis on group projects--almost the entire program. Heavy use of calculators. Even as a supplementary program, it provides such insufficient coverage of the [California] Standards that it is unacceptable. This holds for all grade levels and all strands, including Number Sense, which is the only strand that is even partially covered."

The report goes on to note, "It is explicitly stated that the standard algorithms for addition, subtraction, and multiplication are not taught." Like CMP and Everyday Math, Number Power was rejected for adoption by the state of California.

Interactive Mathematics Program, or IMP, an "exemplary" high school curriculum, has such a weak treatment of algebra that the quadratic formula, normally an eighth- or ninth-grade topic, is postponed until the 12th grade. Even though probability and statistics receive greater emphasis in this program, the development of these topics is poor. "Expected value," a concept of fundamental importance in probability and statistics, is never even correctly defined. The Teacher's Guide for "The Game of Pig," where expected value is treated, informs teachers that "expected value is one of the unit's primary concepts," yet teachers are instructed to tell their students that "the concept of expected value is nothing new ... [but] the use of such complex terminology makes it easier to state complex ideas." (For a correlation of lowered SAT scores with the use of IMP, see Milgram's paper at ftp://math.stanford.edu/pub/papers/milgram.)

Core-Plus Mathematics Project is another "exemplary" high school program that radically de-emphasizes algebra, with unfortunate results. Even Hyman Bass--a well-known supporter of NCTM-aligned programs and a harsh critic of the open letter to Secretary Riley--has conceded the program has problems. "I have some reservations about Core Plus, for what I consider too shallow a coverage of traditional algebra, and a focus on highly contextualized work that goes beyond my personal inclinations," he wrote in a nationally circulated e-mail message. "These are only my personal views, and I do not know about its success with students."

Milgram analyzed the program's effect on students in a top-performing high school in "Outcomes Analysis for Core Plus Students at Andover High School: One Year Later," based on a statistical study by G. Bachelis of Wayne State University. According to Milgram, "...there was no measure represented in the survey, such as ACT scores, SAT Math scores, grades in college math courses, level of college math courses attempted, where the Andover Core Plus students even met, let alone surpassed the comparison group [which used a more traditional program]."

And then there is MathLand, a K-6 curriculum that ED calls "promising" but that is perhaps the most heavily criticized elementary school program in the nation. Like Everyday Math, it has no textbooks for students in any of the grades. The teacher's manual urges teachers not to teach the standard algorithms of arithmetic for addition, subtraction, multiplication, and division. Rather, students are expected to invent their own algorithms. Numerous and detailed criticisms, including data on lowered test scores, appear at http://www.mathematicallycorrect.com.

How could they be so wrong?

Perhaps Galileo wondered similarly how the church of Pope Urban VIII could be so wrong. The U.S. Department of Education is not alone in endorsing watered-down, and even defective, math programs. The NCTM has also formally endorsed each of the U.S. Department of Education's model programs (http://www.nctm.org/rileystatement.htm), and the National Science Foundation (Education and Human Resources Division) funded several of them. How could such powerful organizations be wrong?

These organizations represent surprisingly narrow interests, and there is a revolving door between them. Expert panel member Steven Leinwand, whose personal connections with "exemplary" curricula have already been noted, is also a member of the NCTM board of directors. Luther Williams, who as assistant director of the NSF approved the funding of several of the recommended curricula, also served on the expert panel that evaluated these same curricula. Jack Price, a member of the expert panel is a former president of NCTM, and Glenda Lappan, the association's current president, is a coauthor of the "exemplary" program CMP.

Aside from institutional interconnections, there is a unifying ideology behind "whole math." It is advertised as math for all students, as opposed to only white males. But the word all is a code for minority students and women (though presumably not Asians). In 1996, while he was president of NCTM, Jack Price articulated this view in direct terms on a radio show in San Diego: "What we have now is nostalgia math. It is the mathematics that we have always had, that is good for the most part for the relatively high socioeconomic anglo male, and that we have a great deal of research that has been done showing that women, for example, and minority groups do not learn the same way. They have the capability, certainly, of learning, but they don't. The teaching strategies that you use with them are different from those that we have been able to use in the past when ... we weren't expected to graduate a lot of people, and most of those who did graduate and go on to college were the anglo males."

Price went on to say: "All of the research that has been done with gender differences or ethnic differences has been--males for example learn better deductively in a competitive environment, when--the kind of thing that we have done in the past. Where we have found with gender differences, for example, that women have a tendency to learn better in a collaborative effort when they are doing inductive reasoning." (A transcript of the show is online at (http://mathematicallycorrect.com/roger.htm.)

I reject the notion that skin color or gender determines whether students learn inductively as opposed to deductively and whether they should be taught the standard operations of arithmetic and essential components of algebra. Arithmetic is not only essential for everyday life, it is the foundation for study of higher level mathematics. Secretary Riley--and educators who select mathematics curricula--would do well to heed the advice of the open letter.

David Klein is a professor of mathematics at California State University at Northridge.

Marks of a good mathematics program

It is impossible to specify all of the characteristics of a sound mathematics program in only a few paragraphs, but a few highlights may be identified. The most important criterion is strong mathematical content that conforms to a set of explicit, high, grade-by-grade standards such as the California or Japanese mathematics standards. A strong mathematics program recognizes the hierarchical nature of mathematics and builds coherently from one grade to the next. It is not merely a sequence of interesting but unrelated student projects.

In the earlier grades, arithmetic should be the primary focus. The standard algorithms of arithmetic for integers, decimals, fractions, and percents are of central importance. The curriculum should promote facility in calculation, an understanding of what makes the algorithms work in terms of the base 10 structure of our number system, and an understanding of the associative, commutative, and distributive properties of numbers. These properties can be illustrated by area and volume models. Students need to develop an intuitive understanding for fractions. Manipulatives or pictures can help in the beginning stages, but it is essential that students eventually be able to compute easily using mathematical notation. Word problems should be abundant. A sound program should move students toward abstraction and the eventual use of symbols to represent unknown quantities.

In the upper grades, algebra courses should emphasize powerful symbolic techniques and not exploratory guessing and calculator-based graphical solutions.

There should be a minimum of diversions in textbooks. Children have enough trouble concentrating without distracting pictures and irrelevant stories and projects. A mathematics program should explicitly teach skills and concepts with appropriately designed practice sets. Such programs have the best chance of success with the largest number of students. The high-performing Japanese students spend 80 percent of class time in teacher-directed whole-class instruction. Japanese math books contain clear explanations, examples with practice problems, and summaries of key points. Singapore's elementary school math books also provide good models. Among U.S. books for elementary school, Sadlier-Oxford's *Progress in Mathematics* and the Saxon series through Math 87 (adopted for grade six in California), though not without defects, have many positive features.--*D.K.*

For more information

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Los Angeles Times
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MULTIPLYING MATH WOES

Congressional debates over how to dole out federal education dollars usually turn on social issues like whether to incorporate Judeo-Christian values into school lesson plans. This time, however, Washington is inexplicably debating whether students should be learning the most basic of lessons: the multiplication tables. Cultural disagreements are understandable. This one is not.

In international math surveys, U.S. students have consistently lagged behind students in nations like Japan and the Czech Republic that emphasize memorization. Nevertheless, the National Science Foundation continues to lavish most of its yearly grants on school districts that implement so-called nontraditional math programs emphasizing concepts, not facts. In 1977, a foundation official threatened to cut off federal funds unless California education officials reversed a state mandate that third-graders memorize multiplication tables.

Last month, the U.S. Department of Education sent its clearest signal yet to local school districts, issuing a report that praised only nontraditional math programs as "exemplary and promising." The report claims to have used a "rigorous . . . research-based process" to identify effective programs, but math education experts like Wayne Bishop of Cal State Los Angeles assert that its authors, well-known proponents of nontraditional math, based their conclusions on a highly selective set of student achievement tests.

Education Secretary Richard W. Riley ought to withdraw the report, as a coalition of 200 mathematicians and scientists asked in a letter last week. At the very least, he should make it clear that nontraditional math may complement but not replace traditional math.

Some skilled math teachers have managed to inspire students by making the best of nontraditional math's murky goals: "linking past experience to new concepts; sharing ideas; developing concept readiness through hands-on explorations," according to Mathland, a program the Education Department report calls promising and which has strong advocates in Los Angeles schools. But what are teachers to make of this second-grade exercise from Mathland? It asks students to think up a lunch, draw it on paper, then cut out the foods, supposedly in the name of learning division.

Even state Supt. of Instruction Delaine Eastin, who has supported nontraditional math, admits that the method works only when teachers "have a deep understanding of mathematics content," and such teachers are in short supply in California classrooms. To help correct that, Gov. Gray Davis should promise to sign a pending bill by Sen. Hilda Solis (D-La Puente) that would forgive more of the college loans of teachers who take courses that increase their math proficiency. But California's efforts to improve math education are being undercut by the federal Education Department's funding of nontraditional math nearly to the exclusion of traditional math.

If public leaders cannot agree on the importance of lessons as basic as the multiplication tables, it's fair to ask whether they are leaders at all.

Mathematically Correct

"There is a mathematically correct solution"
PO Box 22083, San Diego, CA 92192-2083
http://www.mathematicallycorrect.com

Testimony to the United States House of Representatives Committee on Education and the Workforce

January 21, 1998

Thank you, Chairman Goodling and Members of the Committee, for the opportunity to comment on the planned Voluntary National Test in Mathematics.

I am a biomedical research statistician at the Department of Veterans Affairs Medical Center in San Diego affiliated with UCSD, and I have had extensive exposure both to mathematics and to the principles of measurement and test construction. I served on the 1997 California Mathematics Framework Committee. I am a member of the San Diego Grade-Level Mathematics Standards Committee and of math textbook adoption committees. I was an author of the Algebra exam that the Mathematics Council of Western Pennsylvania used in their 1997 competition for middle school students. However, I am not representing any of these organizations in my comments to you.

I come, instead, as a cofounder and representative of Mathematically Correct, a parents' advocacy group for mathematics education, and as the father of two children in public school. Mathematically Correct was founded in 1995 by parents who were frustrated by the weaknesses we saw in math education in our public schools. We quickly found that we were not alone in our frustration, and we are now supported by parents and mathematicians across the United States. Indeed, through our voluntary efforts, we have become the most widely recognized voice for parents' concerns about mathematics education in the country.

Our Call for Voluntary Examinations in Mathematics

In April of 1996, Mathematically Correct released a position paper[1] calling for voluntary regional or national examinations covering the standard contents of each secondary course. It was, and still is, our feeling that the establishment of clear expectations, and the publication of examination results based on these expectations, could go a long way toward improving mathematics education in this country. Two important points we noted are:

> 1) The course expectations and the related examinations should be prepared by committees of mature, well-established mathematicians; and,
>
> 2) The examinations should be externally developed and graded, independent of the existing educational infrastructure in this country.

A Good Idea Goes Wrong

Given the similarity of our recommendations to the plan for a national test in mathematics, it might be expected that we would be strongly supportive of such a plan. In reality, however, we find the plan for the test so objectionable that we are strongly against it. In August of 1997, Mathematically Correct wrote letters to President Clinton[2] and to the test design committee[3] outlining our criticisms of the test plan.

To put it bluntly, we feel that the planned test would be worse than no test at all. It is difficult to clarify the detailed reasons for our objections in a short statement, but some of the central issues can be highlighted.

A Schism in Math Education

Whether you are aware of it or not, there is great controversy about math education in America today. The members of Mathematically Correct quickly discovered that the weak programs our children encountered were stimulated by what has been called a "reform" movement in math education. Although praised by certain groups of educators, many parents see the products of this "reform" as dumbed-down math programs. Although promoted by flowery but empty rhetoric from some education groups, these inadequate programs are responsible for the birth of Mathematically Correct in the first place.

One cannot fully appreciate the inferiority of these programs merely on the say-so of irate parents. I urge you to embark on an educational experience of your own. Look at Glencoe Interactive Mathematics: Activities and Investigations[4]. The third book in this program, units 13 to 18, is the 8th grade text for this series. This program was highly rated in California and is now in use in many of our public schools. Yet, the "reform" math covered in this series is grossly inadequate. In fact, these are the worst math books I have ever seen.

Unfortunately, the committee that designed the specifications for the national test in mathematics was heavily laden with individuals known to be tied to the very "reform" movement that brought such textbooks into existence. There is far too much wrong with this "reform" to be detailed here. The point, however, is that parents see the "reform" as a serious threat to rigorous math education, yet the design committee was strongly biased in favor of it. With Professor Dossey at the helm, they produced an unbalanced test plan that is headed in the wrong direction - one that would specifically support the movement we find so objectionable.

Test Characteristics Make a Real Difference

Without thorough study, it is difficult to understand how a mathematics test could be slanted in one way or another to any great extent. Let me summarize just one example that is documented on our web site[5]. Here, traditional and "reform" introductory algebra programs were run simultaneously in the same school. At the end of the year, students were evaluated in two ways - with a regular, objective final exam and with California's Golden State Exam. The Golden State Exam is used for achievement recognition and is modestly slanted to favor the "reform" programs.

The results were that 55% of the traditional program students earned A or B grades on the final, while only 11% of the "reform" program students earned A or B grades. Also, 80% of the "reform" program students earned D or F grades, while only 31% of the traditional program students did this poorly. The traditional final exam shows a huge difference between the programs.

In contrast to these findings, the proportion of students receiving either honors or high honors on the Golden State Exam was the same for both math programs. Not only that, but more "reform" program students earned recognition on the Golden State Exam than earned a C or better on the regular final exam. Clearly, the nature of the test dramatically alters our conclusions.

Recognizing that the tests can be designed in a way that hides dramatic differences in student achievement, we need to look at some of the specific problems with the National Voluntary Test plan.

Arithmetic and Algebra

There are two important areas in mathematics that are seriously slighted in so-called "reform" math and in the planned exam. These are known as arithmetic and algebra - subjects that parents perceive as vital for student success. Even President Clinton, in his ten-point call to action for American education, states that, "...every 8th grader should know basic math and algebra."[6]

With respect to the operations of arithmetic, the design committee intentionally avoids testing basic computational skills under the assumption that all students will know these things. Unfortunately, this is far from universally true among American 8th graders. Indeed the National Assessment of Educational Progress (NAEP) indicates that 21% of 8th graders are not proficient at the level of adding, subtracting, multiplying and dividing whole numbers and solving one-step problems[7]. Failing to directly address these basic skills appears to be nothing less than an attempt to hide weakness even at the lowest levels.

The exam ought to reflect the President's prescription that all 8th graders will learn Algebra. This is an important goal for our country. We cannot monitor our progress toward that goal if it is not measured. In spite of calling 25% of the test "Algebra," the content addressed is not the content of an introductory algebra course at all - it doesn't even cover pre-algebra. It appears that the test planners want to fool the country into believing that these American students have learned algebra. By ignoring the demanding content of algebra, the design will fail to measure success at the level of our international competition. It will also fail as a vehicle to promote greater mathematics achievement.

We have a link on the Mathematically Correct web site to samples of math problems given to Japanese 12-year-olds[8]. The level of these problems obviously exceeds what is planned for American 8th graders. This is clear evidence that the planned exam is not designed to measure or promote high levels of achievement.

Calculators, Guessing and Non-Standard Administration

Under the guise of allowing multiple solution strategies, an effort is being made to design test items that promote guessing rather than more powerful and general analytical methods. This works against the goal of promoting accuracy, clear thinking processes, and careful work. This is one of the ways "reform"- friendly tests can be constructed to disguise limited achievement.

Similarly, the more successful countries discourage the use of calculators in the early grades, yet the test design would effectively promote their use. Although these "reform" educators believe that calculators promote math learning in young students, over 80% of the public believes that their use should be limited[9, 10]. It is distressing to read the minutes of the design committee where we learn that they want even more calculator use than the public might accept, so they planned to build to even more calculator use over time. Flagrant disregard and disrespect for the opinions of parents and the general public cannot be tolerated.

Worse yet, the policy on calculator use, and even the time permitted for the exam, seems to allow differing examination conditions from one locale to another. Thus, the test-administration will be inconsistent or non-standard, greatly reducing the validity of scores for comparison purposes.

Unreliable Methods and Credit for Trying

The test plan calls for a large number of hand-scored items despite the fact that these items are known to suffer from subjective grading and low reliability. The fraction of total points dedicated to these items is high as they are given more weight than objectively scored items. It is likely that subjectively-scored questions will have a substantial, and artifactual, impact on scores.

Unreliable, subjective items are endorsed by the so-called "reform" educators. These items open the door to giving credit for wrong answers, as long as the scorers feel that the student had a good approach to the problem. In fact, this method not only leads to inconsistency in grading, but graders may be influenced by the students' political correctness, attitudes, and degree of social insight. These characteristics have no place in evaluating math achievement. Yet, it is this very sort of encroachment that contributed to the downfall of the "reform"-oriented CLAS test in California.

Subjective items also bring a significant increase in the cost to administer the test and delay the return of results to the concerned parties. For these reasons, the number of subjectively-scored items and the fraction of total points they represent should be greatly reduced.

Psychometrically Incorrect

Non-standard conditions of administration and subjective and unreliable grading techniques are clear psychometric errors. They are inconsistent with established, reliable assessment practices. These weak forms of assessment should not be considered to be valid for making inferences regarding the achievement of individual students. This places the test design in direct conflict with the stated goals of the testing program.

Unlike the NAEP, which is intended to provide data on large, aggregate units, the test plan is supposed to provide meaningful data on individuals. But the range of individual achievement is necessarily much broader than for aggregate units such as means for entire states. For such a plan to succeed, it needs to cover an even broader range of achievement than the NAEP - both higher and lower. For a test to be diagnostically useful, it would also benefit from a greater number of more specifically defined sub-scale scores. None of these features are provided.

Finally, the notion that the examination can and should be modified over time, as suggested by the design committee, is also a serious mistake. We cannot track progress over the years if the target keeps moving. The intention of the committee to use changes to the examination as a scheme to gradually pull Americans toward their own philosophy is an insult and is inappropriate if we are to measure our progress as a nation.

What is Correct for California?

After leaping head-first into the "reform" movement and falling to the bottom of the achievement ladder, California is now setting clear and ambitious goals. Many have noted that our newly adopted Math Standards[11] expect a higher level of achievement than those of any other state, and I would have to agree with them. Our Standards and Draft Framework [12] are geared toward completing the primary school math topics by grade 7 with named content areas, like algebra and geometry, starting in grade 8. We are committed and legally bound to developing tests that are designed to measure the content of the our state Standards.

Importantly, our algebra standards are real algebra, not just algebra in name only. While we want all of our students to learn introductory algebra in grade 8, it is critical that they learn the real thing. Algebra in name only for 8th grade students is nothing more than the basis of an advertisement or public relations campaign, and is not what we are interested in. Our standards-based exam in algebra will be an algebra test. The planned voluntary test is not.

Baring unforseen and drastic changes in the national test plan, the 8th grade math test will be of no value to California. If it is funded and developed as planned, Mathematically Correct will recommend that our own state not waste student time by participating. We would make the same recommendation to any other state that sincerely wants to move their students to a high level of achievement.

Alternatives to the Test Plan

The management of the national test plan has been moved to the National Assessment Governing Board (NAGB), the semi-independent group that oversees the NAEP. This is a move in the correct direction. It is even possible that the NAGB will institute a wholesale revision to repair the test plan. Whether or not the Board has the fortitude to correct the flawed design is open to question. The same educators that are behind the "reform" movement will pressure the Board as well. Even the NAEP faces these pressures. The two math consultants for the NAEP Mathematics Framework itself are none other than Professor Dossey and Cathy Seeley, both listed as "program conceptualizers" for the text [13] that has come to be known as "Rain Forest" Algebra [14], [15]. Given this situation, we are not optimistic about the prospects for a wholesale revision to repair the test design. The NAGB would have to be willing to take drastic steps if there is to be any hope of a balanced approach to a valid, reliable, and useful test plan.

There has also been talk of using existing tests as an alternative, noting that many states already conduct their own assessments. It is clear that these off-the-shelf tools are less than optimal. They do not address specific course contents and do not measure up to the achievement goals our country should be striving for. However, the Voluntary National Test, as planned, does not improve upon this situation. In many ways, it actually makes things worse. Mathematically Correct would rather see existing assessment tools or no test at all than the use of the Voluntary National Test as planned.

Our Conclusion

As representatives of concerned parents throughout the country, the members of Mathematically Correct implore you to take every effort to address these issues. We could benefit from a well-structured national assessment device, but we clearly don't need a tool to further promote the "reform" agenda in math education. Short of a balanced, major overhaul in design, the test can be expected to do more harm than good for the mathematical education of our nation's youth.

Respectfully Submitted
by
Paul Clopton
Member and Cofounder,
Mathematically Correct

Notes

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[3] Comments on the National 8th Grade Mathematics Test, Mathematically Correct

[4] Interactive Mathematics: Activities and Investigations, David Foster, Sandie Gilliam, Jack Price, et. al., Glencoe Division, Macmillan/McGraw-Hill, Westerville, Ohio, 1995.

[5] Effectiveness of CPM vs Traditional Math, Robert W. Haswell

[6] President Clinton's Call to Action for American Education in the 21st Century

[7] NAEP 1996 Mathematics Report Card for the Nation and the States

[8] Japanese Math Challenge, Pacific Software Publishing, 1996.

[9] What Do Parents and the Public Think About Our Schools?, Public Agenda.

[10] 1996 State of the Union Address, Albert Shanker, President, American Federation of Teachers.

[11] State Board of Education Ad Hoc Draft California Mathematics Standards

[12] Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve, Draft, Sept. 5, 1997

[13] Focus on Algebra [Addison-Wesley Secondary Math: An Integrated Approach], R.I. Charles and A. G. Thompson, et. al., Addison-Wesley, Menlo Park, California, 1996.

[14] 'Rain Forest' Algebra Course Teaches Everything but Algebra, Marianne Jennings

[15] More on Addison-Wesley Focus on Algebra, Richard Askey

February 13, 2000

From: Concerned Parents of Reading (CPR), MA
To: The State Board of Education
Re: Proposed Changes to the State Math Frameworks Standards

Dear State Board of Education:

Concerned Parents of Reading (CPR) was formed by a group of parents who became concerned about Reading's implementation of the "Chicago Math Program" Everyday Mathematics following the presentation of the program at the Birch Meadow Elementary School Parents' Night in April 1997. As parents we had noticed that the computational skills of our younger children (K-4) did not seem to be as advanced as their older siblings when they were at the same grade level. We soon discovered that high grades did not necessarily indicate mathematical competency. Our children overly relied upon the use of calculators and, when challenged with the simplest of problems, resorted to finger counting. Further review revealed that textbooks were replaced by disposable journals and that, even at the third and fourth grade level, our children didn't fully know their multiplication tables. There was a lot of writing about mathematics in the journals, but that less emphasis was placed on the ability to perform math computational skills. (Chicago Math is one of the "constructivist" math programs allegedly designed to enhance "problem solving skills" with a de-emphasis of computational skills.)

Concerned Parents of Reading's research led us to uncover California's experience with the "New New Math" programs. Beginning in 1985, California adopted the first of the new standards in math structure which led to fundamental changes in the way math was taught. (This actually preceded the National Council of Teachers in Mathematics (NCTM) standards which were adopted in 1989.) California modified their Mathematics Frameworks in 1992. Charles Sykes, the author of, "Dumbing Down Our Kids" (St. Martin's Press 1995), devotes a chapter to reviewing of the California Experience and the NCTM Standards. (Enclosed on a separate sheet is information, taken from his book, that is particularly applicable to our experiences in Reading and in Massachusetts as a whole.) CPR learned that the results of these early changes in Californian mathematics education were so pernicious that the 1991 National Assessment of Educational Progress (NAEP) rated the math skills of California's students in the bottom third of participating states. The damage was done in California but Concerned Parents of Reading believed our schools need not make the same mistake, or so we naively thought at the time.

On May 14th, 1997, CPR met with School Superintendent Dr. Haratunian and Assistant Superintendent Dennis Richards to discuss our concerns and our findings. At Dr. Haratunian's request CPR presented a discussion list : 1. CPR's first request was the testing of all fourth graders for mathematics achievement by the California Achievement Test and to evaluate the results before committing to expansion of The UCSMP program. 2. CPR's second request was the formation of a Mathematics Task Force composed of teachers, educators and parents with higher levels of mathematics and science skills. (Everything we have learned about " New New Math", the NCTM Standards and the UCSMP program convinces us now more than ever that a Task Force is necessary to review Reading Math Standards and their implementation.) 3. CPR's third request was to review how the UCSMP (Everyday Mathematics) was selected for Reading's school system.

What became of the above requests? Only after six months of hard lobbying were the 5th graders tested by the newly revised Stanford "9", a NCTM aligned examination. Unfortunately, both the school administrators and the school committee declined to take any action on the test data. Concerned Parents of Reading had to go to the State, under the Freedom of Information Act, to obtain the previous Stanford 8 scores which represent the record of Reading's earlier, older math curriculum. (An enclosed chart details a summary of the specific data we obtained.)

Reading's math performance was in the 80 percentiles in the years 1993-1995. A downward trend appears in 1996 and 1997. The first students utilizing the "Chicago Math" program were tested in 1997 with the Stanford 9. This downward trend continues in the "Chicago Math" program with computational skills dipping to the 67th percentile in 1997 and Total Math dropping to 73 percentile. The same group of 1997 students were re-tested in 1998 and the data shows a continuing drop in their computational skills.

Concerned Parents of Reading had only asked that a balanced math program be used in the Reading Schools, one that stressed both concepts and computational skills and that grade by grade learning goals and objectives be achieved! Instead, our parent group was stonewalled. Data was denied which repeatedly forced us to obtain information under the State's Freedom of Information Act, including access to an anonymous teacher survey done by the school administration.. The anonymous survey revealed that 86 % of our elementary school teachers felt the program had major problems. Some central themes expressed by the teachers were: the material is too scattered, there is too much skipping around, moving on without mastery, too much material, no clear identification of what's essential and the list continues. Yet, the superintendent had released a report on Reading's math program, prior to CPR obtaining the real surveys, which distorted the results of the teacher survey and reported they actually loved the program! The superintendent denied our parent group copies of the original teachers' surveys until, with help from the State, our group finally obtained the information.

The objective test results indicating problems with Chicago Math were also ignored. Today the "Chicago Math" program continues to be used from K-6 grade in Reading, in spite of the objective data, which shows it to be not only inferior to our old curriculum but also more expensive. The 1999 curriculum report to the School Committee by the Elementary School Math coordinator did admit that "modifications and adaptations" are being done to the "Chicago Math" program. However, no explanations of what these "modification and adaptations" were discussed. Curriculum changes following the 1989 NCTM standards have been expanded through at least Grade 9, at great expense to the community with questionable results.

As parents we applaud the Board's decision to revise the State Mathematical Frameworks, to be more specific and to specify grade by grade learning objectives. I would be glad to supply any additional supporting documents that the Board would like to examine. The current Mathematical Frameworks have perpetuated poor math skills and, as our experience in Reading illustrates, change has to come from a higher authority than the local school board and school administration if Massachusetts children are to receive adequate math education in the public schools.

Sincerely,

Robert L. Mandell, DMD,MMSc.
Former Chairman of Concerned Parents of Reading, MA

READING'S STANFORD 8 (OLD MATH) VS. STANFORD 9 (NEW MATH)

CPR obtained **Stanford "8"** examinations for the years 1993-1997 Intermediate #2 Grade 5 @ 5.5-6.5 given at 5.8 (April). Results expressed as Percentile Rank-Stanine. These Stanford 8 scores represent the older curriculum.

Year	**Total Math** (118 Items)	**Concepts Math** (34 Items)	**Math Computation** (44 Items)	**Math Applications** (40 Items)
1993	87-7	84-7	81-7	89-8
1994	81-7	76-6	76-6	83-7
1995	83-7	75-6	81-7	84-7
1996	77-7	72-6	73-6	81-7
1997	75-6	72-6	70-6	76-6

The following Stanford "9" results display the new "Chicago Math" curriculum. The examination was the Intermediate #1 Grade 5 @ 4.5-5.5 given at 5.2 (October). Results Percentile Rank-Stanine.

Year	Total Math (78 Items)	Problem Solving (48 Items)	Procedures (30 Items)
1997	73-6	76-6	67-6

Results for Grade 6 October 1998 and October 1999 (Grade 6@ Level 5.5-6.5 given 6.2 October) Intermediate Level #2 (represents three extra months of study)

Year	Total Math (78 Items)	Problem Solving (48 Items)	Procedures (30 Items)
1998	75-6	80-7	64-6
1999	76-6	80-7	67-6

Excerpts from Charles Sykes Book

Charles Sykes, the author of, "Dumbing Down Our Kids" (St. Martin's Press 1995), devotes chapter nine entitled "Why Johnny can't add, subtract, multiply, or divide (but still feels good about himself) (pp. 114-126) to a review of the California Experience and the NCTM Standards (pp. 117-118).

In that year, the NCTM launched the new new math revolution by issuing a comprehensive set of new standards for teaching math. The new standards describe "a vision" for school mathematics that insists that the nations schools embrace a curriculum for teaching mathematics " that capitalizes on children's intuitive insights and language: and that is guided not by the standards of a recognized discipline but by the "children's intellectual, social, and emotional development..." This means that from now on teacher should spend less time on "developmental work that emphasized symbol manipulation and computational rules, and that rely heavily on paper and pencil worksheets...". Such outdated approaches of teaching mathematical rules, the new standards sniffed, "do not fit the natural learning patterns of children and do not contribute to important aspects of children's mathematical development

Sykes then begins to discuss What's in and What's out for the K-4 grades

What's in

- The Standards call for teacher to devote more time and attention to
- Cooperative work
- Discussion of mathematics
- Questioning
- Writing about Mathematics
- Content integration
- Exploration of chance
- Problem-solving strategies
- Use of Calculators and computers
- The NCTM standards want teacher to give decreased attention to:

What's out

- Early attention to reading, writing and ordering numbers symbolically
- Complex paper-and -pencil computations
- Addition and subtraction without renaming
- Isolated treatment of division facts
- Long divisionLong division without remainders
- Paper- and-pencil fraction computation
- Rote practiceRote memorization of rules
- One answer and one methodWritten practice
- Teaching by telling

For fifth through eighth graders, the NCTM standards proposed de-emphasizing

- Relying on outside authority (teacher or an answer key)
- Memorizing rules and algorithms
- Practicing tedious paper- and pencil computations
- Finding exact forms of answers
- Memorizing procedures, such as cross multiplication.

.... the NCTM's standards envision a dramatically expanded role for calculators.......
........"Calculators", the NCTM standards declare "must be accepted at the k-4 level as valuable tools for learning mathematics." The standards argue that calculators have made "computational proficiency" obsolete. And ,since schools have failed in the past to teach students to master basic computational skills without their aid, the standards read, "we might argue that further effort toward mastering computational skills are counterproductive." In other words, the standards are a declaration of surrender.

The results of these early changes in Californian mathematics education were so pernicious , that the 1991 National Assessment of Educational Progress (NAEP) rated the math skills of California's students in the bottom third of participating states. These poor results caused local math activist parents groups to organize and their efforts over the next several years, caused The California Board of Education to reexamine the 1992 Math Frameworks and in 1996, a New Mathematics Advisory Program was released. **The Advisory made it clear that basic mathematics skills should be used routinely and automatically, that these skills should be practiced regularly, and that they should be committed to memory**. This Advisory amounts to a repudiation of the NCTM frameworks. However, the damage has been done to California's students and it will take years to repair it. The San Francisco Examiner reported, 2/28/1997, results from the U.S. Department of Education, that **California 's fourth graders scored fourth worst in the United States.** California has now revised and adopted (1999) their math standards so that clearly defined goals and learning objective and skills are specified for mastery at each grade level. Please note that the new California Frameworks do not tell one how to teach only that each grade level should master certain skills

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Hyman Bass Attacks Massachusetts Deputy Commissioner of Education Sandra Stotsky For Her Support of Arithmetic

Introduction

In November, 1999, Hyman Bass, professor of mathematics at the University of Michigan and at that time incoming president of the American Mathematical Society, slandered Sandra Stotsky, the Deputy Commissioner for Academic Affairs and Planning for the State of Massachusetts in a nationally distributed e-mail message, and has consistently refused to retract and apologize for what he said despite a total lack of evidence to support his slanderous statements. In his message, he harshly criticized the Open Letter to U.S. Education Secretary Richard Riley that was endorsed by more than 200 mathematicians and other scholars, including Sandra Stotsky. Professor Bass' vitriolic letter was posted with his permission, on national list serves, and included the statement:

> [Sandra Stotsky's] ideological and uninformed opposition to "constructivist ideas" has reached the incredible state where she is opposed to inclusion of discussion of "Classical Greek constructions" as being "constructivist pedagogy." Is this what serious mathematicians want to associate themselves with?

In a subsequent message, also circulated nationally with Professor Bass' permission, he quoted Professor Maurice Gilmore, the source for his accusation, as follows:

> The State [of Massachusetts] has appointed a woman, Sandra Stotsky, who is so ignorant of mathematics that she debated for 30 minutes with us to remove "classical Greek constructions" from our document because it was "Constructivist pedagogy." There is much more, not surprisingly about algorithms. To her, algorithms are purely operative for the 4 arithmetic operations with integers, no more than that.

Professor Bass' accusations are completely fraudulent. Dr. Stotsky has given tapes and transcripts to Professor Bass which record the meeting in question. They prove that Bass' accusations are without any basis in reality. They are complete fabrications. Stotsky has consistently and strongly supported the inclusion of Euclidean geometry in the schools, and there is a paper trail to prove this. The term "classical Greek constructions" was never used by Dr. Stotsky nor by anyone at the meetings she attended, nor was any other phrase with any similar meaning ever uttered by her or anyone else at the one meeting with Professor Gilmore she ever had, as the tapes and transcripts of that meeting prove. Moreover the phrase does not appear in any of the draft versions of the Massachusetts Mathematics Framework.

Professor Bass' refusal to retract the false charges he disseminated about Sandra Stotsky have implicitly lent support to the campaign by former mathematics framework panelists Maurice Gilmore, Carol Greenes, Margaret Kenney, and Deborah Hughes-Hallett (and the other five members) to prevent the standard algorithms of arithmetic from being taught in Massachusetts elementary schools. Deputy Commissioner Stotsky insisted that knowledge of these standard algorithms of arithmetic be included as requirements in the Mathematics Framework. Her position in support of arithmetic, and her endorsement of the open letter to Secretary Riley make her a target for Hyman Bass.

In spite of a massive, concerted campaign by the mathematics framework panel and their allies to portray Deputy Commissioner Stotsky as someone who wanted simply to return to rote memorization of mathematical formulas and to prevent students from acquiring "deep mathematical understanding," she has proceeded with constructive revisions of the Massachusetts Mathematics Framework. Changes in the draft mathematics framework were made with assistance from Professor Wilfried Schmid of the Department of Mathematics at Harvard University. The revised Mathematics Framework now requires knowledge of the standard algorithms of arithmetic for elementary school students in Massachusetts and knowledge and understanding of the quadratic formula. The remaining changes are relatively minor, mostly to make the standards clearer and more specific. A recent op-ed in the Boston Globe by Stanley Spiegel, associate professor of mathematics at the University of Massachusetts at Lowell, summarizes the mathematical issues connected with the revisions of the framework.

Hyman Bass has been provided with the evidence we have described and upon which we elaborate below. In spite of repeated requests by Sandra Stotsky that he retract his inflammatory and false accusations, he has declined to do so.

Mathematically Correct supports Sandra Stotsky's efforts to promote sound mathematics education in Massachusetts, including her position in support of arithmetic in elementary school and algebra in eighth or ninth grade. Among the many proponents of "fuzzy math," Hyman Bass' circulated statements are being cited as "proof" that Deputy Commissioner Stotsky is a benighted fool. Bass' slander campaign is not only grossly unfair to Stotsky, it creates barriers to institutionalizing sound mathematics education policies in Massachusetts. It is therefore important to set the record straight on this matter.

The Story

In early October, 1999, the U.S. Department of Education released a report to the nation's 16,000 school districts which designated 10 mathematics programs for K-12 as "exemplary" or "promising." The following month an open letter endorsed by more than 200 scholars and education leaders calling for the withdrawal of these recommendations was sent to Education Secretary Richard Riley. The letter was published as a full page ad in the November 18 edition of The Washington Post. Among the signatories are many of the nation's most accomplished scientists and mathematicians. Department heads at 16 universities including Caltech, Harvard, Stanford, and Yale, along with two former presidents of the Mathematical Association of America also added their names in support, as did seven Nobel laureates and winners of the Fields Medal, the highest award in mathematics. The open letter was covered by the leading newspapers in the United States and a congressional hearing was convened on account of it.

In addition to the many scientists and mathematicians, several prominent education leaders at the state and national levels added their names to the open letter. Among these was Sandra Stotsky, Deputy Commissioner of Academic Affairs for the state of Massachusetts. Dr. Stotsky was in the midst of coordinating changes to the Massachusetts Mathematics Framework when the open letter was released. Based upon the advice of many mathematicians and a report of the American Mathematical Society, she insisted that the standard algorithms of arithmetic (i.e. the standard procedures for addition, subtraction, multiplication, and division of numbers) be included in the elementary school grades, even though the Framework Panelists she "inherited" argued against them and supported child invented algorithms via discovery learning. These mathematics education advisors even went so far to deny the very existence of standard algorithms of arithmetic in direct contradiction to the American Mathematical Society report.

Shortly after the release of the Open Letter to Education Secretary Riley, Professor Hyman Bass circulated a long condemnation of it in which he defended the "exemplary" and "promising" curricula and denounced the authors of the letter and Mathematically Correct, as well as a member of the California State Board of Education. His concluding sentence was,

> What appear to be very sensible reservations about what the Department of Education did, becomes in fact part of a veiled and systematic assault on the professional education community, in which Askey and Wu are perhaps inadvertently allied with people who have much more insidious political agendas.

The "insidious political agendas" that two of the co-authors of the Open Letter, Professors Richard Askey and Hung-Hsi Wu, were being used for were never identified by Bass, but they presumably include the teaching of the standard algorithms of arithmetic to elementary school children.

In a subsequent message dated Sept. 24, 1999, that was nationally circulated with his permission, he added his condemnation of Deputy Commissioner Stotsky. Specifically, Bass wrote:

> Mathematically Correct, an important agent in promoting this Open Letter, has been politically active around the country. In Massachusetts it is allied with efforts of the Deputy Commissioner of Education, Sandra Stotsky, to review proposed revisions to the State Framework. Her ideological and uninformed opposition to "constructivist ideas" has reached the incredible state where she is opposed to inclusion of discussion of "Classical Greek constructions" as being "constructivist pedagogy." Is this what serious mathematicians want to associate themselves with?

Hyman Bass had never spoken to or communicated in any way with Sandra Stotsky prior to spreading this attack on her. He did not even include her as a cc on his initial email message. When Dr. Stotsky learned of this message from Richard Askey, she wrote to back to him:

> Absolutely bizarre. I have never said or written anything whatsoever regarding 'Classical Greek constructions.' I wouldn't have even known what was being referred to if you hadn't explained that it was related to geometry. Can't imagine what it could even be a distortion of. If anything, I have been concerned about the teaching of Euclidean geometry, wanting to be sure it's there in the curriculum.
>
> I sure would like to know Hy Bass's source for this bit of specious nonsense.

Askey asked Bass on November 25, 1999, for the basis of his condemnation of Deputy Commissioner Stotsky. His reply was:

> Dear Dick,
>
> What I said was taken from a message from Maurice Gilmore. Let me quote below the relevant passages.
>
> Hy
>
> ______________________________
>
>> Dear Hy,
>> Not that we know each other. But I am former Chair here at Northeastern University, ...
>>
>> I was an algebraic topolgist in the past. Now I work in secondary math reform in Boston. I was Andy Gleason's "replacement" on the Panel to revise the State Frameworks in Mathematics. Andy helped write the first version. Both he and I worked with math teachers at every level. Deb Hughes-Hallett was also on the panel I served on. I write because the process here is rapidly being politicized by Mathematically Correct from California.
>>
>> The State has appointed a woman, Sandra Stotsky, who is so ignorant of mathematics that she debated for 30 minutes with us to remove "classical Greek constructions" from our document because it was "Constructivist pedagogy." There is much more, not surprisingly about algorithms. To her, algorithms are purely operative for the 4 arithmetic operations with integers, no more than that.
>>
>> She is Deputy Commissioner and will be in charge of the review of our document. We are taking steps to have the review be a public review, as the current set-up is isomorphic tho the situation which occurred in California.
>>
>> I am no "constructivist pedagog", I respect Connected Mathematics and work to implement it, along with extra work on both fractional and algebraic skill development, which the reform community here all are doing.

This extraordinary accusation is a complete fabrication. Sandra Stotsky met with Maurice Gilmore for the first time on July 27 and has not met with him since. Attending that July 27 meeting were David Driscoll, Commissioner of Education for Massachusetts, and the other math advisors serving on the Framework Panel, Carol Greenes and Deborah Hughes Hallett. The math advisors were fighting against the inclusion of the standard algorithms of arithmetic in the mathematics framework. Because of the official nature of this meeting, it was recorded and transcribed. Nothing resembling the phrase "classical Greek constructions" was uttered by anyone during the meeting and this can be verified by listening to the tapes or reading the transcript.

However, during this meeting and the following recorded dinner conversation, the math advisors explained several times that they had great mathematical expertise as they argued against the teaching of the standard arithmetic algorithms. They claimed that there is no such thing as standard arithmetic algorithms. At one point in the conversation, one of them claimed that the standard algorithm for adding numbers is mathematically incorrect! It was never explained by these allies of Hyman Bass how the standard procedure for adding two numbers could both be incorrect and not exist. Nor was it explained how a report of the American Mathematical Society could support the teaching of the standard algorithms if they don't exist. If the other two Framework Panelists present for that meeting had any reservations about these assertions of their colleague, they failed to express them.

A series of exchanges between Deputy Commissioner Stotsky, Professor Bass, and others occurred via email. There can be no doubt that Bass understood that his evidence against Stotsky had completely collapsed. Rather than admit that what he said was a mistake, he posted a message on a national education listserve. In response to information Stotsky sent him, he wrote,

> Though this further information was very helpful to me in understanding her perspective on issues, I saw nothing in it that was inherently inconsistent with the possibility that she might have said what is cited above.

A few days later, Bass sent Stotsky a copy of this message and he included some additional comments. He wrote:

> I trust that you have seen the "clarification" message that I sent to Jerry Becker. In case you haven't I shall now forward it to you. It was sent to Becker on Nov 28, but it only appeared a couple of days ago, because of email backup. In that message I state that I still lacked confirming information about what happened in the review panel meetings. Since that time I have been able to speak to other panelists, and I am persuaded that the essentials of what I reported, as clarified, is correct. What I was told by the panelists, and what you yourself state in your memo, seem to be entirely consistent.

In his "clarification," Bass included Stotsky's June 10 Memo, which asserted her support of Euclidean geometry and also criticisms of constructivist approaches to mathematics education. Stotsky had previously sent an elaboration of her views on constructivism to Bass.

In the meantime, Bass' allies in Massachusetts were also busy. Carol Greenes, a Professor of Mathematics Education Boston University and a Framework Panelist opposed to teaching the standard arithmetic algorithms sent a letter dated December 13, 1999, with a long copy list, to Education Commissioner Driscoll criticizing Stotsky's position and objecting to the process of revisions. David Driscoll sent a letter of reply to Dr. Greenes with copies to the same list of recipients. In his letter, Commissioner Driscoll explained again the process for revising the Mathematics Framework by quoting procedures previously sent to Dr. Greenes. His letter explained that in accordance with policies of the Department of Education,

> ...we are pleased that Professor Wilfried Schmid of the Department of Mathematics at Harvard University has agreed to assist us in the preparation of the final draft with respect to the accuracy and adequacy of the mathematical content of the standards. Groups of teachers at different grade levels and from a variety of school districts will also be assisting us in deciding upon the right level at which to pitch the standards in K-8, an issue on which we have already received a wide range of comments.

Professor Schmid made a number of improvements to the draft Framework, most of them minor. Commissioner Driscoll's letter also rebutted Professor Bass' invented scenario that portrayed Deputy Commissioner Stotsky as an idiot. The Commissioner wrote:

> It is my hope that the revision process we have planned will be carried out without distractions. I regard as unfortunate the totally unfounded charge in an earlier letter from a member of the mathematics revision panel that one of my Deputy Commissioners supposedly "debated for 30 minutes" to remove the phrase "classical Greek constructions" from the draft of the mathematics document because she failed to understand that it referred to Euclidean geometry. Not only was there no such phrase in any of our mathematics documents, which several members of my staff have attested to, a tape recording of the meeting at which this "debate" supposedly took place provides unequivocal evidence that there was no discussion whatsoever about "classical Greek constructions" of any kind. Please note that the Department tape records some committee meetings for future reference, in the absence of a stenographer, with the explicit permission of those attending.

Stotsky sent a formal letter to Bass dated January 18, 2000. The letter once again asked Bass to act honorably and retract his previous damaging fabrications. Stotsky's letter once again reminded him that the only meeting that she and Professor Gilmore ever had was on July 27 and that meeting was recorded. A full transcript accompanied the January 18 letter. Nothing resembling what Bass had written had occurred.

Bass' response via email a few days later was as follows:

> Dear Sandra,
>
> I just returned from some travel and found your letter accompanied by the transcript of the July 27, 1999 session of the Math Framework Review Panel. It is my understanding that the discussion of geometric constructions occurred in an earlier meeting, one not attended by Prof. Gilmore. He apparently reported on that exchange based on reports from others who had attended that session, which included Carol Greenes and Peg Kenney.
>
> I can't be sure about the exact accuracy of this second hand reporting, but it would help me assess the situation further if you could supply a transcript of that meeting.

> Best wishes,
> Hyman

The meeting on June 10 that Bass obliquely refers to in this email note to Stotsky, was not recorded. It was an informal meeting in Stotsky's Office to discuss the June 10 memo that Bass included in his earlier December 1 letter to the education listserve. Department of Education staff members were present, however, and they have confirmed that no discussion of the kind Bass described had occurred. Gilmore was not present at that meeting. This contradicts Gilmore's original statement to Bass that was nationally circulated. Recall from above that Gilmore had written:

> The State has appointed a woman, Sandra Stotsky, who is so ignorant of mathematics that she debated for 30 minutes with us to remove 'classical Greek constructions' from our document because it was "Constructivist pedagogy."

The key word is "us." That means Gilmore was included and that forces the meeting to be July 27 since that is the only time Gilmore and Stotsky were ever together. Bass' December 1 letter included the following paragraph:

> Prior to giving Jerry Becker permission to distribute my letters, Dick Askey had seen this and called it to the attention of Sandra Stotsky, who vehemently denied saying any such thing. Askey asked me for my evidence for this. I explained that it came in a message from a mathematician who was part of the Massachusetts Mathematics Framework Review Panel, and who was at the meeting where this was said to occur. I sent to Askey the relevant passages from that message.

The key phrase here is "from a mathematician who was part of the Massachusetts Mathematics Framework Review Panel, and who was at the meeting where this was said to occur. "

Rather than admit his obvious error, Hyman Bass is now pretending that Sandra Stotsky committed the actions he invented on another occasion that was not recorded. Bass now expects Stotsky to jump through more hoops to try to convince Bass that she never did what he accuses her of with only Gilmore's discredited lie.

It is time that Professor Hyman Bass admits that he was wrong, apologizes to Sandra Stotsky, and fully retracts his fabrications.

Notes

Note 1

The American Mathematical Society NCTM 2000 Association Resource Group's Second Report published in the Notices of the American Mathematical Society February 1998 contains the following passage:

> Standard algorithms may be viewed analogously to spelling: to some degree they constitute a convention, and it is not essential that students operate with them from day one or even in their private thinking; but eventually, as a matter of mutual communication and understanding, it is highly desirable that everyone (that is, nearly everyone -- we recognize that there are always exceptional cases) learn a standard way of doing the four basic arithmetic operations. (The standard algorithms need not be absolutely unique, just as there are variant spellings between, say, the U.S. and England, but too much variation leads to difficulties.) We do not think it is wise for students to be left with untested private algorithms for arithmetic operations -- such algorithms may only be valid for some subclass of problems. The virtue of standard algorithms -- that they are guaranteed to work for all problems of the type they deal with -- deserves emphasis.
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> We would like to emphasize that the standard algorithms of arithmetic are more than just "ways to get the answer" -- that is, they have theoretical as well as practical significance. For one thing, all the algorithms of arithmetic are preparatory for algebra, since there are (again, not by accident, but by virtue of the construction of the decimal system) strong analogies between arithmetic of ordinary numbers and arithmetic of polynomials. The division algorithm is also significant for later understanding of real numbers. For all its virtues, decimal notation suffers a significant drawback over, say, standard notation

for fractions: decimal numbers (meaning decimal fractions with finitely many terms) do not allow division. This can be remedied at the cost of using infinite decimal expansions, but this is a big leap, and the general infinite decimal is not rational. To understand that rational numbers correspond to repeating decimals essentially means understanding the structure of division of decimals as embodied in the division algorithm. We do not see that naive use of calculators can be of much help here: the length of repeat of a decimal will typically be comparable to the size of the denominator, so that 7/23 or 5/29 will not reveal any repeating behavior on standard calculators.

Note 2

David P. Driscoll
Commissioner of Education

December 17, 1999

Carole Greenes
Professor of Mathematics Education
Boston University
605 Commonwealth Avenue
Boston, MA 02215

Dear Professor Greenes and Members of the Revision Panel:

Thank you for your letter of December 13 inquiring about the process to be used by the Department of Education in responding to the public comments on the draft of the Mathematics Curriculum Framework released to the public in September and in preparing the final version for the Board of Education's approval in February. I share your desire that your comments be considered. To that end, as the November 9 letter sent to all Panel members indicated:

> After the period of public comment is over, the chair of the Curriculum Framework Revision Panel may wish to convene a meeting of Panel members to review all of the feedback. (This will not be an obligatory meeting.) The members of the Panel may then, singly or as a group, present their comments, inquiries, or suggestions to Dr. Sandra Stotsky, Deputy Commissioner for Academic Affairs and Planning, who will be in charge of the final revision, editing, and formatting of the document.
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> The Department of Education will then determine the scope of the revisions to be made and the resources to be drawn upon, based on the Panel members' suggestions and on the Department's own review of the feedback. Outside experts may be drawn upon to assist in the preparation of the tentative final draft. Before this draft is presented to the Board of Education for formal approval, Panel members will be sent copies and given the opportunity for comment.
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> Once completed, copies of the tentative final draft will be sent to the Board of Education for its review and action.

This is the process the Department of Education will be following for the revision of the other curriculum frameworks being revised this year - science and technology/engineering, history and social science, and the English language arts. It is my intention to make sure that we present to the Board the very best Mathematics Curriculum Framework we can in February. To that end, we are pleased that Professor Wilfried Schmid of the Department of Mathematics at Harvard University has agreed to assist us in the preparation of the final draft with respect to the accuracy and adequacy of the mathematical content of the standards. Groups of teachers at different grade levels and from a variety of school districts will also be assisting us in deciding upon the right level at which to pitch the standards in K-8, an issue on which we have already received a wide range of comments.

It is my hope that the revision process we have planned will be carried out without distractions. I regard as unfortunate the totally unfounded charge in an earlier letter from a member of the mathematics revision panel that one of my Deputy Commissioners supposedly "debated for 30 minutes" to remove the phrase "classical Greek constructions" from the draft of the mathematics document because she failed to understand that it referred to Euclidean geometry. Not only was there no such phrase in any of our mathematics documents, which several members of my staff have attested to, a tape recording of the meeting at which this "debate" supposedly took place provides unequivocal evidence that there was no discussion whatsoever about "classical Greek constructions" of any kind. Please note that the Department tape records some committee meetings for future reference, in the absence of a stenographer, with the explicit permission of those attending.

We look forward to producing a first-rate Mathematics Curriculum Framework for Massachusetts teachers and students but will be successful only if we focus on the issues that relate to the teaching and learning of mathematics for our students.

Sincerely yours,

David P. Driscoll
Commissioner of Education

c. Margaret Kennedy, Professor of Mathematics, Boston College
Maurice Gilmore, Professor of Mathematics, Northeastern University
Anne Collins, Principal Investigator for Partnerships Promoting Student Achievement in Mathematics
Deborah Hughes Hallett, Professor of Mathematics, University of Arizona
Victor Steinbok, Doctoral Candidate in Mathematics Education, Boston University
Barbara Haig, K-12 Curriculum Chair, Elementary Mathematics, Teacher, Northborough Public Schools
Gisele Zangari, Mathematics Instructor, Boston University Academy
Jacqueline Rivers, Director Math Power, Northeastern University
The Honorable Paul Cellucci, Governor, Commonwealth of Massachusetts
The Honorable Thomas Finneran, Speaker of the House, Commonwealth of Massachusetts
The Honorable Robert Antonioni, Senate Chair, Joint Committee on Education, Arts and Humanities
The Honorable David Donnelly, Representative Vice-Chair, Joint Committee on Education, Arts and Humanities
James A. Peyser, Chairman, Massachusetts Board of Education
Sandra Stotsky, Deputy Commissioner of Academic Affairs and Planning, Department of Education, Commonwealth of Massachusetts
Representative Paul Haley
Senator Cynthia Creem
Representative Ronny Sidney
Representative Ruth Balser
Representative Paul Demakis
Dr. Perry Davis, President, Massachusetts Association for Supervision and Curriculum Development
Lynn Ryan, President, Massachusetts Elementary School Principals' Association
Carol Woodbury, President, Massachusetts Parent Teacher Association
Dr. Russell E. Goyette, President, Massachusetts Secondary School Administrators' Association

Note 3

January 18, 2000

Professor Hyman Bass
Department of Mathematics
3864 East Hall
University of Michigan
Ann Arbor, MI 48109-1109

Dear Professor Bass:

I am writing to you, again, to request that you provide your colleagues with an unambiguous explanation and retraction of the erroneous statement that you published, in which you attributed to me an "ideological and uninformed opposition to 'constructivist ideas' [that] has reached an incredible state where she is opposed to inclusion of discussion of "Classical Greek constructions' as being 'constructivist pedagogy.'" Your characterization is wrong, and it has no basis in fact.

As you indicated in an e-mail on November 25, 1999, you copied the statement from a personal e-mail that Professor Maurice Gilmore of Northeastern University sent to you sometime last fall. Professor Gilmore, whom I have met only once (at the July 27,1999 meeting of the Mathematics Framework Panel in Massachusetts) has since apologized to me for its tone. In fact, the transcript of the July 27, 1999 meeting of the Mathematics Panel, a copy of which I have enclosed for your review, clearly shows that there was no mention of "Classical Greek constructions" at that meeting (indeed, the phrase does not appear in any Department documents), and there was no discussion whatsoever of the content of Euclidean geometry. The words "constructivist" and "constructivism" do not even appear in the transcript. Further, contrary to what Professor Gilmore claims in his December 15 letter to me and contrary to what you claim in a personal December e-mail to me (based, you said, on conversations with other Panel members), there was no mention whatsoever (never mind a discussion) of geometric constructions created by a "straightedge" or "compass." These words do not appear at all in the transcript of the meeting. Nor have they ever been mentioned in any conversation about the mathematics standards in which I participated.

However you choose to word an explanation and retraction for your colleagues, it should be made clear that the transcript of that meeting proves conclusively that there is no basis to Professor Gilmore's charge, and that you made a grievous error in judgment (and interpretation) in broadcasting that mischaracterization of my views.

Sincerely yours,

Sandra Stotsky, Ed.D.
Deputy Commissioner

C: Professor Maurice Gilmore